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A COMPRESSIVE SOFTENING MODEL FOR CONCRETE

G. Markeset,
Norwegian Defence Construction Service, Technical Division, Test and Development Section, Oslo, Norway

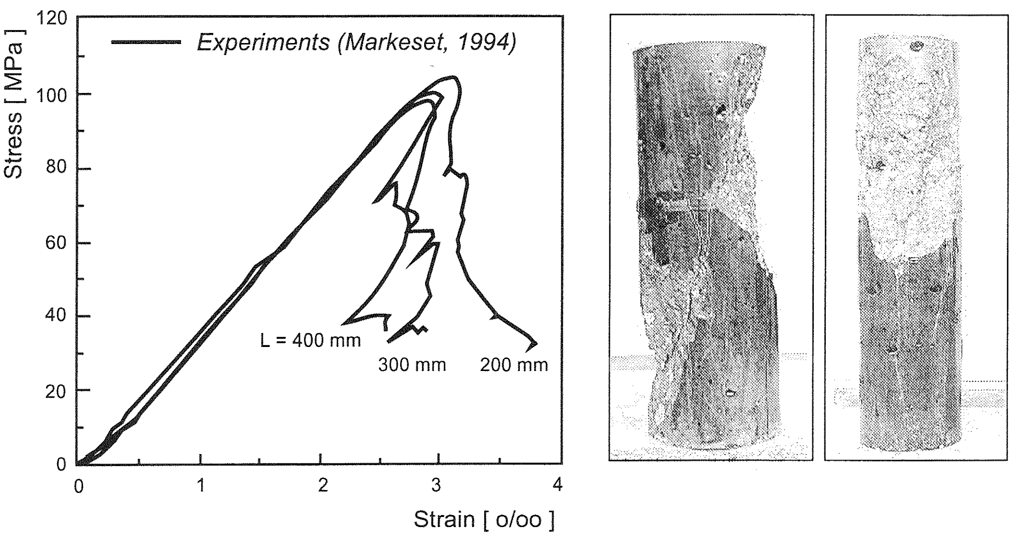
Abstract
This paper presents a constitutive macromechanical model for concrete in compression. The model, named the Compressive Damage Zone (CDZ) model, is based on the hypothesis of compressive failure within a zone of limited length. In this damage zone, the failure mode is a combination of distributed axial splitting and localized deformation. Within the damage zone the post-peak behaviour is described by means of two curves, one related to the distributed splitting cracking and one to the localized deformation. The compressive strain caused by the axial splitting cracks is assumed to be proportional to the tensile fracture energy G_F. It is shown through compressive tests of specimens of different length and width that the proposed failure hypothesis is realistic.

1 Introduction

Material models for compressive failure of concrete are normally based on uniaxial compressive stress-strain curves obtained from tests, where uniform deformation of the concrete specimens is assumed. While this assumption is reasonable for the ascending branch of the stress-strain curve, this is not the case for the descending branch. In fact, experimental studies have shown that the deformation after peak stress is more or less localized to certain zones (Van Mier (1984), Rokugo and Koyanagi (1992), Vonk (1993), Markeset (1994)). The measured strain will then depend on the position and the length of the strain gauge. The descending branch of the stress-strain curve becomes thus size dependent. The influence of specimen length on the complete stress-strain curves of high strength concrete cylinders with diameter 100 mm is shown in Fig. 1a).

Compressive failure is always accompanied by lateral deformations. The lateral deformations have to do with longitudinal distributed splitting cracks, which form and expand during the failure process. In addition to the longitudinal splitting cracks localized shear deformations may also occur, (Fig. 1b).

In contrast to the compressive softening models proposed by Hillerborg (1988) and Bazant (1989), the model proposed by Markeset (1993), takes both the localized shear deformation and the deformation due to splitting cracks into account. This gives a model consisting of three curves and introducing the tensile fracture energy in the splitting cracks as an important parameter. This model, named the Compressive Damage Zone model, is described in the next section.



a) b)

Fig. 1. Behaviour of high stength concrete cylinders (diameter 100 mm) a) Influence of specimen length on the complete stress-strain curve and b) Formation of shear band and damage zones

2 The Compressive Damage Zone model

2.1 Phenomenological basis for the model

In the Compressive Damage Zone (CDZ) model (Markeset (1993)), it is assumed that failure of a relatively slender concrete specimen takes place within a damage zone of limited length L^d, whereas the parts outside this zone are unloaded during failure. The deformation of the specimen under centric compression is described by means of three curves shown in Fig. 2.

The first curve is the stress-strain curve for the material loaded up to the compressive strength f_c and then unloaded. This curve is valid for the concrete material in the whole specimen.

The second curve shows the relationship between the stress and the average additional strain ε_d within the damage zone, related to the formation of longitudinal cracks and a corresponding additional lateral strain within this damage zone.

The third curve is a stress-deformation curve, related to localized deformations. These deformations are in Fig. 2 illustrated as shear deformations in an inclined shear band, but they may also take place in other ways.

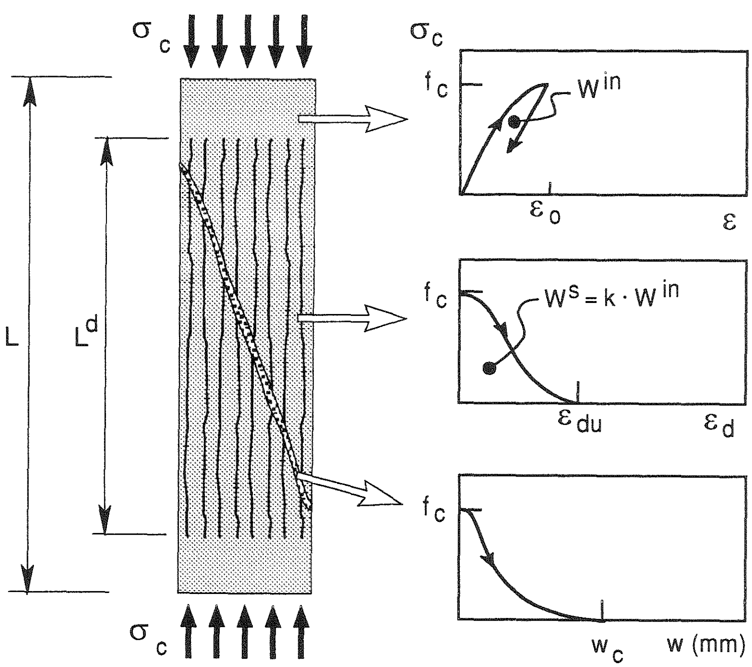


Fig. 2. Illustration of the CDZ model on a specimen loaded in uniaxial compression

The most fundamental new concept in the CDZ model is the introduction of the second curve in Fig. 2. The formal description given by this curve is of course very approximate, as the additional strain here is described as constant within the damage zone and zero outside this zone. The same formality holds also for the corresponding additional lateral strain. In reality the lateral strain as well as the longitudinal strain can be expected to have a maximum at the centre of the damage zone and to decrease towards the ends of this zone. In spite of this approximation, the curve is considered to give a reasonable overall description of the behaviour.

The lower curve in Fig. 2 starts from $\sigma = f_c$, implying that the localized deformation is assumed to take place at the same time as the damage zone starts to develop. It is, however, likely that this localization does not take place until some of the strain ε_d has developed. This needs to be investigated further. So far the approach shown in Fig. 2 has been applied.

2.2 The complete stress-strain curve

The total deformation of the specimen shown in Fig. 2 is

$$\Delta L = \varepsilon L + \varepsilon_d L^d + w \tag{1}$$

and the corresponding average strain then becomes:

$$\varepsilon_m = \varepsilon + \varepsilon_d \frac{L^d}{L} + \frac{w}{L} \tag{2}$$

This equation, which is valid for L greater than L^d, is illustrated in Fig. 3. For a specimen of length L equal to or less than the damage zone length L^d the average strain becomes:

$$\varepsilon_m = \varepsilon + \varepsilon_d + \frac{w}{L} \tag{3}$$

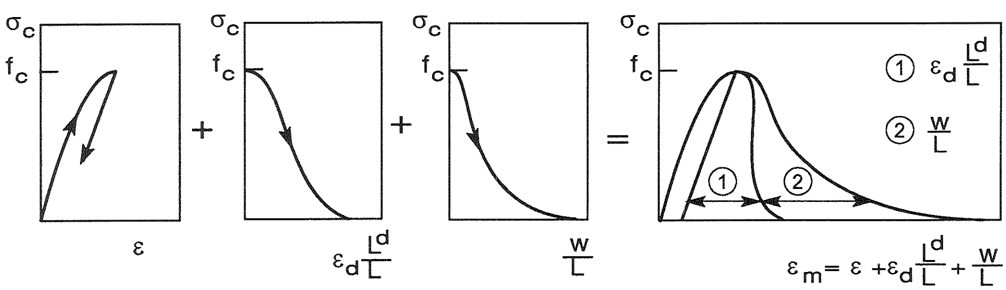


Fig. 3. Composition of the complete stress-strain curve

2.3 Influence of tensile fracture energy G_F

The development of the longitudinal cracks is accompanied by energy absorption. The complete opening of one such crack may be assumed to absorb the same amount of energy as the opening of a pure tensile crack. This absorbed fracture energy per unit crack area is denoted G_F (Hillerborg et al. (1976)).

The development of longitudinal microcracks starts before the strength f_c is reached. These microcracks are assumed to cause the inelastic strain corresponding to the upper stress-strain curve in Fig. 2. However, only a part of the fracture energy is consumed in this process. The remaining fracture energy is consumed when the microcracks coalesce and form the open longitudinal cracks at final failure. This part of the fracture energy thus belongs to the second stress-strain curve in Fig. 2. The energy absorptions per unit volume in the two fracture processes are denoted W^{in} and W^{s}, respectively, which correspond to the areas below the two stress-strain curves as shown in Fig. 2.

The total absorbed energy in the longitudinal cracks is assumed to be proportional to the tensile fracture energy G_F. This assumption may be written as:

$$W^{in} + W^{s} = \frac{G_F}{r} \tag{4}$$

where r is a parameter with the dimension of length. Most likely, r is proportional to the average distance between successive longitudinal cracks, as more cracks per unit volume will give a higher energy absorption per unit volume.

It has been further assumed that W^{s} is proportional to W^{in}, that is:

$$W^{s} = k\, W^{in} \tag{5}$$

in which k is a material constant.

Eqs (4) and (5) yield:

$$W^{in} = \frac{G_F}{r(1+k)} \tag{6}$$

If we know the material parameters k and r and the conventional material parameters f_c, E_c and G_F we can determine the two stress-strain curves in Fig. 2. This requires assumptions regarding the shape of the curves, as W^{in} and W^{s} are the areas below these curves.

3 Discussion

3.1 Material parameters

The values of the material parameters used in the CDZ model have been estimated by means of test results from the literature as well as the experiments performed in Markeset (1993). The main parameters are, in addition to the more conventional parameters f_c, E_c and G_F:

- r - Material property related to the distance between successive splitting cracks (mm)
- k - Ratio between energy consumed due to opening of splitting cracks in the post-peak and pre-peak region, respectively
- w_c - Localized deformation (mm)
- L^d - Length of the damage zone (mm)

The value of r is found to be about 1.25 mm for d_{max}=16 mm. Most likely, r increases somewhat with increasing maximum aggregate size.

The value of k is approximately 3 for normal density concrete and 1 for light weight aggregate concrete.

The parameter w_c is a localized deformation, for instance caused by frictional restraint in a shear band. The value of w_c is sensitive to whether the cracks run through or around the aggregate particles. It is found to vary between 0.4 and 0.7 mm for normal density concrete. For light weight aggregate concrete the value of w_c is found to be less than 0.3 mm.

For a specimen exposed to centric compression, the length of the damage zone L^d seems to be of order 2.0 and 2.5 times the smallest lateral dimension (Sangha and Dhir (1972), Kotsovos (1983), Markeset (1994)).

3.2 Size effect in compressive softening

To illustrate the applicability of the CDZ model some uniaxial compression tests taken from Markeset (1994) and Vonk (1993) are simulated by the model, (Figs. 5 and 6). In these calculations a linear softening description is applied as illustrated in Fig. 4. Further, the length of the damage zone L^d is taken as 2.5 times the width of the specimen. This means that for the specimens with slenderness ratio less than 2.5, the average strain in the post-peak region is calculated according to Eq (3), and for the specimens with slenderness ratios higher than 2.5 the average strain is calculated using Eq (2).

The CDZ model is in Fig. 5 compared to the experimental stress-strain curves shown in Fig. 1, in which the curves were obtained on concrete cylinders with diameter 100 mm and lengths 200, 300 and 400 mm, respectively. The fracture energy for the concrete was found to be about 0.15 N/mm. The input parameters to the CDZ model are: k=3, r=1.25 mm, G_F=0.15 N/mm, w_c=0.4 mm.

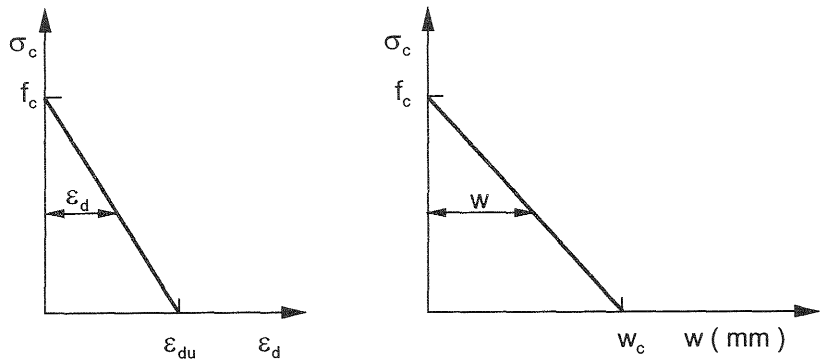


Fig. 4. Linear softening description

The influence of length and width of the concrete prisms on the post-peak energy per unit area is illustrated in Fig. 6. The fracture energy G_F of the concrete was indicated by Vonk (1993) to be about 0.12 N/mm. The concrete strength varied between 40 and 50 MPa. The input parameters to the CDZ model are: k=3, r=1.25 mm, G_F=0.12 N/mm, f_c=45 MPa, w_c=0.4 mm. The tests show that the post-peak energy per unit area increases with increasing specimen length. However, as indicated by the CDZ model, this energy becomes independent of the specimen length when the slenderness is greater than about 2.5.

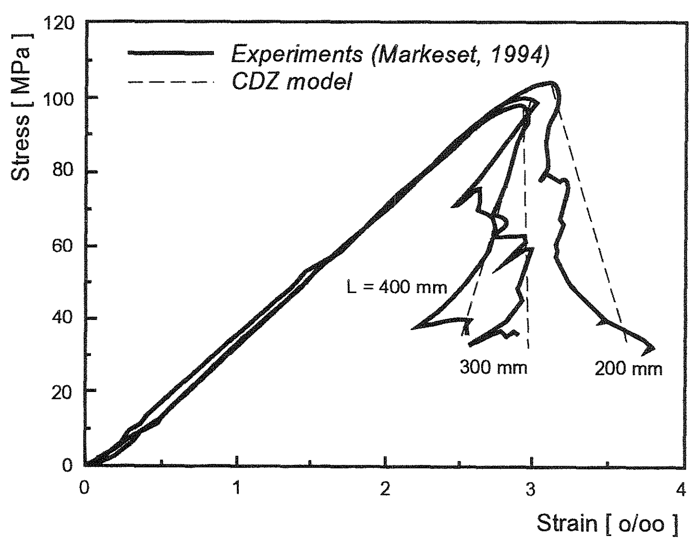


Fig. 5. Influence of specimen length on the stress-strain curve

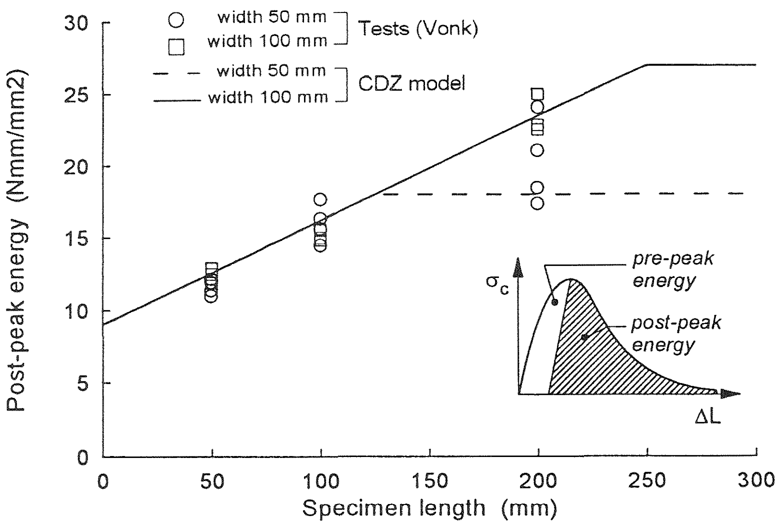


Fig. 6. Size effect in the post-peak energy (Markeset (1995))

4 Conclusions

A compressive softening model for concrete is proposed. The Compressive Damage Zone (CDZ) model, is based on a hypothesis that the compressive failure occurs within a zone of limited length. The post-peak behaviour within this zone is described by means of two curves, one related to distributed axial splitting cracking, and one to localized deformation. Comparison with test data shows that the model can describe the influence of specimen length and width on compressive softening.

According to the CDZ model the steepness of the descending branch of a formal stress-strain curve will:

- increase with increasing compressive strength
- increase with decreasing fracture energy (G_F)
- increase with increasing specimen length
- increase with increasing specimen slenderness

The material parameters applied in the CDZ model are yet uncertain. Further tests are therefore needed to increase the knowledge of material parameters and to check the validity of the CDZ model. One important aspect is to investigate when the localization of deformation takes place, whether it starts prior to, at peak or after peak stress.

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