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FRACTURE PREDICTION FOR LARGE CONCRETE BEAMS USING MEASUREMENTS FROM SMALL NOTCHED CONCRETE SAMPLES

GANG HAN[*], XIANGYU HAN[†] AND XIAOZHI HU[#]

[*] University of Western Australia
Department of Mechanical Engineering
Perth, WA6009, Australia
e-mail: gang.han@research.uwa.edu.au

[†] Southwest University of Science and Technology
School of Civil Engineering and Architecture
Mianyang, Sichuan 62101, China
e-mail: xiangyuhan03@163.com

[#] University of Western Australia
Department of Mechanical Engineering
Perth, WA6009, Australia
e-mail: xiao.zhi.hu@uwa.edu.au

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Abstract: Three-point-bending (3pb) test results of small concrete specimens are used to predict unstable fracture of large concrete beams with shallow or no visible cracks. Small specimens and large beams with any notch depth (0, shallow or deep) are linked together by one simple formula. It models quasi-brittle fracture of concrete and the associated size effect by modelling the notch-tip fracture process zone (FPZ), which is intrinsically linked to the aggregate size. A comprehensive set of concrete fracture data published in the literature was analyzed by the model, which was further substantiated by three new sets of 3pb concrete tests: 48 un-notched, 46 shallow-notched (1 mm) and 44 deep-notched (6 mm) specimens, with span of 160 mm, thickness around 40 mm and width/height around 38 mm. The average aggregate size is 4 and 5 mm for the two concrete mixes. Both FPZ length and width at the peak fracture load were estimated and linked to the average aggregate size, and then FPZ interactions with the specimen/structure boundary were modelled for quasi-brittle fracture and size effect. Statistical functions were added to this "boundary effect model" so that large experimental scatters were defined by the statistical reliability.

1 INTRODUCTION

It is an idea scenario that unstable fracture of large concrete structures with or without visible cracks can be reliably predicted from experimental results of small notched three-point-bending (3pb) specimens of the same concrete mix. Small specimens mean economical to prepare and easy to handle, while the notch pinpoints the fracture location for the stress analysis and modelling of the crack-tip fracture process zone (FPZ).

The FPZ-induced size effect on different fracture behaviours from small specimens to large structures has thus become a focus in the concrete fracture community since the seminal work published in 1983 and 1984 [1,2]. Nowadays, the size effect law (SEL) is probably one of the most recognized size effect models, which has three formulas for un-notched, shallow-notched and deep-notched specimens together with more than 10 fitting parameters [3-5]. Besides many other experiments, two comprehensive tests performed separately by two different groups [6,7] confirm SEL can describe quasi-brittle fracture of concrete and associated size effect.

It will make a big step forward if three SEL formulas can be reduced to one, and one material property or physical constant can be found and used to replace those 10 plus fitting parameters. The recent publication of Chen and Hu [8] has just done that, i.e., a simple linear formula through the origin (0,0) can model all quasi-brittle fracture behaviours from un-notched, shallow-notched and deep-notched specimens of large and small sizes. The slope of the linear function, as the sole parameter, is the tensile strength of concrete as the criterion for the FPZ formation at the notch tip.

The primary objective of this study is to show size effect modelling for concrete fracture can be significantly simplified both experimentally and analytically. One set of 3pb concrete specimens of any convenient size is sufficient for predictions of unstable fracture of large concrete specimens or structures and for determination of both the fracture toughness K_{IC} and tensile strength f_t (criterion for the FPZ formation). Three new sets of 3pb tests (138 in total) for un-notched, shallow-notched and deep-notched specimens of single size (around 160×40×38 mm^3) are performed and analyzed by the new linear model.

2 PREDICTIVE FRACTURE MODEL

2.1 Linear formula for 3pb geometry

A fracture model is predictive if it does not contain any parameter, always relying on curve-fitting of experimental results. In this study, a simple linear model recently reported in [8-10] is adopted. Firstly, the average aggregate size d_{av} of concrete should be known and specified, which can be determined from the concrete mix and resultant aggregate structures of concrete specimens. This d_{av} measurement is critical for differentiation of concrete properties as it can still be varied even if the maximum aggregate d_{max} remains constant for different concrete mixes.

After the average aggregate size d_{av} is determined prior to fracture testing, the tensile strength f_t (the local criterion for FPZ formation at the notch tip) is the only parameter required in the following linear function for 3pb geometry.

$$P_{max} = f_t \cdot A_e \tag{1}$$

Here the equivalent area A_e is a geometry and dimensional parameter, fully determined by the average aggregate size d_{av} and 3pb specimen dimensions [8-10], i.e., span S, thickness B, width/height W and the initial notch a_0 (≥ 0). For the 3pb geometry, A_e is given by:

$$A_e = \frac{W^2 \cdot \left(1-\frac{a_0}{W}\right) \cdot \left(1-\frac{a_0}{W}+\frac{3 \cdot d_{av}}{W}\right)}{1.5\left(\frac{S}{B}\right)\sqrt{1+\frac{a_e}{3 \cdot d_{av}}}} = \frac{W^2 \cdot \left(1-\frac{a_0}{W}\right) \cdot \left(1-\frac{a_0}{W}+\frac{2 \cdot FPZ_L}{W}\right)}{1.5\left(\frac{S}{B}\right)\sqrt{1+\frac{a_e}{FPZ_W}}} \tag{2}$$

Classic LEFM considers the influence of the notch ratio a_0/W through the geometry factor $Y(a_0/W)$. In Eq. (2), the FPZ length and width ratios (FPZ_L/W, a_e/FPZ_W) in comparison with the width/height W and notch a_0 (or a_e) are also considered.

The linear function in Eq. (1) is based on the original asymptotic analysis for a large plate with a short edge crack ($a_0/W << 1$) or "boundary effect model" (BEM) [11].

$$a_e \left[\frac{\left(1-\frac{a_0}{W}\right)^2 \cdot Y\left(\frac{a_0}{W}\right)}{1.12}\right]^2 \cdot a_0 \tag{3}$$

A given notch length a_0 has different effects on fracture depending on whether the specimen is large or limited in size. The equivalent size a_e in Eq. (3) specifies the difference.

For a large plate, $a_0/W \to 0$, the geometry factor $Y = 1.12$ so that $a_e = a_0$ and the large plate solution [11] is recovered.

If the tensile strength f_t is known for a concrete mix, the maximum fracture load P_{max} of large 3pb structures or specimens can be predicted directly by the simple linear model Eq. (1) for any crack or notch size $a_0 \geq 0$. That is the size or width W in Eq. (1) can be small for test samples or large for structures.

2.2 Linear model with statistical reliability

f_t measurements from direct tensile tests can be easily influenced by pre-existing micro-defects [12]. It is inevitable that the intrinsic f_t (for FPZ formation) is higher than the tensile strength $f_{t\text{-}tensile}$ measured from direct tensile tests, due to micro-pores, weak interfaces and unfavorable aggregate and sand structures at certain locations (weak sections). Eq. (1) provides an alternative way to determine the intrinsic f_t from small 3pb specimens, i.e., a f_t measurement is obtained directly from a fracture load P_{max}. Since scatters are inevitable due to the highly heterogeneous aggregate structures at the notch tip, a normal distribution is used to determine the mean μ (= f_t) and standard deviation σ. Eq. (1) can then be rewritten as:

$$P_{max} = (\mu \pm 2\sigma) \cdot A_e \qquad (4)$$

It should be reiterated that there is no specific requirement for the initial notch a_0, which can be 0 for the un-notched case, or > 0 for shallow- and deep-notch cases. In other words, the three SEL formulas for un-notched, shallow-notched and deep-notched specimens have been effectively reduced to one linear formula in Eq. (1). Furthermore, the strict condition of geometrically similar specimens imposed for size effect tests is no longer required. The statistical reliability range specified in Eq. (4) is also beyond the scope of SEL, which is not considered even the three SELs contain more than 10 fitting parameters.

3 EXPERIMENTAL REVIFICATIONS

3.1 Comprehensive results from literature

3pb tests [6] with the size W = 40, 93, 215 and 500 mm, and the initial notch and size ratio a_0/W for un-notched (0), shallow-notched (0.025 and 0.075) and deep-notched (0.15 and 0.3) specimens form 5 sets of geometrically similar specimens (with constant a_0/W ratios). These tests can also be regrouped into 4 sets of specimens with the constant size W of 40, 93, 215 and 500 mm [8]. Therefore, in total, 9 sets of comprehensive data for concrete fracture can be used to test the applicability of a fracture model, as done in [8].

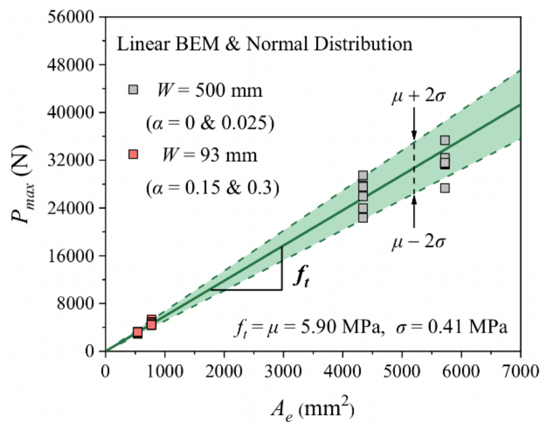

Figure 1: Predictions of large un-notched and shallow-notched ($a_0/W = 0$ and 0.025) concrete beams with size $W = 500$ mm from small deep-notched ($a_0/W = 0.15$ and 0.3) samples size $W = 93$ mm.

Those 3pb specimens can also be randomly selected and grouped, e.g., not geometrically similar, different sizes or with different initial notches. Figure. 1 shows such a case, i.e., small deep-notched (a_0/W = 0.15 and 0.3) samples with size W = 90 mm, and large un-notched and shallow-notched (a_0/W = 0 and 0.025) samples with size W = 500 mm are grouped together. The linear boundary effect mode (BEM) with 95% reliability band from Eq. (4) with d_{av} = 5 mm describes the fracture behaviors of both small and large specimens (not geometrically similar, different in size and initial notch and size ratio). It means test results from small notched 3pb samples can be used to predict unstable fracture of large concrete samples (or structures).

The results in Figure. 1 are significant as it shows that fracture loads of un-notched large 3pb specimens (or structures) can be predicted from experimental results of small notched 3pb specimens. The equivalent area A_e can be evaluated for any large 3pb specimens or structures, then the predicted fracture loads are indicated by the linear relation.

Furthermore, Figure. 1 shows the fracture loads of large 3pb specimens or structures with limited crack growth (a_0/W = 0.025) can also be predicted. Concrete structures with limited surface cracks may be classified into this category.

3.2 Small 3pb tests with one size *W*

The prediction illustrated in Figure. 1 relies on the prior knowledge of the tensile strength f_t – the criterion for the formation of FPZ at the notch tip. If f_t is not known, one set of small 3pb tests with or without notch can be performed first to determine f_t.

New experiments of 3pb specimens have been performed for this study, to determine f_t of commercial concrete plates with d_{av} = 4 mm. The specimen span S = 160 mm. 48 un-notched specimens have thickness B from 33.7 – 40.9 mm and size W from 38.1 to 43.9 mm. Those variations in B and W measurements are due to cutting, but they can be considered by Eq. (4).

46 shallow-notched specimens with a_0 = 1 mm were also prepared. B varies from 32.9 – 38.8 mm and W from 36.2 to 42.2 mm. For comparison, 44 deep-notched specimens with a_0 = 6 mm were prepared, with B from 35.4 – 39.6 mm and W from 36.7 to 42.7 mm.

All the experimental results together with 95% reliability band from Eq. (4) are shown in Figure. 2(a). Three mean points for un-notched, shallow-notched (1 mm) and deep-notched (6 mm) specimens are shown in Figure. 2 (b). There is hardly any error between the three mean points as they are all from large numbers of tests. This proves any small specimens with or without notch (shallow or deep) can all be used to determine the tensile strength f_t from 3pb experiments. With the knowledge of f_t, Eq. (4) becomes a predictive model, i.e., the fracture load P_{max} can be predicted by the linear formula for any 3pb geometry and size.

It should be emphasized that the small 3pb tests for the results in Figure. 2 are easy to prepare and preform. In comparison with more than 40 specimens for each group in Figure. 2, the two groups of small 3pb tests in Figure. 1 have only around 8 specimens per group. Yet, reliable estimations of f_t were still obtained, indicating around 20 small 3pb specimens should be sufficient for f_t estimations.

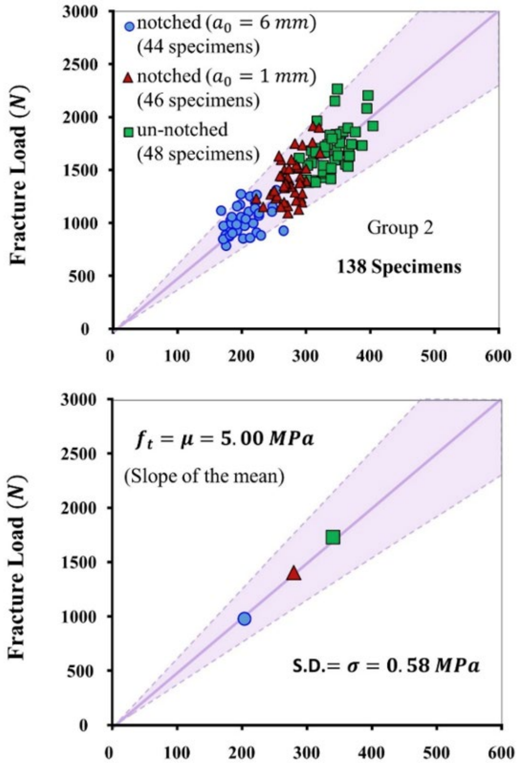

Figure 2: (a) Three groups of small 3pb test results for un-notched, shallow-notched (1 mm) and deep-notched (6 mm) specimens of constant size W (around 40 mm), as modelled by Eq. (4). (b) The means from the three 3pb groups, showing the same tensile strength f_t = 5 MPa (the slope of the mean line) is determined by the linear BEM or Eq. (4).

4 DISCUSSIONS

4.1 Geometrically similar specimens

Typical size effect experiments require geometrically similar specimens, i.e., a_0/W = constant, and the size range for W should be as wide as possible for reliable curve fitting. For instance, two separate comprehensive concrete tests [6,7] selected the size range for W from 40 to 500 mm.

Three SELs are used for curve fitting for un-notched (a_0/W = 0), shallow-notched (a_0/W < 0.1) and deep-notched (a_0/W > 0.1) geometrically similar specimens. Because of the demanding SEL requirements, the large number of single group 3pb test is only 11 for W = 40 mm and a_0/W = 0.075. The tensile strength f_t estimation from this group of 11 specimens [8] is shown in Figure. 3(a).

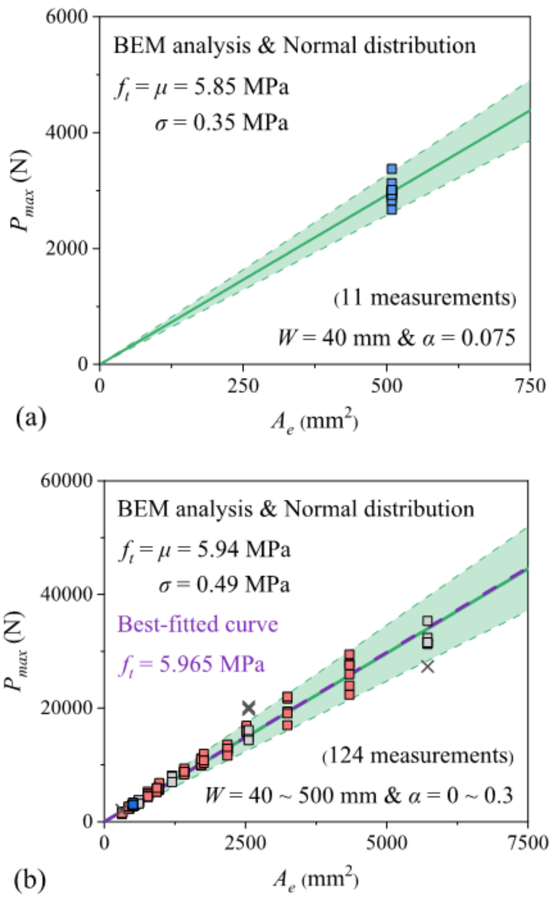

Figure 3: (a) f_t from 11 specimens of $W = 40$ mm. (b) f_t from total 124 specimens with W from *40* to *500* mm and various notch lengths.

The tensile strength f_t estimation from the total 124 specimens is shown in Figure. 3(b). The relative error between the two estimation is less than 2%. In Figure. (1), the estimated f_t = 5.9 MPa, the relative error in comparison with the results in Figure. 3 is less than 1%. Since consistent f_t estimations can be obtained from various combinations of the experimental results as shown in Figure. 1 and 3, the strict geometry similarity condition for size effect tests can be relaxed. This will significantly simplify size effect experiments and modelling.

4.2 Simple tests using one-size specimens

Since one group of small 3pb tests of any convenient size is sufficient for size effect experiments, more samples can be prepared and tested as shown in Figure. 2(a). Based on the results in Figure. 1 to 3, around 20 specimens can ensure a reliable estimation of the tensile strength f_t, and then reliable predictions of the fracture load P_{max} for different large 3pb specimens or structures.

In conjunction with the linear BEM with the statistical reliability function shown in Eq. (4), a single convenient size for small 3pb tests with or without the initial notch as shown in Figure. 2 has considerably simplified the size effect experiments and modelling.

4.3 Commonly used SEL for curve fitting

The original SEL [2] for deep-notched geometrically similar specimens ($a_0/W > 0.1$) is most commonly used for curve fitting because it only contains two fitting parameters, B_0 and W_0.

$$\sigma_N = \frac{B_0 \cdot f_t}{\sqrt{1 + \frac{W}{W_0}}} \tag{5}$$

The nominal strength σ_N is determined by the standard strength of materials theory using the maximum fracture load P_{max} without consideration of the initial notch. However, the tensile strength f_t cannot be determined by this SEL from curve fitting because $B_0 \cdot f_t$ cannot be separated during curve fitting unless Eq. (5) is treated as a three-parameter formula.

Recently and as early as 2008 [8,13], it is shown that the initial BEM proposed in 2000 [11] can be rearranged into the following form, i.e.,

$$\sigma_N = \frac{B_{0-BEM} \cdot f_t}{\sqrt{1 + \frac{W}{W_{0-BEM}}}} \tag{6}$$

Akin to Eq. (5), this BEM also contains two fitting parameters, $B_{0\text{-}BEM}$ and $W_{0\text{-}BEM}$. If Eq. (5) and (6) are used for curve fitting, there is no difference between SEL and BEM. That is although the size effect modelling and boundary effect formulation were originated from different thoughts and assumptions, they actually arrived at the same solution.

The linear and improved BEM in Eq. (4) with one parameter (material constant f_t) is simpler than Eq. (5) and (6). If the tensile strength f_t is known, there is no need for curve fitting. As a result, BEM becomes a predictive model.

It should also be mentioned that Eq. (2) in the framework of SEL is limited to deep-notched geometrically similar specimens with $a_0/W > 0.1$ (a_0/W = constant for a selected set of test samples). BEM specified in Eq. (4) can be used for any 3pb specimens of any size and with any initial notch length, i.e., $a_0/W \geq 0$.

4.4 "Hall-Petch" relation for brittle solids

There is a reason why three SELs are required for un-notched, shallow-notched and deep-notched specimens. This is because neither the strength criterion nor fracture toughness criterion applies due to the limited notch and FPZ ratio, a_0/FPZ, which is a direct consequence due to the strong influence of FPZ and highly heterogeneous aggregate structures.

The classic Hall-Petch relation [14,15] established the relation between the yield strength σ_Y and average grain size d_G for metals, i.e.,

$$\sigma_Y = \sigma_0 + k \cdot \frac{1}{\sqrt{d_G}} \tag{7}$$

The two fitting parameters, σ_0 and k, need to be determined experimentally through curve fitting for a given metal.

Similar to Eq. (7), the "Hall-Petch" relation for brittle heterogeneous solids like concrete has recently been established to derive the linear BEM, Eq. (1) or (4), i.e.,

$$K_{IC} = 2 \cdot f_t \sqrt{3 \cdot d_{av}} \tag{8}$$

Or

$$f_t = 0 + \frac{K_{IC}}{2\sqrt{3}} \cdot \frac{1}{\sqrt{d_{av}}} \tag{9}$$

Comparing Eq. (7) and (9), it is clear that the new "Hall-Petch" relation for brittle solids has one less parameter ($\sigma_0 = 0$), and the previous un-known parameter k is specified explicitly by the fracture toughness K_{IC}. The linkage between f_t and K_{IC}, specified by the Eq. (8), is the reason why Eq. (1) or (4) can deal with both notched and un-notched specimens.

The classic Hall-Petch relation [14,15] proposed in early 50's (attracting over 9000 citations) is limited to the grain size range from 20 nm to 200 μm. The new "Hall-Petch" relation for brittle solids and composites is valid for the microstructure range from the atomic scale (< 1 nm) all the way up to 200 mm for large engineering structures, e.g., dam concrete [10]. Eq. (8) is proven to be valid for a wide range of brittle solids including single crystal silicon, fine and coarse-grained ceramics, rock and concrete, bone and fiber composites [10,16,17]. With the assistance of the new "Hall-Petch" relation, the three SELs have been reduced to one linear formula [8], as shown in Eq. (1) or (4).

Eq. (2) shows how Eq. (1) models both FPZ width and length, and how the average aggregate size d_{av} is linked to the FPZ. For very large size W, FPZ_L/W can be ignored, and only the a_0/FPZ_W ($a_e = a_0$) needs to be considered. That is only the crack blunting effect of FPZ_W needs to be considered for large structures.

If $a_0 = a_e = 0$, Eq. (1) can be simplified as follows, i.e.,

$$P_{max} = f_t \cdot \frac{W^2 \cdot \left(1 + \frac{3 \cdot d_{av}}{W}\right)}{1.5\left(\frac{S}{B}\right)} \tag{10}$$

If $d_{av}/W = 0$, the classic stress analysis result is obtained. If 10% error is acceptable using the common stress analysis for homogeneous materials, $3 \cdot d_{av}/W < 0.1$ or $W > 30 \cdot d_{av}$ should be ensured. Under such a condition, heterogeneous concrete can be treated roughly as a "homogeneous material". Eq. (10) has recently been confirmed for plain concrete specimens without notch, which also explains the ASTM standard for 3pb tests of laminar carbon fiber composites [18].

4.5 Notch width influence on K_{IC}

Typically, the initial notch introduced in concrete specimens do not have a sharp crack tip, but 1 – 4 mm wide. However, if the notch width n_w is reasonably narrow in comparison with the FPZ_W (around three times of the average aggregate size or $3 \cdot d_{av}$), the notch width influence can be estimated as follows [10,17]:

$$K_{IC} \approx K_{IC-NW} \cdot \sqrt{\frac{3 \cdot d_{av}}{3 \cdot d_{av} + n_w}} \quad (11)$$

In this study, the notch width n_w is 1 mm and $d_{av} = 4$ mm, Eq. (11) shows the toughness ratio is 96%. Therefore, the notch width influence can be ignored.

The simple approximation of the notch effect in Eq. (11) is possible only because the fracture process zone width FPZ_W is modelled in Eq. (1) and (2). Since most fracture models emphasize the FPZ_L influence, the notch width effect specified in Eq. (11) is unique to BEM, which models both FPZ_L and FPZ_W.

5 CONCLUSIONS

It is already 40 years since the original SEL [1,2] was proposed on quasi-brittle fracture of concrete. It is over 20 years since the simple asymptotic analysis of BEM on interactions between FPZ and specimen boundary [11] was proposed. It is expected that useful progress in size effect modelling and experiments should be made after such a lengthy period of time.

This study and recent work [8] have shown the 3 SELs with multiple fitting parameters can indeed be reduced to one simple linear function as shown in Eq. (1). This linear BEM is fully specified by the aggregate size d_{av} and tensile strength f_t (criterion for the FPZ formation).

This study further shows the size effect experiments using the gematrically similar specimens of different sizes can be replaced by testing one set of small 3pb specimens of any convenient size, i.e., as shown in Figure. 2.

This linear BEM Eq. (4) can explicitly specify the statistical reliability band for experimental scatters in concrete fracture data, inevitable due to highly heterogeneous aggregate structures. The simplicity of Eq. (4) with its statistical function is unique among numerous models proposed for concrete fracture [1-5, 7, 19-24].

The new "Hall-Petch" relation for brittle solids and composites as given in Eq. (9) is critical for the derivation of the linear BEM shown in Eq. (1) and (4). While the classic Hall-Petch relation for metals is limited to the grain size range from 20 nm to 200 μm, the new "Hall-Petch" relation for brittle solids and composites is valid for microstructures from the atomic scale (< 1 nm) to around 200 mm for dam concrete [10]. Furthermore, the new "Hall-Petch" relation covers various brittle solids and composites as summarized in [10].

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