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NON-LOCAL ANISOTROPIC DAMAGE COUPLED TO PLASTICITY

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Abstract: In this paper, a novel approach is introduced for handling orthotropic plasticity coupled with damage, using second gradient formulations based on several directional diffusion tensors. This method automatically identifies the crack location and opening through the curvature of the non-local profiles of principal plastic strains. The paper includes an application demonstrating the numerical objectivity of the obtained solution.

1 INTRODUCTION

The durability of concrete structures relies on various factors, one of which is the crack opening. Cracks in concrete structures create pathways for the ingress of fluids and aggressive chemicals, such as chloride or carbon dioxide, leading to conditions able to initiate reinforcement corrosion. International standards limit crack openings to mitigate this issue. While design standards offer empirical relationships for common structural elements like beams to assess crack openings, applying these relationships to non-conventional structures, such as containment structures of nuclear plants or dams, remains challenging. The Finite Element Method (FEM) is an interesting approach used in this work to predict crack openings in such complex structures.

Accurately determining crack openings allows for better modeling of the hydro-mechanical behaviour of concrete, leading to improved assessments of its durability. Numerous studies have shown that permeability evolves with the opening of localized cracks [1]–[4]. Managing crack opening and reclosing is also crucial during cyclic loading of cracked structures, such as dealing with seasonal variations in water tank levels behind a dam or the effects of seismic events and wind. During these phases, the stiffness of concrete becomes anisotropic, with stiffness being lost in the direction of crack opening and recovered when cracks reclose. To account for this orthotropic and unilateral effect of cracks on mechanical behaviour, Finite Element models can employ an anisotropic damage model coupled with plasticity. This type of model captures the strain localization phenomena interpreted as a crack formation.

When implementing this type of model in finite element software using linear interpolation functions for the finite elements, the local method based on Hillerborg's principle [5] can be used to ensure the dissipation of the correct amount of fracture energy [6]. However, if quadratic finite element interpolation functions are used, the implementation of this method becomes less straightforward. As a result, alternative methods like XFEM [7]–[10], EFEM [11], [12] or non-local methods have to be used. Among non-local methods, the integral non-local method [13] and the second gradient method [14]–[18] are both based on the works of [19].

In this work, it is proposed to extend the non-local method from the classical framework of isotropic state variables to second-order tensorial variables. For this purpose, the multivariable Helmholtz formulation, as clarified in [20], is adapted to consider several phase-fields. Two numerical implementation methods are possible: (1) each term of the plastic strain tensor is treated separately, resulting in six scalar differential equations of the Helmholtz form, to which the non-local method is applied directionally using anisotropic diffusion matrices expressed in the fixed coordinate system; (2) only three diffusion matrices need to be computed, each one associated with one principal direction of the plastic strain tensor. The second method reduces the number of Helmholtz problems to three, but it requires reassessing the diffusion matrices at each sub step due to the potential rotation of principal directions, which increases computational time. The three or six equations, depending on the selected method, are solved numerically using a non-local solver nested in the global equilibrium convergence loop.

Furthermore, an analytical analysis of the directional Helmholtz formulation reveals that the non-local principal strains can directly predict localized crack openings and their most probable positions. After clarifying the method, one application is presented to discuss the advantages and limitations of the non-local tensorial formulation, particularly concerning the determination of localized crack openings and positions.

2 GENERAL PRINCIPLES

In this work, the general principles are described assuming radial loading. Nevertheless, the method remains easily transposable for any type of loading. The total strain increment in a principal direction $j = (1,2,3)$ is denoted $\Delta\varepsilon_j$ and expressed in Equation (1):

$$\Delta\varepsilon_j = \Delta\varepsilon_j^{el} + \Delta\varepsilon_j^{pl} \tag{1}$$

where $\Delta\varepsilon_j^{el}$ and $\Delta\varepsilon_j^{pl}$ are the elastic and plastic strains increments, respectively.

The effective stress increment $\Delta\tilde{\sigma}_i$ is related to the elastic strain increment and the Hooke tensor $\mathbb{H}_{ij}$ (Equation (2)):

$$\Delta\tilde{\sigma}_i = \mathbb{H}_{ij} \cdot \Delta\varepsilon_j^{el} \tag{2}$$

A damage variable was introduced by [21] for isotropic states. In this paper, an anisotropic damage is considered. The principal values of tensile damage D_i^t are introduced in Equation (3):

$$\sigma_i = \tilde{\sigma}_i^+ \cdot (1 - D_i^t) + \tilde{\sigma}_i^- \tag{3}$$

where $\tilde{\sigma}_i^+$ and $\tilde{\sigma}_i^-$ are the tensile and compressive stresses, respectively, in a principal direction $i = (1,2,3)$.

2.1 Tensile damage law

The principal tensile damage value D_i^t is related to the localized crack opening by the damage evolution law (Equation (4)):

$$D_i^t = 1 - \exp\left(-\left(\frac{w_i^{pl,max}}{w_{kt}}\right)^m\right) \tag{4}$$

where $w_i^{pl,max}$ is the maximal crack opening in a principal direction of tension (Equation (5)). In Equation (4), the damage value must not decrease in order to respect the second principle of thermodynamics.

$$w_i^{pl,max} = \max_t\left(w_i^{pl}\right) \tag{5}$$

where t is the kinematic time for loading and w_i^{pl} is the crack opening.

In addition, in Equation (4), w_{kt} and m are parameters: w_{kt} controls the fracture energy and m allows to control the shape of the post-peak branch of the behaviour law, as illustrated in Figure 1.

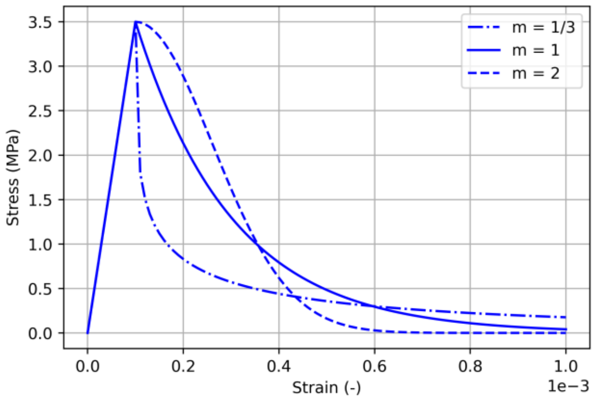

Figure 1: Stress-strain behaviour law implemented at Gauss integration points for different values of m

2.2 Non-local formulation and crack openings assessment

The non-local method used in this model is the second gradient originally proposed by [14]–[18] in a context of isotropic state context. In this work, the method will be applied successively to each principal plastic strain ε_i^{pl} in order to obtain a one-dimensional diffusion in each principal direction of tension (Equation (6)).

$$-\mathrm{div}\left(\overline{\overline{D}} \cdot \overline{grad}\left(\hat{\varepsilon}_i^{pl}\right)\right) = \varepsilon_i^{pl} - \hat{\varepsilon}_i^{pl} \qquad (6)$$

where $\hat{\varepsilon}_i^{pl}$ is the non-local plastic strain in a principal direction of tension and $\overline{\overline{D}}$ a diffusion tensor (Equation (7)).

$$\overline{\overline{D}} = \frac{l_D^2}{2} \cdot (\vec{e}_i \otimes \vec{e}_i) \qquad (7)$$

where l_D is the diffusion length of the non-local method. The formulation in Equation (6) allows the local plastic strain to be diffused along a main tensile direction in proportion to l_D.

The conservative nature of the Helmholtz formulation of Equation (6), which considers a zero flow boundary condition at infinity, leads to the solution shown in Figure 2.

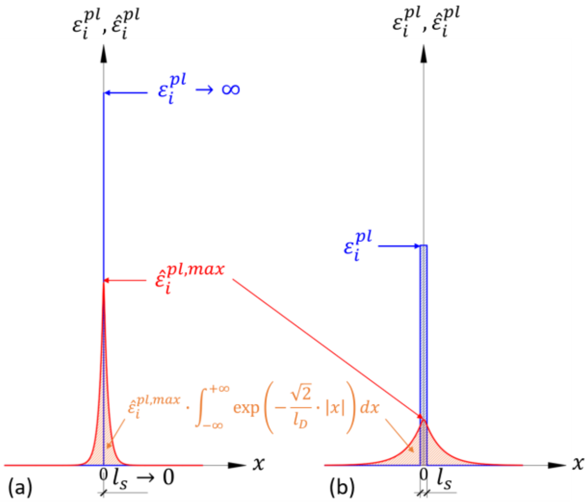

Figure 2: Local and non-local plastic strains along a principal direction x associated to the principal value ε_i^{pl}, (a) theorical solution for a Dirac source and (b) solution for a finite source

In this work, the local source is assumed to occur in a localized region, whose width is equal to l_s. During the localization process, the plastic strain source is localized, leading through Equation (6) to a non-local plastic deformation field. This field is maximal at the position of the source, as presented in Figure 3.

By considering in Equation (8) that the local plastic strain is constant on a finite width l_s:

$$\varepsilon_i^{pl}(x) = \begin{cases} \varepsilon_i^{pl} \; \forall \; x \in \left[-\frac{l_s}{2}; \frac{l_s}{2}\right] \\ 0 \; otherwise \end{cases} \qquad (8)$$

the crack opening can be expressed by Equation (9):

$$w_i^{pl} = \varepsilon_i^{pl} \cdot l_s \approx \sqrt{2} \cdot l_D \cdot \hat{\varepsilon}_i^{pl,max} \qquad (9)$$

The crack opening corresponds to the integral of the non-local strain along a principal direction of tension and depends on two variables: the diffusion length l_D and the maximum value of the non-local strain $\hat{\varepsilon}_i^{pl,max}$.

According to the literature, the crack opening is located where the non-local strain value is maximal [22], [23]. In this formulation, the analytical solution shows that the source is located where the second derivative of the non-local strain is negative, as illustrated in Figure 3. This allows automatically finding the position of the crack and its opening.

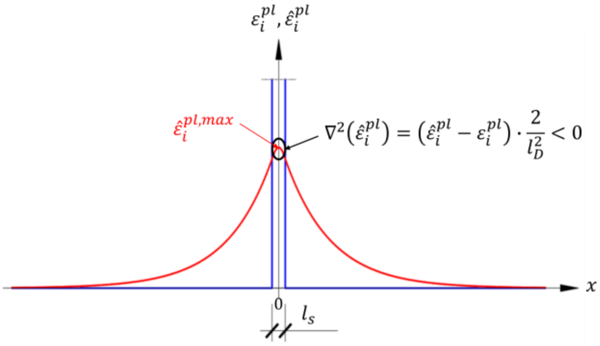

Figure 3: Determination of the crack position and its opening in the structure

Moreover, within the finite element framework, the diffusion length of the non-local method l_D must be chosen to guarantee a satisfactory level of accuracy of the numerical approximation of the solution of Equation (6). l_D can be defined by the user according to the sizes of the mesh and the structure. On the one hand, a minimum of six finite elements must be included in the non-local length l_D as suggested in [24]. On the other hand, the diffusion length must have a small value in comparison to the size of the structure to avoid undesirable boundary effects.

2.3 Local source of plastic strain

The yield surface describing a plasticity yield is defined in the principal directions of tension. In this model, a set of three orthogonal stress criteria in the principal directions of tension is used to define an orthotropic Rankine multi-criterion, as explained in [6].

$$f_i^R = \tilde{\sigma}_i - \tilde{R}^t \leq 0 \qquad (10)$$

where f_i^R is the yield function and $\tilde{R}^t$ the tensile strength of the material. The local plastic strain increment $\Delta\varepsilon_i^{pl}$ in the principal direction of tension i is defined in equation (11):

$$\Delta\varepsilon_i^{pl} = \Delta\lambda \cdot \frac{\partial f_i^R}{\partial \tilde{\sigma}_i} \qquad (11)$$

where $\Delta\lambda$ is the plastic multiplier. Finally, the loading-unloading conditions are the followings (Equation (12)):

$$f_i^R \leq 0, \qquad \Delta\lambda \geq 0, \qquad \Delta\lambda \cdot f_i^R = 0 \quad (12)$$

2.4 Fracture energy

In this model, the same fracture energy in tension Gf_i^t, expressed in Equation (13), must be dissipated whatever the mesh used.

$$Gf_i^t \approx R^t \cdot \int_0^\infty (1 - D_i^t) \cdot dw_i^{pl,max} \qquad (13)$$

By replacing D_i^t by its definition given in Equation (4) and calculating analytically the integral of the fracture energy, w_{kt} can be expressed in equation (14):

$$w_{kt} \approx \frac{Gf_i^t \cdot m}{R^t \cdot \Gamma\left(\frac{1}{m}\right)} \qquad (14)$$

where Γ is the Gamma function. Thus, for given values of m and l_D, w_{kt} is evaluated in order to verify the fracture energy.

The behaviour law implemented at each Gauss point defined in Equation (3) is plotted on Figure 4 for different values of l_D.

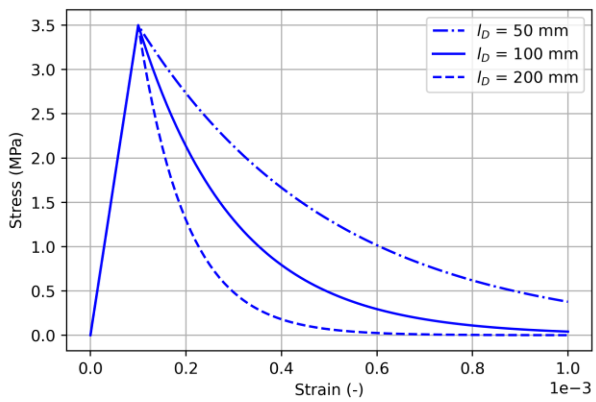

Figure 4: Stress-strain behaviour of a Gauss point for various values of l_D ($m = 1$)

In this model, the non-local diffusion length l_D is not a material parameter. The only intrinsic material parameters are the Hooke coefficients, the tensile strength R^t, the fracture energy Gf_i^t, and the brittleness exponent m.

3 APPLICATION – UNIAXIAL LOADING TEST IN TENSION

This application concerns a structure subjected to uniaxial loading in tension. This application deals with the ability of the model to regularize the structural response and to obtain mesh independent solutions, whatever the interpolation functions used in the finite element model. This application highlights also the ability of the model to find the position of the localized crack and to assess its crack opening by itself, i.e. during the sub stepping process instead of after the calculus by postprocessing.

3.1 Presentation of the test

In this test, a uniaxial displacement is applied to a concrete bar discretized successively with linear 8-nodes and quadratic 20-nodes cubic finite elements, as illustrated in Figure 5. The imposed uniaxial displacement varies linearly from 0 to $100\,\mu m$. The particularity of this structure is its varying geometry along the loading axis to provoke a localized crack. The structure has a symmetrical geometry with only one element in the middle. When tensile strength is reached, only the finite element in the middle of the structure is supposed to be damaged. Different mesh discretizations have been used in calculations and results were compared.

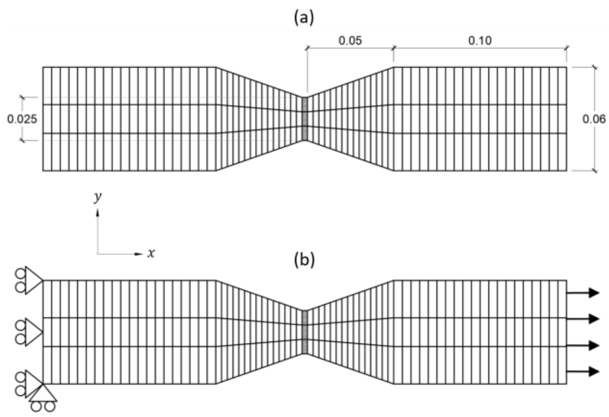

Figure 5: Schema of the concrete bar: 2D views with dimensions in meters (a) and boundary conditions (b)

The material characteristics are presented in Table 1.

Table 1: Material parameters of the concrete bar

Parameter	Notation	Value	Unit
Young modulus	E	30 000	MPa
Poisson's ratio	ν	0.20	–
Tensile strength for	R^t	3.50	MPa
Strain at R^t	$\varepsilon^{peak,t}$	$1.17 \cdot 10^{-4}$	m/m
Fracture energy in tension	Gf^t	$9.00 \cdot 10^{-5}$	MJ/m^2

3.2 Results

First, a series of tests was carried out by choosing different mesh sizes with a constant diffusion length l_D ($l_D = 25mm$). Next, the computed responses of the model by choosing successively linear ($CUB8$) and quadratic ($CU20$) finite elements are compared. All the simulated load-displacement curves are presented in Figure 6. It is worth noting that the load-displacement responses of the beam are reasonably mesh independent. As expected, the difference of responses obtained with quadratic finite elements between the finest and the coarsest meshes is smallest than the ones obtained with linear finite elements.

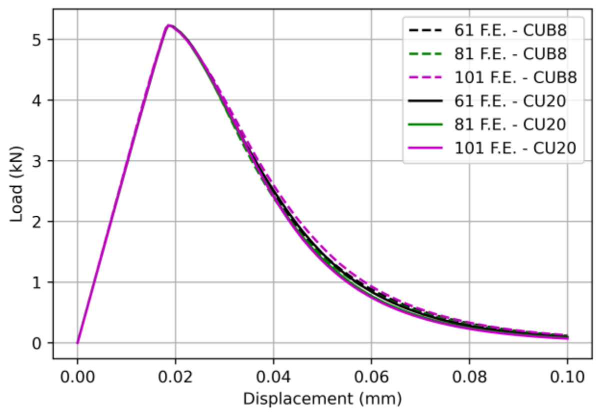

Figure 6: Load-displacement behaviour of the bar for various mesh sizes with a constant value of l_D and using $CUB8$ and $CU20$ finite elements

Figure 7 shows the non-local plastic strain $\hat{\varepsilon}_{xx}^{pl} \approx \hat{\varepsilon}_1^{pl}$, the crack opening $w_1^{pl} = \sqrt{2} \cdot l_D \cdot \hat{\varepsilon}_1^{pl,max}$ and the tensile damage D_1^t fields, plotted on a deformed mesh. As expected, the crack opening field shows clearly that the crack is positioned in the central finite element. Moreover, the diffusion length l_D has to be handle with care. As mentioned previously, if the l_D value is too small, in other words, if there is not a minimum of six elements in l_D, the quality of the numerical response of the material will be degraded. On the contrary, if the l_D value is too large in comparison to the structure size, boundary effects can appear.

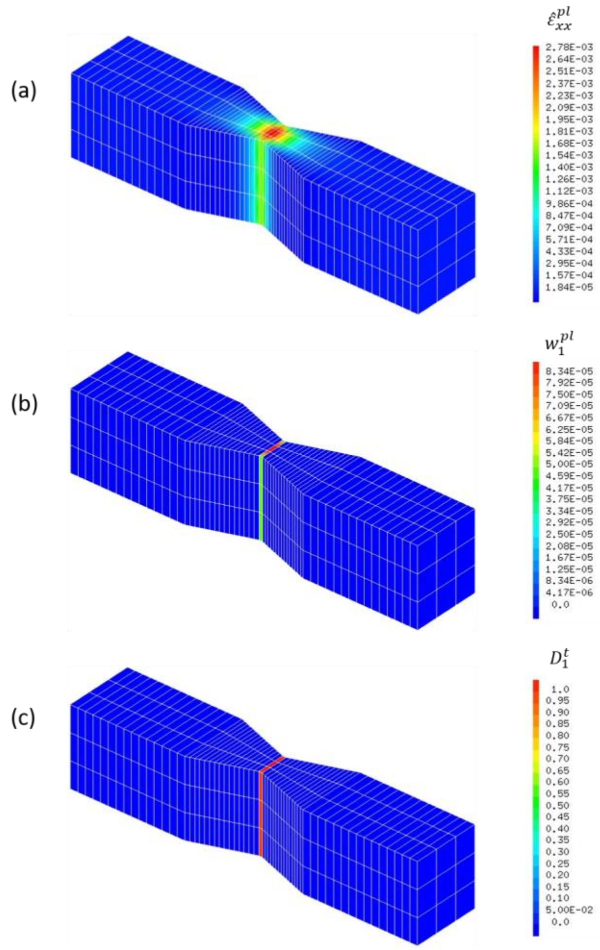

Figure 7: Non-local plastic strain tensor component $\hat{\varepsilon}_{xx}^{pl}$ (a), crack opening w_1^{pl} (b) and damage D_1^t (c) plotted on the mesh of the deformed beam, corresponding to the last step computed in Figure 6 (61 FE ; CUB8 ; $l_D = 25mm$)

4 CONCLUSIONS

This study demonstrates the possibility of generalizing the non-local theory from the classical framework of the isotropic state variables to the second-order tensorial variables. The method, implemented in a finite element software, is based on a second gradient formulation with directional diffusion tensors. The orthotropic formulation of non-local plasticity coupled with damage offers several advantages compared to the isotropic non-local formulation.

One principal advantage is the absence of plastic strain field diffusion in parts of the structure that are normally undamaged, which is not the case in the isotropic non-local formulation. This ensures that plastic strains are localized only in regions where damage has occurred, making the orthotropic formulation more accurate in representing localized damage.

Another benefit is the facilitated assessment of crack position and opening. Analytically, the crack position corresponds to the zone where the curvature of the non-local strain is negative. Numerically, it corresponds to the zone where the local plastic strain exceeds the non-local one. This information allows for a precise determination of the location and size of the cracks.

Once the crack has localized, the crack opening can be approximated by multiplying the non-local plastic strain by the diffusion length (l_D) times $\sqrt{2}$. This approximation remains valid as long as the zone with negative curvature is small compared to the diffusion length (l_D), which is satisfied when there are at least six elements within l_D.

The localized crack openings are then used to calculate damages, which can be applied to the effective principal stresses to obtain macroscopic stresses. Importantly, the value of l_D, which affects the softening behaviour of the material in tension at Gauss points, can be chosen independently by the users without altering the numerical results of the entire structure.

A future perspective of this work involves computing the permeability of the material as a function of the crack opening. This would lead to the formulation of a poro-mechanical finite element method regulated by this non-local orthotropic formulation. Such an approach could provide valuable insights into the behaviour of materials with localized damages and contribute to more accurate predictions in the analysis of concrete structures and their durability.

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