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DETERMINATION OF K^S_{Ic} AND $CTOD_c$ FROM PEAK LOADS AND RELATIONSHIP BETWEEN TWO-PARAMETER FRACTURE MODEL AND SIZE EFFECT MODEL

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Abstract
A simple method, named peak-load method, is proposed for determination of material fracture parameters K^S_{Ic} and $CTOD_c$ of the two-parameter fracture model (TPFM) from the peak loads of three or more distinct specimens. This method is verified by beam bending and split tension tests. A quantity called specimen distinction number is proposed to quantitatively distinguish specimens. The larger the differences in specimen distinction number among all the test specimens, the more confident the results obtained by the method. By using the peak-load method, the peak loads measured in previous fracture tests for applying the size effect model (SEM) can be used to determine K^S_{Ic} and $CTOD_c$. The relationship between K^S_{Ic} and $CTOD_c$ in TPFM and G_f and c_f in SEM is established. The equivalency between the fracture models can provide appreciable flexibility for research and engineering practice.

1 Introduction

The two-parameter fracture model (TPFM) (Jenq and Shah, 1985) introduces two material fracture parameters K^S_{Ic} and $CTOD_c$ to characterize the fracture properties of concrete. A three-point bend beam has been suggested for measuring values of K^S_{Ic} and $CTOD_c$ by RILEM (1990). In the test the initial compliance and the compliance at the peak load must be measured. Therefore a closed-loop testing machine is required to record load versus crack mouth opening displacement relation in loading and unloading.

However, a closed-loop test system may not be available in many laboratories and worksites. Thus Tang, Ouyang and Shah (1995) have proposed a simple procedure to determine K^S_{Ic} and $CTOD_c$ from peak loads of three or more distinct specimens. This method is named peak-load method to distinguish it from the test method recommended by RILEM, which is called compliance method. In this paper, a quantity is proposed as a measure of confidence level for K^S_{Ic} and $CTOD_c$ determined by the peak load method. Based on it, another quantity called specimen distinction number is proposed to distinguish specimens in the sense of the proposed method. With the peak-load method, it becomes possible to use existing data of the measured peak loads for applying another popular fracture model of concrete, the size effect model (SEM) (Bažant and Kazemi, 1990), to determine K^S_{Ic} and $CTOD_c$ and examine the relationship between the two fracture models. Ouyang, Tang and Shah (1995) have studied tensile specimen with a center crack and three-point bend beam and proved that K^S_{Ic} and $CTOD_c$ of TPFM and G_f and c_f of SEM can be converted from each other with no significant numerical differences.

2 Peak-load method for two-parameter fracture model

According to TPFM, the critical stress intensity factor K^S_{Ic} and the critical crack tip opening displacement $CTOD_c$ uniquely determine the critical nominal stress σ_{Nc} and the effective critical crack length a_c. The failure criterion of a concrete structure is then simultaneous satisfaction of the following two equations:

$$K_I(\sigma_{Nc}, a_c) = K^S_{Ic} \, , \tag{1}$$

$$CTOD(\sigma_{Nc}, a_c) = CTOD_c \tag{2}$$

where calculation on the left side of Eqs. (1) and (2) can be expressed through an analytical or numerical solution based on linear elastic fracture mechanics (LEFM). Solution of K^S_{Ic} and $CTOD_c$ from measured σ_{Nc} values through Eqs. (1) and (2) is the basis of the peak-load method. If one applies Eqs. (1) and (2) to two different specimens, the following four equations are obtained:

$$K_I^1(\sigma_{Nc}^1, a_c^1) = K_{Ic}^S \tag{3}$$

$$CTOD^1(\sigma_{Nc}^1, a_c^1) = CTOD_c \tag{4}$$

$$K_I^2(\sigma_{Nc}^2, a_c^2) = K_{Ic} \tag{5}$$

$$CTOD^2(\sigma_{Nc}^2, a_c^2) = CTOD_c \tag{6}$$

where superscripts 1 and 2 denote two different specimens. If critical nominal stresses σ^1_{Nc} and σ^2_{Nc} for two different specimens are measured, Eqs. (3) to (6) provide four simultaneous equations for four unknowns of a^1_c, a^2_c, K^S_{Ic}, and $CTOD_c$. To avoid mathematical difficulty, an efficient procedure for determining K^S_{Ic} and $CTOD_c$, in conjunction with a^1_c and a^2_c, is suggested subsequently.

With σ^1_{Nc} known, Eqs. (3) and (4) can be rewritten as

$$K_{Ic}^S = K_I^1(a_c^1) \ , \tag{7}$$

$$CTOD = CTOD(a_c^1) \ . \tag{8}$$

Likewise, Eqs. (5) and (6) can be rewritten similarly. Combination of Eqs. (7) and (8) can be considered the parametric form of a function for $CTOD_c$ in terms of K^S_{Ic} with a^1_c as the parameter for specimen 1. The Cartesian form of the function is

$$CTOD_c = f^1(K^S_{Ic}) \ . \tag{9}$$

A similar equation can be obtained for specimen 2. If tests of two different specimens were conducted with "exact" measurement of σ_{Nc} and specimen dimensions, the intersection point of the two $CTOD_c$ - K^S_{Ic} curves would be the solution.

Because of randomness of concrete properties and errors in measurement, at least three different specimens should be tested for statistics data modeling. Based on the $CTOD_c$ - K^S_{Ic} curve of each specimen, an average of $CTOD_c$ values can be calculated. Then solution of K^S_{Ic} and $CTOD_c$ can be found by applying the least-squares error criterion, that is, by minimizing the following function:

$$f(K^S_{Ic}) = \sum_{i=1}^{n} (CTOD_c^{avg} - CTOD_c^i)^2 \tag{10}$$

where n is the number of specimens, $CTOD^{avg}_c$ is the average value of $CTOD_c$ of all n specimens, and $CTOD^i_c$ is the $CTOD_c$ value for the ith specimen. In the function f in terms of K^S_{Ic}, the value of K^S_{Ic} at which the minimum f occurs is the solution of material parameter K^S_{Ic}, designated by K^S_{Ic}*. By substituting K^S_{Ic}* into the average $CTOD_c$ - K^S_{Ic} curve, the solution of $CTOD_c$, designated by $CTOD_c$*, is obtained. The symbol * is used to indicate the values determined by the peak-load method. The parameter K^S_{Ic} rather than $CTOD_c$ is chosen as the variable in a simple one-dimensional optimization, because of the fact that values of K^S_{Ic} obtained from different specimens of the same batch of concrete with the compliance method are very close to one another but values of $CTOD_c$ show considerable scattering (Karihaloo and Nallathambi, 1991). Originally, the sample standard deviation, s, instead of f, was suggested to minimize (Tang et al., 1995). Since $f = (n - 1)s^2$ with n as a constant for a set of specimens, results of K^S_{Ic}* and $CTOD_c$* obtained by minimizing f and s make no difference.

The peak-load method is based on the proposition that K^S_{Ic} and $CTOD_c$ are materials constants. Therefore the statistics modeling involved in the peak-load method is not to average the data scatter due to randomness of material properties and errors of measurements, but to select an average value of $CTOD_c$ whose variance is the least among all the possible values of $CTOD_c$ for different specimens. In order to use the peak-load method, the test specimens should be so different that the $CTOD_c^{avg}$ - K^S_{Ic} curve is not significantly disturbed by the random errors. In minimizing f for K^S_{Ic}* and $CTOD_c$*, these determined values are of higher confidence when Δf is higher for a given small ΔK^S_{Ic}. Thus, the following quantity β can be used to measure the confidence for K^S_{Ic}* and $CTOD_c$*:

$$\beta = \sum_{i=1}^{n} (k^{avg} - k^i)^2 \tag{11}$$

$$k = \left[\frac{dCTOD_c}{dK^S_{Ic}}\right]_{K^S_{Ic}=K^S_{Ic}*} \tag{12}$$

because $\Delta f = \beta(\Delta K^S_{Ic})^2$. The quantity β, equal to half the curvature of the f - K^S_{Ic} curve at the minimum f, is named confidence level for K^S_{Ic} and $CTOD_c$ determined by the peak-load method. Thus specimens can be approximately distinguished by the value of k, in the sense of the peak-load method. The quantity k is named specimen distinction number. By comparing LEFM formulas for K_I and $CTOD$, one can find how k depends on specimen geometry, specimen size and notch length. With these two proposed quantities, specimens can be well designed so as to enhance the confidence for K^S_{Ic} and $CTOD_c$ determined by the peak-load method.

3 Experimental verification of peak-load method

To verify the proposed peak-load method, eight three-point bend beams and six split tension cylinders from the same batch of mortar mix were prepared and tested. Proportion of water, cement and aggregate in the mix was 1 : 2.5 : 5. The maximum size of the aggregate was 3 mm. The notch in the beam specimen was formed by sawing, whereas it was precast for cylinder specimens. All the beams were 50.8 mm deep and 25.4 mm wide with the ratio of the support span to the specimen depth of 4:1. All the cylinder specimens were 102 mm in diameter and 31.8 mm in thickness. Other dimensions of the specimens are presented in Tables 1 and 2.

All the beams were tested according to the compliance method to obtain K^S_{Ic} and $CTOD_c$, which are shown in Table 1 along with values of the peak load P_c. The average values of K^S_{Ic} and $CTOD_c$ from all the beam tests but the last test in Table 1 are 0.883 MPa·m$^{1/2}$ and 0.00754

Table 1. Dimensions of three-point bend beams and values of K^S_{Ic} and $CTOD_c$ obtained by the compliance method

group	a_0 (mm)	P_c (N)	K^S_{Ic} (MPa • m$^{1/2}$)	$CTOD_c$ (mm)	k (10^{-4} MPa^{-1} • m$^{1/2}$)
1	10.2	784.1	0.911	0.0103	0.290
		717.2	0.768	0.0073	
2	17.8	547.5	0.827	0.0073	0.281
		535.0	0.787	0.0065	
		653.4	1.033	0.0102	
3	25.4	430.9	0.894	0.0050	0.241
		387.0	0.962	0.0062	
		450.7	0.740	0.0022	

Table 2. Dimensions and peak loads of split tension cylinders

specimen group	notch length $2a_0$ (mm)	P_c (kN)	k (10^{-4} MPa$^{-1}\cdot$m$^{1/2}$)
1	10.2	23.6	0.171
2	17.8	23.3	0.204
		19.8	
3	25.4	18.0	0.230
		17.3	
		17.1	

mm, respectively. Data of the last test are not included in the average because $CTOD_c$ from the test deviates largely from the average. The elastic modulus obtained from these beam tests was E = 27.8 GPa. In split tension of the cylindrical specimens, only the peak loads were recorded (Table 2). Wooden load-bearing strips used in the tests were 10 mm wide.

The specimens were grouped according to the notch length. Based on the average peak load of each specimen group the $CTOD_c$-K^S_{Ic}curve was established by using LEFM formulas for the beam (Jenq and Shah, 1985) and the cylinder (Tang, 1994). Then the f-K^S_{Ic} curve was constructed. Figs. 1 and 2 show the $CTOD_c$-K^S_{Ic}curves and the f-K^S_{Ic}

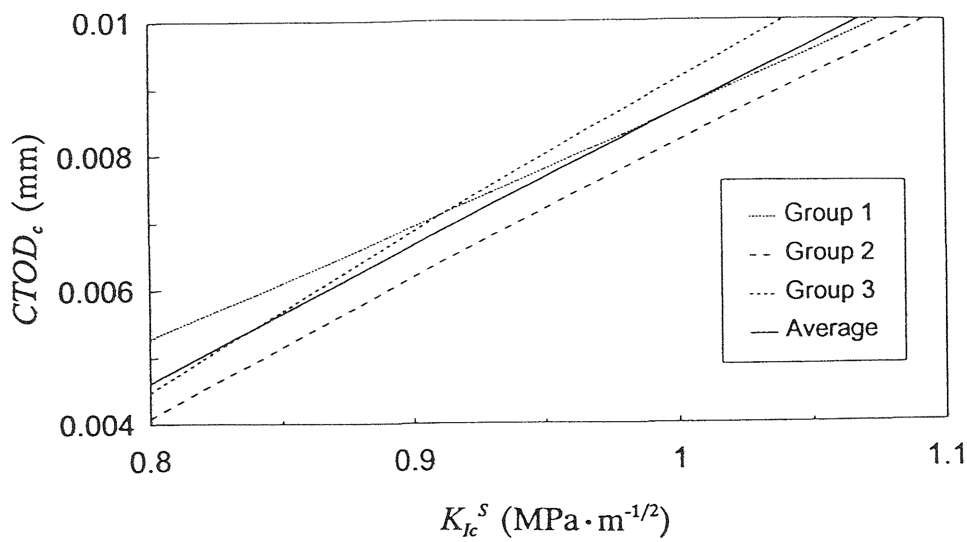

Fig. 1. K^S_{Ic} - $CTOD_c$ curves for distinct groups of split tension cylinders

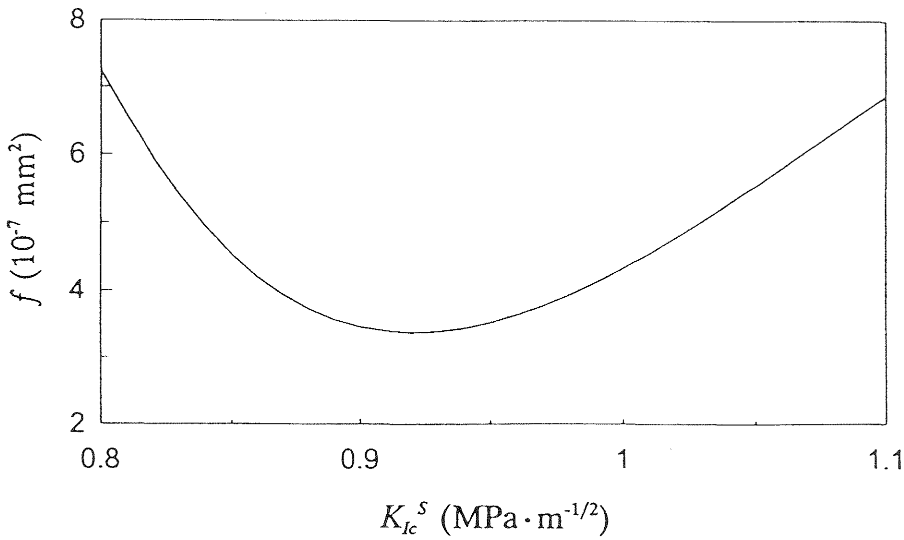

Fig. 2. f - K_{Ic}^S relation for split tension cylinder

relation for the split tension cylinders, respectively. The values of K_{Ic}^{S*} and $CTOD_c$* obtained from either and both of beams and cylinders, with the value of β, are shown in Table 3. The value of k for each specimen group at the value of K_{Ic}^{S*} obtained from the beams and from the cylinders, respectively, is shown in Tables 1 and 2. It is seen that different notch lengths make different values of k for the specimen of the same shape and same size. It is also seen that the split tension cylinder provides apparently different k values than the bend beam and therefore combination of these two types of specimens enhances the confidence for the values of K_{Ic}^S and $CTOD_c$ determined.

Table 3. Values of K_{Ic}^S and $CTOD_c$ obtained from three-point bend beams and split tension cylinders

test method	specimens	K^S_{Ic} (MPa · m$^{1/2}$)	$CTOD_c$ (mm)	β (10^{-4} GPa^{-2} · m)
compliance	beams	0.883	0.00754	N/A
peak load	beams	0.929	0.00820	0.135
peak load	cylinders	0.921	0.00709	0.177
peak load	beams and cylinders	0.881	0.00739	1.40

4 Relationship between TPFM and SEM

By applying both the two-parameter fracture model (TPFM) and size effect model (SEM) to the infinitely large specimen with the infinitely-long initial crack, the following relationship can be obtained:

$$G_f = \frac{(K_{If})^2}{E'} = \frac{(K_{Ic}^{s})^2}{E'} \tag{13}$$

$$CTOD_c = \frac{K_{If}}{E'}\sqrt{\frac{32c_f}{\pi}} = \sqrt{\frac{32G_f c_f}{E'\pi}} \tag{14}$$

where $E' = E$ for plane stress and $E' = E/(1-\nu^2)$ for plane strain, E is Young's modulus, and ν is Poisson's ratio. Since the initial crack, a_0, is usually related to the material defect, the length of the fracture process zone may not be small compared to the material defect even for a very large specimen. Therefore a more general relationship between $CTOD_c$ and c_f needs to be considered.

In the tensile specimen with a center crack, if the specimen is infinitely large, the crack opening displacement (COD) can be expressed as follow,

$$COD = \frac{4\sigma_f}{E'}\sqrt{a^2-x^2} \tag{15}$$

Denoting the critical crack length $a_c = a_0 + c$, one can obtain the process zone length:

$$c = \frac{E'\pi CTOD_c^2}{32G_f} + \sqrt{a_0^2 + \left[\frac{E'\pi CTOD_c^2}{32G_f}\right]^2} - a_0 \tag{16}$$

In the three-point bend beam, the formulas based on the finite element results (Jenq and Shah, 1985) leads to

$$c = \sqrt{0.292a_0 + 0.057\frac{E'CTOD_c^2 a_0}{G_f} + 0.0038\left[\frac{E'CTOD_c^2}{G_f}\right]^2} + \frac{0.061E'CTOD_c^2}{G_f} - 0.540a_0 \tag{17}$$

When the specimen is very large, the boundary does not restrict the development of the process zone, the process zone length in Eqs. (16) and (17) should be equal to c_f. These equations were verified with the data from Bažant and Pfeiffer (1987). The results are shown in Fig. 3, where K_{Ic}^S and $CTOD_c$ of TPFM were obtained by the peak-load method, c_f was obtained by the size effect method, and all the marks represent the process zone length calculated with Eq. (16) or (17). All the values of the process zone lengths are quite close to one another. In other words, the two fracture models are equivalent to each other.

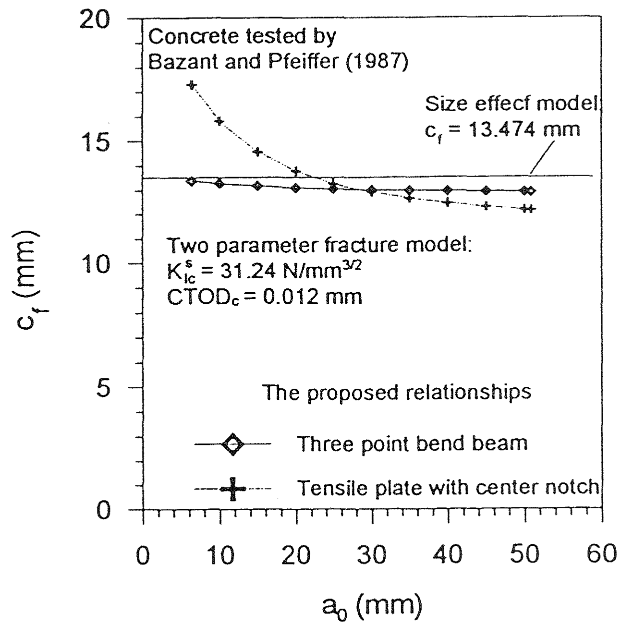

Fig. 3. Values of c_f by proposed relationships

5 Conclusions

The simple method of determining K_{Ic}^S and $CTOD_c$ from the measured peak load of distinct specimens is a reliable method. The values of K_{Ic} and $CTOD_c$ obtained by the peak-load method and obtained by the method recommended by RILEM match each other very well. By using the specimen distinction number and the confidence level as the criterion, specimens can be well designed to achieve confident results. Relationship between the fracture parameters of the two-parameter fracture model and the size effect model is studied. As concluded, these parameters of the different models can be derived from each other. The two models are equivalent.

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