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PARIS LAW BASED MODELLING OF MIXED MODE FATIGUE CRACK PROPAGATION IN CONCRETE-CONCRETE INTERFACE

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Abstract. In structural engineering, the integrity of concrete structures when subjected to cyclic loads remains a significant concern, more so in the case of repaired structures or mass concrete frameworks, wherein inherent interfacial imperfections exist. Hence, it is essential to model the fatigue crack propagation in concrete bi-material interface in order to facilitate the evaluation of the structural integrity of concrete constructions. This study considers the intrinsic mixed mode character of the interface and proposes a generalised model based on Paris, law to predict fatigue crack propagation in concrete-concrete interface by introducing equivalent stress intensity factor instead of mode-I stress intensity factor. The prediction model is validated using the experimental data on concrete-concrete interface from existing literature.

1 INTRODUCTION

Various concrete structures experience fatigue loading during their operational lifespan, including concrete pavements and bridge decks that endure repetitive vehicle loads. Gravity dams, piers, and other offshore structures are susceptible to fatigue or cyclic loading due to periodic variations in temperature or cyclic wave and wind loads. Under fatigue loading conditions, damage accumulation occurs gradually, leading to the formation of microcracks, subsequently resulting in a deterioration of strength. The feasibility of replacing most of these structures is typically limited, leading to the implementation of repair strategies. These repair methods entail the application of new materials onto the existing structures, thereby enhancing their strength and extending their operational lifespan. Although these repair techniques are widely utilised and highly developed, it is crucial to exercise caution to ensure optimal compatibility between the new and existing materials, as crack propagation at the interface is rather complex and it is weaker than the materials present on either side of the interface [1].

Due to the inherent mixed-mode property of a crack at an interface, the potential crack is governed by the *mode-mixity* ratio i.e. the ratio of stress intensity factor in mode-II to stress intensity factor in mode-I, the crack follows the complete interfacial path for a low mode-mixity ratio [2] and for a higher mode-mixity ratio, it may either propagate first along the interface and kink into one of the materials or it may directly kink in to one of the materials [3]. Nevertheless, the trajectory of crack propagation is influenced not solely by the stress field magnitude at the crack tip, but also by the properties of the material and interfaces involved [4]. The stress intensity factor based criterion, proposed in Kishen and Singh [5], has been employed by many researchers to determine the crack propagation path in the interface between rock and concrete for a Mode-I dominated [2] i.e. low *mode-mixity* ratio. However, it is important to consider Mode-II fracture when analysing the propagation of interfacial cracks under mixed Mode I-II stress circumstances. Since the estimation of interface resistance becomes overstated if only Mode I is considered [3].

So far, extensive research has been undertaken to examine the fatigue fracture behaviour of concrete. Bažant and his group [6–9] have proposed a size-adjusted model for characterising fatigue crack propagation in concrete by incorporating the concept of transitional sizes of concrete in traditional Paris' law. Slowik et al. [10] introduced an empirical model that effectively computes fatigue crack propagation under variable amplitude loading, considering the influence of overload effects. Sain and Chandra Kishen [11, 12] improved this model by further considering the impact of loading frequency on fatigue crack propagation. In their study, Kim and Kim [13] demonstrated a positive correlation between the fatigue crack propagation rate and the strength of concrete. The material constants in Paris' law were adjusted by considering the influence of the grade of concrete strength. Researchers have also attempted to construct analytical models for predicting crack propagation under the influence of fatigue loading by employing the fundamental concepts of dimensional analysis and the theory of intermediate asymptotic. The recent publication in this field, incorporated change in thermodynamic state [14] and considered the effect of maximum aggregate size [15]. .

The fracture behaviour of a bi-material interface under fatigue loading may exhibit greater complexity as compared to intact concrete due to the presence of mismatched material properties on each side of the interface. Shah et al. [16] experimentally studied the effect of elastic mismatch on fatigue crack propagation and Zhao et al. [17] conducted experimental investigation to examine the behaviour of the rock-concrete interface subsequent to pre-fatigue loading.

The recent notable efforts to expand the scope of the Paris' law to include mixed-mode crack propagation in metals [18] and plain concrete [17] shows that conventional compliance calibration approach becomes impractical in mixed mode condition, therefore there is a need to establish a new method to extend the Paris' law to accommodate interface fractures. In this paper attempt has been made to explore of this imperative research gap, the objective is to develop a generalised model based on Paris' law introducing the equivalent stress intensity factor (ΔK_{eq}) instead of stress intensity factor in mode-I (ΔK_I). Finally the model is then validated using the experimental data from [16].

2 Mixed-mode Stress Intensity Factor in Bi-Material Interface

The interface crack tip experiences both in-plane normal and shearing tractions, despite the application of pure Mode I loading. For a crack between two isotropic elastic material shown in Figure 1 the interfacial SIFs for the mode I and II, i.e. K_1 and K_2 were given in [19] as:

$$K_1 = \sqrt{a}\left\{\frac{\sigma[\cos(\epsilon\log 2a) + 2\epsilon\sin(\epsilon\log 2a)]}{\cosh\pi\epsilon} + \frac{\tau[\sin(\epsilon\log 2a) - 2\epsilon\cos(\epsilon\log 2a)]}{\cosh\pi\epsilon}\right\} \tag{1}$$

$$K_2 = \sqrt{a}\left\{\frac{\tau[\cos(\epsilon\log 2a) + 2\epsilon\sin(\epsilon\log 2a)]}{\cosh\pi\epsilon} - \frac{\sigma[\sin(\epsilon\log 2a) - 2\epsilon\cos(\epsilon\log 2a)]}{\cosh\pi\epsilon}\right\} \tag{2}$$

where a is the crack length, σ and τ are the normal and shear stresses along the interface, respectively. The oscillatory parameter ϵ is written in terms of Dundur's elastic mismatch parameter β .

$$\epsilon = \frac{1}{2\pi}\ln\left[\frac{1-\beta}{1+\beta}\right] \tag{3}$$

β is dependent on the material properties as:

$$\beta = \frac{\mu_1(1-2v_2) - \mu_2(1-2v_1)}{2[\mu_1(1-v_2) + \mu_2(1-v_1)]} \tag{4}$$

where μ and υ are the shear modulus and Poisson's ratio, respectively, suffix 1 and 2 denote material 1 and material 2, respectively

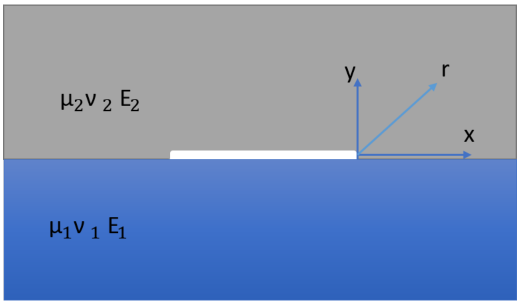

Figure 1: Geometry of an interface crack

The analytical solution of the geometric factor of Mode-I Stress Intensity Factor for a homogeneous material under three-point bend loading is given as [20]:

$$F_1\left(\frac{a}{D}\right) = \frac{1.99 - \left(\frac{a}{D}\right)\left(1-\frac{a}{D}\right)(\gamma)}{(1+2\frac{a}{D})(1-\frac{a}{D})^{\frac{3}{2}}} \quad (5)$$

where,

$$\gamma = \left[2.15 - 3.93\frac{a}{D} + 2.7\left(\frac{a}{D}\right)^2\right]$$

Since the above geometric factor is valid for a homogeneous material a correction function has been introduced in [21] to take into account the effect of mismatch in elasticity of bi-material interface and size of specimen:

$$K_I = K_1 F_1\left(\frac{a}{D}\right) F_2\left(\frac{E_2}{E_1}, \frac{a}{D}\right) \quad (6)$$

where,

$$F_2\left(\frac{E_2}{E_1}, \frac{a}{D}\right) = Q_1 + Q_2\left(\frac{E_2}{E_1}\right) + Q_3\left(\frac{E_2}{E_1}\right)^2 \quad (7)$$

$$Q_1 = 0.975 + 0.074\left(\frac{a}{D}\right) - 0.062\left(\frac{a}{D}\right)^2 \quad (8)$$

$$Q_2 = 0.023 - 0.067\left(\frac{a}{D}\right) + 0.056\left(\frac{a}{D}\right)^2 \quad (9)$$

$$Q_3 = -0.001 + 0.003\left(\frac{a}{D}\right) - 0.002\left(\frac{a}{D}\right)^2 \quad (10)$$

$F_1\left(\frac{a}{D}\right)$ is the geometric factor homogeneous material under three point bend loading and $F_2\left(\frac{E_2}{E_1}, \frac{a}{D}\right)$ is the correction factor considered for bi-material interface. In this study the geometric and correction factor for Mode-II SIFs is considered unity.

3 Prediction Model for Fatigue Crack Propagation in Concrete-Concrete Interface

The Paris' law has been extensively employed to predict the fatigue crack propagation in concrete by relating crack propagation rate and Mode-I stress intensity factor. Other forms of the same have been introduced by incorporating more parameters that affect crack propagation. The Paris' law has been modified to characterise the fatigue crack propagation in mixed mode condition [22], as shown in Equation 11

$$\frac{da}{dN} = C(\Delta K_{eq})^m \quad (11)$$

Through out the literature different forms of ΔK_{eq} have been formulated such as:

- Based on energy concept for mixed mode fracture by Irwin [23]

$$\Delta K_{eq} = (\Delta K_I^2 + \Delta K_{II}^2)^{1/2} \quad (12)$$

- Based on fatigue crack propagation in metal derived by Tanaka [24]

$$\Delta K_{eq} = (\Delta K_I^4 + 8\Delta K_{II}^4)^{1/4} \quad (13)$$

- Richard et al. [25] proposed

$$\Delta K_{eq} = 0.5\Delta K_I + 0.5\left[\Delta K_I^2 + 4(\alpha_1 \Delta K_{II})^2\right]^{1/2} \quad (14)$$

where value of empirical constant α_1 is set to 1.155.

Table 1: Material Properties [16]

Specimen Designation	Compressive strength N/mm^2	Poisson's ratio	Elastic modulus N/mm^2
A	34	0.2	30000
B	45	0.19	32000
C	54	0.18	34000
D	66	0.17	35000

- Based on the Dugdale's model Chen and Keer [26] suggested

$$\Delta K_{eq} = \left[(\Delta K_I^2 + 3\Delta K_{II}^2)^3\right] \times \left[(\Delta K_I^2 + \Delta K_{II}^2)\right]^{1/8} \tag{15}$$

It is to be noted that Equations 12-15, are derived for metals, therefore after critical analysis done by Jia et al. [22] concluded the direct applicability of Equation 12.

The experimental data for this work has been taken from Shah et al. [16] which included three sizes of geometrically similar specimen namely small, medium and large. Interfaces were provided at the centre along with the notch with different strengths of concrete on either side of the interface. The material properties of the different concrete mixes is shown in Table 1. These were tested under step wise amplitude fatigue loading until failure. The empirical constants C and m are extracted from the fatigue crack crack growth curve given by Shah et al. [16] and are presented in Table2. A model is then developed as given in Equation 11. Now the process of predicting mixed mode fracture is discussed here. First step involves the separation of variables given in Equation 16

$$dN = \frac{1}{C(\Delta K_{eq})^m} da \tag{16}$$

Equation 16 is then integrated form 0 to N_f and right hand side from the initial notch to the unstable crack length.

$$\int_0^{N_f} dN = \int_{a_o}^{a_f} \frac{1}{C(\Delta K_{eq})^m} da \tag{17}$$

By using the method of integration by limit of sum we have integrated the right hand side of Equation 17 as shown:

$$N(a) = (a - a_0) \lim_{n\to\infty} \frac{1}{n} \sum_{a=a_o}^{a_0+(n-1)h} \frac{1}{C(\Delta K_{eq})^m} \tag{18}$$

Equation 18 gives the number of fatigue cycle corresponding to the crack length, by using the inverse interpolation crack length for specific number of load cycle is predicted.

Table 2: Paris' Constants

Specimen Designation	C	m
AA	$3.0X10^{-4}$	0.90
AB	$1.73X10^{-4}$	1.52
AC	$1.75X10^{-4}$	2.10
AD	$6.0X10^{-4}$	2.00

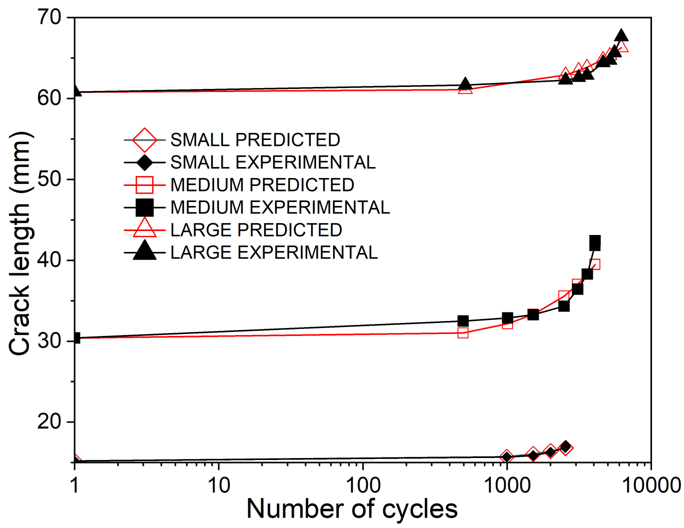

Figure 2: Crack length versus number of cycles for AA specimen

Figures 2-5 shows plot between crack length versus number of cycles for all three sizes of interface AA, AB, AC and AD respectively. It can be noted that predicted model show reasonable agreement with the experimental data of Shah et al. [16]. Since the experiment conducted by Shah et al. had very low *mode mixity* ratio, it was mostly Mode-I dominated, thus due to the limited experimental data, the accuracy of the predicted model needs to be further verified by more extensive experimental data. Therefore, more experiments incorporating the mixed-mode loading condition to better understand fatigue crack propagation in concrete-concrete interface is required.

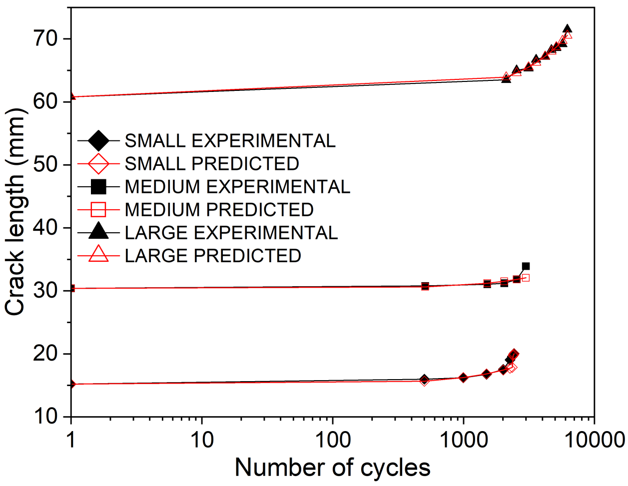

Figure 3: Crack length versus number of cycles for AB specimen

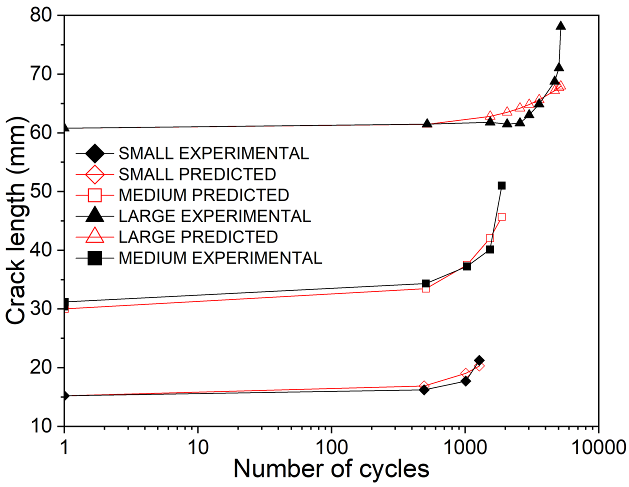

Figure 4: Crack length versus number of cycles for AC specimen

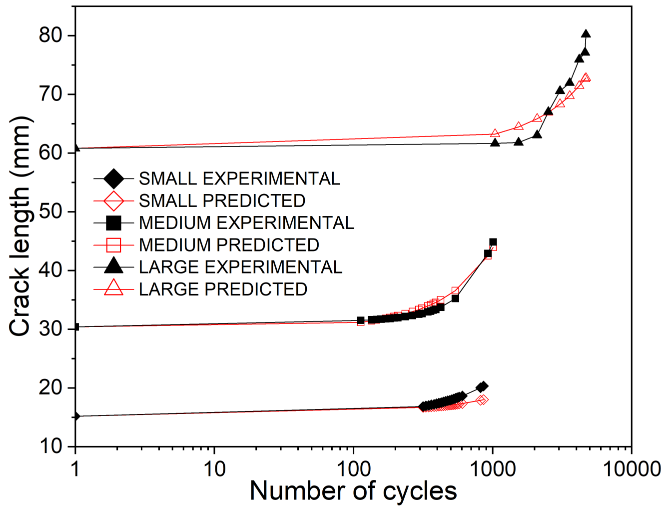

Figure 5: Crack length versus number of cycles for AD specimen

3.1 Conclusions

In this study, Paris' law based fatigue crack propagation model has been proposed where, the general mode-I stress intensity factor has been replaced by equivalent stress intensity factor, in an attempt to incorporate mixed-mode nature of bi-material interface crack propagation. The model was then validated by using the experimental data from Shah et al. [16]. The model shows a good agreement with the experimental results, however extensive experimental studies are suggested to better validate the accuracy and generality of the proposed model.

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REFERENCES

[1] Santosh G. Shah, Bhasya, and J. M. Chandra Kishen. Tension-softening properties for concrete-concrete interfaces. *Aci Structural Journal*, 108:725–734, 2011.

[2] Hong Zhong, Ean Tat Ooi, Chongmin Song, Tao Ding, Gao Lin, and Hongjun Li. Experimental and numerical study of the dependency of interface fracture in concrete–rock specimens on mode mixity. *Engineering Fracture Mechanics*, 124:287–309, 2014.

[3] Volker Slowik, J.M. Chandra Kishen, and Victor E. Saouma. Mixed mode fracture of ce-

mentitious bimaterial interfaces;: Part i: Experimental results. *Engineering Fracture Mechanics*, 60(1):83–94, 1998.

[4] Wei Dong, Dong Yang, Binsheng Zhang, and Zhimin Wu. Rock-concrete interfacial crack propagation under mixed mode i-ii fracture. *Journal of Engineering Mechanics*, 144(6):04018039, 2018.

[5] JM Chandra Kishen and Konjengbam Darunkumar Singh. Stress intensity factors based fracture criteria for kinking and branching of interface crack: application to dams. *Engineering fracture mechanics*, 68(2):201–219, 2001.

[6] Zdenek P Bazant and Kangming Xu. Size effect in fatigue fracture of concrete. *ACI Materials Journal*, 88(4):390–399, 1991.

[7] Zdenek P Bazant and William F Schell. Fatigue fracture of high-strength concrete and size effect. *ACI Materials Journal*, 90:472–472, 1993.

[8] Kedar Kirane and Zdeněk P Bažant. Microplane damage model for fatigue of quasibrittle materials: Sub-critical crack growth, lifetime and residual strength. *International Journal of Fatigue*, 70:93–105, 2015.

[9] Kedar Kirane and Zdeněk P Bažant. Size effect in paris law and fatigue lifetimes for quasibrittle materials: Modified theory, experiments and micro-modeling. *International Journal of Fatigue*, 83:209–220, 2016.

[10] Volker Slowik, Giovanni Angelo Plizzari, and Victor E Saouma. Fracture of concrete under variable amplitude fatigue loading. *Materials Journal*, 93(3):272–283, 1996.

[11] Trisha Sain and JM Chandra Kishen. Prediction of fatigue strength in plain and reinforced concrete beams. *ACI structural journal*, 104(5):621, 2007.

[12] Trisha Sain and JM Chandra Kishen. Probabilistic assessment of fatigue crack growth in concrete. *International Journal of Fatigue*, 30(12):2156–2164, 2008.

[13] Jin-Keun Kim and Yun-Yong Kim. Fatigue crack growth of high-strength concrete in wedge-splitting test. *Cement and Concrete Research*, 29(5):705–712, 1999.

[14] KM Pervaiz Fathima and JM Chandra Kishen. A thermodynamic framework for fatigue crack growth in concrete. *International journal of fatigue*, 54:17–24, 2013.

[15] Sonali Bhowmik and Sonalisa Ray. An improved crack propagation model for plain concrete under fatigue loading. *Engineering Fracture Mechanics*, 191:365–382, 2018.

[16] Santosh G Shah, Sonalisa Ray, and JM Chandra Kishen. Fatigue crack propagation at concrete–concrete bi-material interfaces. *International journal of fatigue*, 63:118–126, 2014.

[17] Xiaoyu Zhao, Wei Dong, Binsheng Zhang, and Yiming Wang. Fracture properties of rock-concrete interface after fatigue loading. *Journal of Materials in Civil Engineering*, 35(4):04023033, 2023.

[18] S Sajith, KSRK Murthy, and PS Robi. Experimental and numerical investigation of mixed mode fatigue crack growth models in aluminum 6061-t6. *International Journal of Fatigue*, 130:105285, 2020.

[19] JR Rice and George C Sih. Plane problems of cracks in dissimilar media. 1965.

[20] H. Tada, P. Paris, and G. Irwin. *The Stress Analysis of Cracks Handbook*. Paris Productions, Hellerton, PA, 2nd edition, 1985.

[21] Wenyan Yuan, Wei Dong, Binsheng Zhang, and Hong Zhong. Investigations on fracture properties and analytical solutions of fracture parameters at rock-concrete interface. *Construction and Building Materials*, 300:124040, 2021.

[22] Mengdi Jia, Zhimin Wu, Rena C Yu, and Xiaoxin Zhang. Modeling of mixed mode i–ii fatigue fracture of concrete based on paris law. *Journal of Materials in Civil Engineering*, 35(5):04023078, 2023.

[23] G. R. Irwin. Analysis of Stresses and Strains Near the End of a Crack Traversing a Plate. *Journal of Applied Mechanics*, 24(3):361–364, 06 2021.

[24] Keisuke Tanaka. Fatigue crack propagation from a crack inclined to the cyclic tensile axis. *Engineering Fracture Mechanics*, 6(3):493–507, 1974.

[25] H. A. Richard, M. Fulland, and M. Sander. Theoretical crack path prediction. *Fatigue & Fracture of Engineering Materials & Structures*, 28(1-2):3–12, 2005.

[26] Wei-Ren Chen and L. M. Keer. Fatigue Crack Growth in Mixed Mode Loading. *Journal of Engineering Materials and Technology*, 113(2):222–227, 04 1991.