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ENERGY DISSIPATION APPROACH TO CHARACTERIZE THE FRACTURE BEHAVIOR OF CONCRETE UNDER FATIGUE LOADING

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Abstract. Concrete is a quasi-brittle material which may undergo repetitive loading during its service life, presenting challenges in understanding its fracture behavior. The fracture process zone (FPZ) ahead of the crack tip behaves differently under fatigue loading than monotonic loading, making it crucial to comprehend the energy dissipation associated with the FPZ. Concrete members exhibit a two-stage stiffness degradation process under repetitive load cycles, with the first stage being the toughening stage (FPZ development stage) that resists crack propagation and the second stage being the stage in which catastrophic failure takes place once the FPZ is fully developed. Therefore, to better understand the fracture behavior of concrete under cyclic loading, it is important to consider stiffness degradation. This work proposes an analytical expression that predicts energy dissipation and stiffness degradation approach as functions of crack length. This expression can be utilized to characterize the damage behavior and fatigue life of structures. In this study, specimens with a center notch have been utilized to develop formulations. Furthermore, the formulation for stiffness degradation has been developed by relating the dissipated energy with the work done by the externally applied load. This expression has been used to determine the fully developed fracture process zone (FPZ) length at the switchover point when stiffness degradation begins to increase.

1 INTRODUCTION

Concrete materials are known for their quasi-brittle behavior, mainly attributed to the presence of an inelastic damage zone ahead of the crack tip, referred to as the fracture process zone (FPZ) [2, 3, 8, 10]. The FPZ encompasses various toughening mechanisms, including aggregate bridging, micro-cracking, crack deflection, and crack branching, which contribute to the quasi-brittle nature of concrete. This zone plays a crucial role in exhibiting size effect behavior in concrete and renders linear elastic fracture mechanics (LEFM) inapplicable. The nature of loading governs the modes of concrete failure, making the characterization of the fracture process zone a vital research topic. While a fully developed FPZ is commonly associated with peak loads under increasing monotonic loading conditions, its behavior under repetitive loading differs significantly [12]. The literature presents divergent views on the characterization of the fracture process zone under cyclic loading. Some studies employing the Digital Image Correlation (DIC) technique indicate a larger FPZ under cyclic loading compared to monotonic loading for quasi-brittle rock [11–13], while others claim non-existence of FPZ under fatigue loading based on Acoustic Emission (AE) analysis [5, 6]. Conversely, certain investigations predict comparable FPZ sizes under both cyclic and monotonic loading for plain concrete. The characterization of FPZ in plain concrete under repetitive loading using experimental techniques remains particularly challenging. This paper reviews and discusses the existing literature on the characterization of the fracture process zone under repetitive loading conditions in plain concrete, highlighting the discrepancies and challenges associated with different experimental approaches.

In this study, the expression for dissipated energy has been derived for predicting the evolution of the fracture process zone throughout repetitive loading. A stiffness degradation approach [7] has been adopted for developing the formulations for the critical energy dissipation and fully developed fracture process zone.

2 INTYERNAL ENERGY DISSIPATION, W_I

Under the conditions of fatigue loading, repetitive loading cycles induce localized, irreversible microstructural alterations accompanied by some elastic deformation. Notably, during the initial stages of load application, the extent of permanent damage is relatively low, with the predominant deformation being elastic in nature. These cyclic microstructural changes accumulate over time, leading to the formation of a permanent localized damage zone commonly referred to as the fracture process zone (FPZ). The FPZ arises due to the interplay of various toughening mechanisms within concrete. As the damaged zone advances ahead of the crack tip, it grows incrementally with each load cycle until it reaches a fully developed stage, at which point unstable crack propagation ensues. Throughout the stable state of crack propagation, the energy expended to overcome the resistance offered by cohesive forces has been quantified as internal energy or dissipated energy, a parameter that has been computed in this section.

2.1 Cohesive stresses

Considering linear cohesive stress distribution with the crack opening, cohesive stress at the bottom-most fiber of the beam can be expressed as [1, 4],

$$\sigma_b = f_t \left(1 - \frac{a}{l_f}\right) \tag{1}$$

here, a is the crack length, f_t is the tensile strength of concrete, and l_f is the fully developed fracture process zone length beyond which traction-free crack occurs. Cohesive stress at any arbitrary distance x from the bottom-most fiber of the beam can be expressed as follows.

$$\sigma_x = f_t \left(1 + \frac{x}{l_f} - \frac{a}{l_f}\right) \tag{2}$$

2.2 Crack mouth opening displacement

In the context of fatigue loading, crack growth behavior has been found to exhibit two distinct regimes, namely stable and unstable crack propagation [2, 3, 7, 9]. Stable crack growth arises from the presence of a significant fracture process zone influenced by diverse toughening mechanisms. Following the stable phase, an unstable crack propagation occurs when the fracture process zone reaches its full extent. In the stable crack growth regime, studies [15, 16] have provided an expression for the crack mouth opening displacement as follows.

$$W_m = (W_{m_F} + W_{m_M}) \tag{3}$$

The quantities W_{m_F} and W_{m_M} are the crack opening displacements at the crack initiation point due to cohesive tensile stress and the corresponding moment. Further, the average cohesive stress σ_m and the corresponding moment M_σ can be expressed as follows.

$$\sigma_m = \frac{1}{D} \int_0^a \sigma_x dx \tag{4}$$

$$M_\sigma = B \int_0^a \sigma_x r dx \tag{5}$$

where $r = \frac{D}{2} - x$ is the lever arm distance. Here, the lever arm distance r will change during the load cycle. For simplification purposes, this change has been ignored here; however, eventually, its effect will be incorporated into the integration process.

The study carried out by Zhang et al. [17] has reported that crack mouth opening displacement due to cohesive tensile force W_{m_F} and corresponding moment W_{m_M} can be expressed as follows, where α is the ratio of crack length and specimen size $(\frac{a}{D})$.

$$W_{m_M} = 24s\frac{M_\sigma}{BDE}V_1 \tag{6}$$

$$V_1 = 0.8 - 1.7\alpha + 2.4\alpha^2 + 0.66(1+2\alpha) \tag{7}$$

$$W_{m_F} = 4\frac{\sigma_{avg_x}}{E}\alpha D V_2 \tag{8}$$

$$V_2 = \left(1.46 + 3.42\frac{t^2}{2}\right)(1+t^2) \tag{9}$$

Further, the crack opening displacement W_x at any arbitrary distance x from the crack initiation point can be written as follows.

$$W_x = W_m \times V_3 \tag{10}$$

$$V_3 = 1 - 0.5\left(0.081\frac{x^2}{a^2} + 0.919\frac{x}{a}\right) - 0.5\left(1.49\frac{x}{D} - 1.49\frac{x^2}{aD}\right) \tag{11}$$

After substituting W_{m_F} and W_{m_M} in the expressions Equation 3 and 10, the functional dependency of crack mouth opening displacement W_m and crack opening W_x at any arbitrary distance, x has been shown with the following expressions.

$$W_m = f_1(E, f_t, D, a, l_f) \tag{12}$$

$$W_x = f_2(E, f_t, D, a, x, l_f) \tag{13}$$

2.3 Work done through cohesive force, W_I

In the context of crack propagation, internal energy represents the energy stored within the material. For stable crack propagation, the internal energy in the structure is equivalent to the energy exerted by the externally applied load. To calculate the internal energy resulting from cohesive stresses resisting crack opening, a typical crack profile shown in Figure 1 has been considered, where point 'A' denotes the crack tip and 'BC' represents the crack opening at the initiation point. The internal energy, denoted as W_I for a crack length a, can be determined by evaluating the total work performed by the cohesive forces. As the cohesive force varies along the crack length, the total internal energy can be obtained through elemental integration, which involves multiplying the cohesive force with the corresponding crack opening. By considering a strip of width dx located at a distance x from the bottom in the total crack length a, the applied force f_{dx} in the dx strip can be expressed as follows.

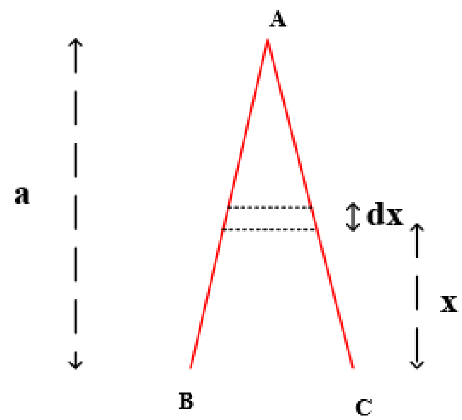

Figure 1: Crack opening profile

$$f_{dx} = B\sigma_x dx \tag{14}$$

The total internal energy due to cohesive force to create a total damage zone of length a can be written as the corresponding work done $dw_{f_{dx}}$

in dx strip for a crack increment da to grow from 0 to a can be written as follows.

$$dw_{f_{dx}} = f_{dx} W_{x_a} da \tag{15}$$

where, W_{x_a} is the crack opening rate at a distance x from the bottom fiber of the beam, which can be expressed as following.

$$W_{x_a} = \frac{d(W_x)}{da} \tag{16}$$

The total work done, $w_{f_{dx}}$ for a crack to grow from 0 to a in a dx strip can be written as follows,

$$w_{f_{dx}} = \int_0^a f_{dx} W_{x_a} da \tag{17}$$

Further, the total work done carried out by cohesive forces can be calculated by integrating $w_{f_{dx}}$ from 0 to a.

$$W_I = \int_0^a w_{F_{dx}} dx \tag{18}$$

It has been shown in expression, Equation **??** that the total internal energy W_I will be a function of material properties, heterogeneity, and specimen size.

$$W_I = B \times f_3(E, f_t, D, a, l_f) \tag{19}$$

3 WORK DONE CARRIED OUT BY EXTERNALLY APPLIED LOAD, W_E

Under repetitive loading conditions, damage occurs due to the accumulation of permanent, localized microstructural changes that take place in each load cycle. These microstructural changes lead to the separation of atomic units and contribute to the formation of a damage zone or fracture process zone. In flexural beams, this damage zone may manifest as vertical deflection. The dissipated energy within the fracture process zone or damage zone during a single cycle can be related to the externally applied load amplitude. Since repetitive loading involves cycling between two stress ranges, i.e., the maximum and minimum load amplitudes, the external energy W_E associated with a single load cycle N has been expressed below [14].

$$W_E = \frac{1}{2}[P_{min} + P_{max}]\delta_N \tag{20}$$

where P_{max} and P_{min} are the maximum and minimum load amplitudes, and δ_N is the incremental plastic deformation in the N^{th} load cycle. Considering $P_{min} = 0$ and P_{max} as P, the above expression can be written as,

$$W_E = \frac{P}{2}(\delta_1 + \delta_2 + \delta_3 ..... + \delta_N) \tag{21}$$

Here, δ_1, $\delta_2 ... \delta_N$ are the inelastic deformations associated with different load cycles. Considering the total cumulative inelastic deformation as Δ, the total external work can be rewritten as,

$$W_E = 0.5P\Delta \tag{22}$$

Furthermore, the vertical deflection at the center of the beam under three-point bending can be evaluated using the crack opening displacement at the initiation point, W_m, span length, L, and specimen size D [15].

$$\Delta = W_m \frac{L}{3.6D} \tag{23}$$

4 FULLY DEVELOPED FRACTURE PROCESS ZONE LENGTH

In this section, an analytical formulation has been presented aiming at assessing the size of the fracture process zone in concrete subjected to fatigue loading. The maximum length of the fracture process zone under repetitive loading conditions is attained when stable crack growth transitions to unstable crack growth. This process zone event can be better elucidated by studying crack growth behavior and the rate of stiffness degradation. During fatigue loading, the crack mouth opening displacement (CMOD) values have been observed to increase at a diminishing rate with an increase in the number of load cycles. Additionally, experimental investigations reveal that CMOD and crack length exhibit similar growth trends. Similarly, the loss in stiffness follows a comparable trend, i.e., it decreases at an increasing rate. As loading progresses, a critical stage is reached where the crack propagation or stiffness loss rate begins to escalate. This stage is commonly referred to as the ”bend-over point” [7], marking the initiation of unstable crack propagation. Beyond the bend-over point, both crack growth and stiffness loss display a catastrophic growth nature. Thus, it is reasonable to consider this bend-over point as the stage when the fracture process zone is fully developed. Consequently, the cohesive stress at the notch tip of the beam becomes zero, initiating the propagation of a traction-free crack.

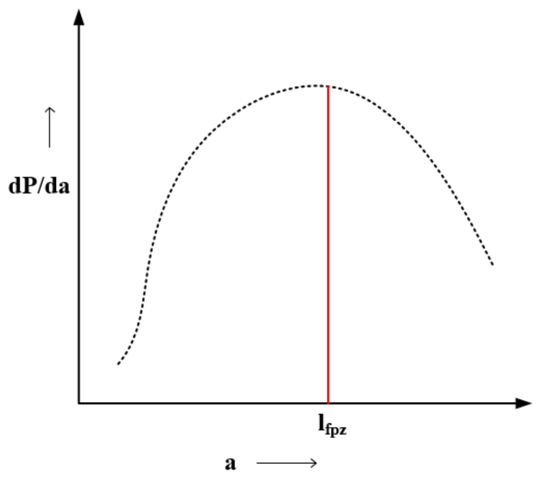

Figure 2: variation of dp/da with crack length

It is crucial to emphasize that during the deceleration stage of the crack growth curve, the crack experiences a slower rate of increase. When considering the maximum and minimum load amplitudes as constant, denoted as $\delta P = P_{max} - P_{min}$, the quantity $\frac{\partial P}{\partial a}$ will exhibit a growing trend with the crack length until the deceleration stage of crack growth concludes. Subsequently, $\frac{\partial P}{\partial a}$ will display a decreasing pattern after the fracture process zone reaches full development. This interpretation is illustrated in Figure 2, where the potential variation of stiffness degradation rate with crack length is depicted.
Therefore, it is reasonable to assert that the fracture process zone achieves full development when the condition $\frac{\partial\left(\frac{\partial P}{\partial a}\right)}{\partial a} = 0$ is met at $a = l_f$ (Figure 2).

Considering energy conservation, the external work done W_E caused due to the application of load will be equal to the internal energy offered by the material.

$$W_E = W_I \tag{24}$$

In the above expression presented in Equation 24, W_E and W_I are substituted from the expression Equation 22 and Equation 19 respectively. Further, considering the minimum load amplitude $P_{min} = 0$ and $P_{max} = P$, $\Delta P = P$, and using $\frac{\partial\left(\frac{\partial P}{\partial a}\right)}{\partial a} = 0$, Equation 24 has been solved.

The solution to Equation 24 yields the formulation for the length of the fracture process zone, l_{fpz} in terms of the size of the structure.

$$l_f = 0.58 \times D \tag{25}$$

The above, formulation produces the linear relation between the fully developed fracture process zone length and the specimen size due to the ignorance of material heterogeneity, which is the future scope of this study. In the above theoretical framework, material heterogeneity has not been considered, which is the primary reason of quasi-brittle nature in concrete, and provides non-linearity, and eventually leads to specimen size effect.

5 RESULT : ENERGY RELEASE RATE

The energy release/dissipation rate G can be expressed as the total work and the produced cracked area ratio. During the stable state of crack propagation, the total work done in the damaged zone will equal to the internal energy W_I induced by cohesive forces. Therefore, the energy release rate G will be the ratio of W_I and the produced crack area Ba. Figure 3, is showing the variation of energy release rate with crack propagation. It may noted, the rate of energy dissipation decreases with increase in crack length, which is the stable state of crack propagation. Once it the crack length reaches the fully developed fracture process zone length, the energy dissipation becomes constant, and unstable crack propagation takes place.

$$G = \frac{W_I}{Ba} = f_4(E, f_t, D, a, l_f) \qquad (26)$$

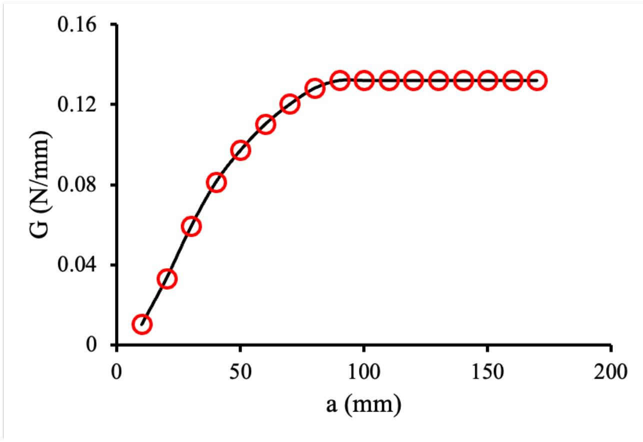

Figure 3: Variation of energy release rate with crack propagation

Further, the critical energy dissipation rate G_c corresponds to the critical state, i.e., when the damage length reaches the maximum fracture process zone length, l_f and the state of crack propagation changes from stable to unstable [7]. Therefore, G_c can be obtained by substituting crack length a with fully developed FPZ length l_f in Equation 26.

$$G_c = f_5(E, f_t, D, l_f) \qquad (27)$$

It may be observed (Equation 27) that the critical energy parameter depends on the material properties, E, f_t, and geometric property D. Here, l_f is the maximum fracture process zone length in cyclic loading conditions.

The above expression, Equation 27, has been developed for an un-notched section; so, for a notched section, specimen size D in the above expression will be replaced by the ligament length $D_l(= D - a_0)$, where a_0 is the initial notch length.

6 CONCLUSION AND FUTURE SCOPE

In this study, a theoretical formulation has been developed to predict energy dissipation with the load cycle, and the fully developed fracture process zone has been expressed analytically. It shows the rate of energy dissipation decreases with an increase in crack length, and it becomes constant when the crack propagation achieves a fully developed fracture process zone length. The proposed expressions consider the material properties such as f_t and E along with the specimen size, which has been further extended to predict the critical energy dissipation when unstable crack propagation starts. The above formulation does not account the influence of material heterogeneity, which is crucial, and it is the future scope of this study.

REFERENCES

[1] Zdeněk P Bažant and Mohammad T Kazemi. Size effect in fracture of ceramics and its use to determine fracture energy and effective process zone length. *Journal of the American Ceramic Society*, 73(7):1841–1853, 1990.

[2] Zdenek P Bazant and Kangming Xu. Size effect in fatigue fracture of concrete. *ACI Materials Journal*, 88(4):390–399, 1991.

[3] Sonali Bhowmik and Sonalisa Ray. An experimental approach for characterization of fracture process zone in concrete. *Engineering Fracture Mechanics*, 211:401–419, 2019.

[4] Arne Hillerborg, Mats Modéer, and P-E Petersson. Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements. *Cement and concrete research*, 6(6):773–781, 1976.

[5] K Keerthana and J M Chandra Kishen. Micromechanics of fracture and failure in concrete under monotonic and fatigue loadings. *Mechanics of Materials*, page 103490, 2020.

[6] K Keerthana and JM Chandra Kishen. An experimental and analytical study on fatigue damage in concrete under variable amplitude loading. *International Journal of Fatigue*, 111:278–288, 2018.

[7] Subramaniam V Kolluru, Edward F O'Neil, John S Popovics, and Surendra P Shah. Crack propagation in flexural fatigue of concrete. *Journal of Engineering mechanics*, 126(9):891–898, 2000.

[8] Bineet Kumar and Sonalisa Ray. A multi-scale based fracture characterization in concrete under fatigue loading using critical energy dissipation. *International Journal of Fatigue*, 165:107165, 2022.

[9] Bineet Kumar and Sonalisa Ray. Nanomechanics based crack growth characterization of concrete under fatigue loading. *Procedia Structural Integrity*, 39:222–228, 2022.

[10] Bineet Kumar and Sonalisa Ray. An analytical approach for the fracture characterization of concrete under cyclic loading. *Materials Today: Proceedings*, 2023.

[11] Jia-Liang Le, Jonathan Manning, and Joseph F Labuz. Scaling of fatigue crack growth in rock. *International Journal of Rock Mechanics and Mining Sciences*, 72:71–79, 2014.

[12] Dongyang Li, Peiyan Huang, Zhanbiao Chen, Guowen Yao, Xinyan Guo, Xiaohong Zheng, and Yi Yang. Experimental study on fracture and fatigue crack propagation processes in concrete based on dic technology. *Engineering Fracture Mechanics*, 235:107166, 2020.

[13] Koji Otsuka and Hidehumi Date. Fracture process zone in concrete tension specimen. *Engineering Fracture Mechanics*, 65(2-3):111–131, 2000.

[14] Xiangqian Fan Xudong Chen, Jingwu Bu. Effect of loading frequency and stress level on low cycle fatigue behavior of plain concrete in direct tension. *Construction and Building Materials*, 2017.

[15] J Zhang and H Stang. Applications of stress crack width relationship in predicting the flexural behavior of fibre-reinforced concrete. *Cement and Concrete research*, 28(3):439–452, 1998.

[16] Jun Zhang, Victor C Li, and Henrik Stang. Size effect on fatigue in bending of concrete. *Journal of materials in civil engineering*, 13(6):446–453, 2001.

[17] Xiao Xin Zhang, C Yu Rena, Gonzalo Ruiz, Manuel Tarifa, and Miguel A Camara. Effect of loading rate on crack velocities in hsc. *International Journal of Impact Engineering*, 37(4):359–370, 2010.