Fracture Mechanics of Concrete Structures,
Proceedings FRAMCOS-2, edited by Folker H. Wittmann,
AEDIFICATIO Publishers, D-79104 Freiburg (1995)

SOME CONSIDERATIONS ON EXPLICIT DAMAGE MODELS INCLUDING CRACK CLOSURE EFFECTS AND ANISOTROPIC BEHAVIOUR

A. Rouquand and C. Pontiroli
D.R.E.T./ E.T.C.A./Centre d'Etudes de Gramat, France

Abstract

Under severe mechanical loading brittle material like concrete can be described favourably using damage models. J. Mazars (1984), S. Ramtani (1990) and C. Laborderie (1991) propose such models established within the framework of thermodynamics. Internal state variables are used to describe cracking or micro cracking process. The number of internal state variables depends on the level of sophistication. Crack closure effect, inelastic strains and induced anisotropic damage can be introduced in the constitutive equations. In order to solve in a more efficient way dynamic engineering problems involving complex two or three dimensional structures, damage model with explicit formulation are recommended. Computation time can be therefore reduced drastically.

Here we present two explicit damage models. The first one uses two scalar variables and takes into account permanent strains. This model is introduced in a finite element program and gives interesting results. The second damage model uses a scalar damage variable for micro cracking effects and a second order tensor variable to model anisotropic tensile damage. Inelastic strains are also introduced. In their respective principal axes inelastic strains and tensile damage variables can be represented with an ellipsoid shape. This model is still under development.

Thermodynamic aspects are presented and the first evolution laws for damage variable are discussed and the model response is presented with fixed load axes.

1 Introduction

In order to determine the vulnerability of buried or half buried reinforced concrete structures submitted to blast wave loading or ground shock loading, the Centre d' Etudes de Gramat (C.E.G.) develops explicit damage models for geologic materials. The purpose of such models is to describe behaviour of reinforced concrete or rock media under severe mechanical loading.

Damage models can be used favourably to simulate the physical mechanism encountered during failure of brittle materials like concrete or rock. Cracking process with the associated loss of stiffness is well describe using internal damage variables. Inelastic strains can also be introduced in the damage model from the same damage state variables. Such models already exist but for practical engineering problems an explicit formulation of the stress strain relation is able to increase drastically the numerical efficiency.

More over crack closure effect, inelastic strains, strain rate effects, and sometime anisotropic initial or induced damage, must be included in the constitutive relations for more realistic predictions. This paper gives some considerations about explicit damage models including such effects.

2 Two scalars explicit damage model

2.1 Stress-strain relation

The first explicit model is a two scalars damage model devoted to concrete behaviour under reverse loading. This model can be presented as an extension of the model developed by J. Mazars (1984). Two scalar damage variables are used instead of one. Figure 1 shows the response of the new model under a cyclic loading. As shown on this figure when the stress σ is greater than a closure stress σ_{ft} a tensile behaviour is obtained and D_t controls the stiffness. On the contrary when σ is lower than σ_{ft} a compressive response is simulated and D_c gives the relative magnitude of the stiffness reduction.

For a two or three dimensional problem, the stress strain relation is:

$$\begin{aligned} \underline{\underline{\sigma}} - \underline{\underline{\sigma}}_{ft} = {} & (1 - D_t)\, \alpha_t \left[\lambda_0 \mathrm{Trace}\,(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}_{ft})\, \underline{\underline{I}} + 2\, \mu_0\, (\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}_{ft}) \right] \\ & + (1 - D_c)\, \alpha_c \left[\lambda_0 \mathrm{Trace}\,(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}_{ft})\, \underline{\underline{I}} + 2\, \mu_0\, (\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}_{ft}) \right] \end{aligned} \quad (1)$$

λo and μo are the Lamé coefficients.

D_t and D_c are the two scalars damage variables.

$\underline{\underline{\sigma}}_{ft}$ is called the closure stress tensor. This stress state defines the transition state between tension and compression.

$\underline{\underline{\sigma}}_{ft}$ can be written as follow:

$$\underline{\underline{\sigma}}_{ft} = (1 - D_c)^2 \underline{\underline{\sigma}}_{ft0} \tag{2}$$

Then the closure stress decreases when compressive damage growth.

$\underline{\underline{\sigma}}_{ft0}$ is the initial closure stress tensor. $\underline{\underline{\varepsilon}}_{ft0}$ is the strain tensor associated to $\underline{\underline{\sigma}}_{ft0}$.

$\underline{\underline{\varepsilon}}_{ft}$ is the closure strain tensor that gives the strain state associated to $\underline{\underline{\sigma}}_{ft}$.

α_c and α_t are scalar values that give respectively the compressive and the tensile part of any loading.

The expression of α_t is obtained from the effective stress tensor $(\underline{\underline{\tilde{\sigma}}} - \underline{\underline{\tilde{\sigma}}}_{ft})$:

$$\underline{\underline{\tilde{\sigma}}} - \underline{\underline{\tilde{\sigma}}}_{ft} = \lambda_0 \mathrm{Trace}\,(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}_{ft})\,\underline{\underline{I}} + 2\mu_0\,(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}_{ft}) \tag{3}$$

also if $\mathrm{Trace}\,(\underline{\underline{\tilde{\sigma}}} - \underline{\underline{\tilde{\sigma}}}_{ft}) \geq 0 \quad \alpha_t = 1$

if $\mathrm{Trace}\,(\underline{\underline{\tilde{\sigma}}} - \underline{\underline{\tilde{\sigma}}}_{ft}) \leq 0 \quad \alpha_t = \dfrac{\left|\sum < \tilde{\sigma}_i >_+\right|}{\left|\sum < \tilde{\sigma}_i >_-\right|}$

α_c is deduced using the next equation:

$$\alpha_c + \alpha_t = 1 \tag{4}$$

From the strain tensor $\underline{\underline{\varepsilon}}$ and from a scalar ε_{fc} (see figure 1), it is possible to get the compressive damage variable D_c and its increment $\dot{D}_c$.

The evolution of damage D_c is controlled by an equivalent strain $\tilde{\varepsilon}_M$ related to the growth of micro crack opening in mode 1 (Mazars, 1984):

$$\tilde{\varepsilon}_M = \sqrt{\sum < \varepsilon_i >_+^2} \tag{5}$$

where ε_i are the principal strains of the tensor $\underline{\underline{\varepsilon}}$.

The closure strain increment $\dot{\underline{\underline{\varepsilon}}}_{ft}$ is obtain from the $\dot{D}_c$ value and from the strain tensor $\underline{\underline{\varepsilon}}$. Integration versus time of $\dot{\underline{\underline{\varepsilon}}}_{ft}$ gives $\underline{\underline{\varepsilon}}_{ft}$ which is not related to tensile stiffness as shown on figure 1.

Then the reversible strain tensor $(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}_{ft})$ allows computation of the tensile damage variable D_t which is controlled by an equivalent strain $\tilde{\varepsilon}$:

$$\tilde{\varepsilon} = \sqrt{\sum < (\varepsilon - \varepsilon_{ft})_i >_+^2} \tag{6}$$

where $(\varepsilon - \varepsilon_{ft})_i$ are the principal strains of the tensor $(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}_{ft})$.

All terms of equation (1) can be obtained directly without any iterative process, even with large strain tensor increment.

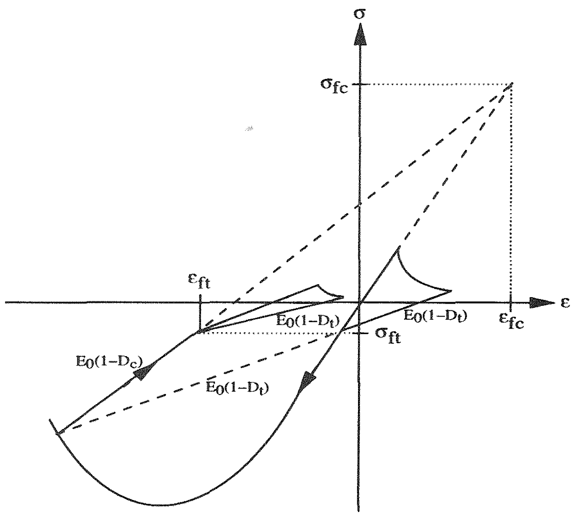

Figure 1. Cyclic behaviour of damage model under uniaxial loading

2.2 Thermodynamic aspects

Under the assumption of proportional loading, the new model can be established within the framework of thermodynamics. Table 1 shows selected state variables and their corresponding associated variables.

Table 1. State variables and associated variables (two scalars model)

State variables			**Associated variables**	
	Observable	Internal		
Strain	$(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}_{ft})$		$\underline{\underline{\sigma}}$	Stress
Tensile damage		D_t	Y_{Dt}	Energy release rate
Compressive damage		D_c	Y_{Dc}	

Still under the assumption of proportional loading (α_c and α_t are constant), the thermodynamic potential is given by :

$$\begin{aligned}\rho\psi = &\frac{1}{2}(1-D_t)\left[\lambda_0 \mathrm{Trace}^2(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}_{ft}) + 2\mu_0(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}_{ft}):(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}_{ft})\right] \\ &+\frac{1}{2}(1-D_c)\left[\lambda_0 \mathrm{Trace}^2(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}_{ft}) + 2\mu_0(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}_{ft}):(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}_{ft})\right] \qquad (7) \\ &+ (1-D_c)^2\left[\lambda_0 \mathrm{Trace}(\underline{\underline{\varepsilon}}_{ft0}) + 2\mu_0(\underline{\underline{\varepsilon}}_{ft0})\right]:(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}_{ft})\end{aligned}$$

Considering isothermal process, the Clausius Duhem relation that ensures positive dissipated energy is given by:

$$\underline{\underline{\sigma}} : \dot{\underline{\underline{\varepsilon}}} - \rho\dot{\psi} \geq 0 \tag{8}$$

As shown by C. Pontiroli (1995), for proportional loading the Clausius Duhem inequality is satisfied if $\dot{D}_t \geq 0$ and $\dot{D}_c \geq 0$. These two conditions are automatically satisfied because damage variables are strictly increasing variables.

3 Anisotropic and unilateral explicit damage model

3.1 Stress-strain relation

It is clear that cracking process introduces a strong anisotropy in material behaviour. Some materials like rocks have an initial anisotropy resulting from a layered aspect of most geologic media. For complex loading with rotation of the load axes the anisotropy should be accounted. Such kind of behaviour can induce significant effect in the material response. Here we present the first developments of an unilateral and anisotropic damage model. As for the previous one, we adopt an explicit formulation in order to preserve, as much as possible, a numerical efficiency. The physical mechanism taking into account are similar to mechanism modelled in the previous two scalars damage model. Here we introduce an anisotropic tensile damage variable $\underline{\underline{D}}_t$ in the form of a second order damage tensor. The stress strain relation of the anisotropic model is given by:

$$\begin{aligned}\underline{\underline{\sigma}} - \underline{\underline{\sigma}}_{ft} &= (1 - D_c)\left[\lambda_0 \mathrm{Trace}\,(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}_{ft})\,\underline{\underline{I}} + 2\mu_0\,(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}_{ft})\right] \\ &- \left[\begin{array}{l}\lambda_0 \mathrm{Trace}\,(\underline{\underline{D}}_t - D_c\underline{\underline{I}})^{1/2}\,(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}_{ft})\,(\underline{\underline{D}}_t - D_c\underline{\underline{I}})^{1/2}\,\underline{\underline{I}} \\ + \; 2\mu_0\,(\underline{\underline{D}}_t - D_c\underline{\underline{I}})^{1/2}\,(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}_{ft})\,(\underline{\underline{D}}_t - D_c\underline{\underline{I}})^{1/2}\end{array}\right]^{+}\end{aligned} \tag{9}$$

Notations are similar to the previous one's. Crack closure effect is simulated using positive part (positive principal values) of tensor inside brackets. A two scalars damage model is recovered replacing the tensile damage tensor $\underline{\underline{D}}_t$ by a scalar D_t.

3.2 Thermodynamic aspects

Selected state variables and their corresponding associated variables are given in table 1 but tensile damage variable is now a second order tensor.

The thermodynamic potential as the following expression:

$$\rho\psi = \frac{1}{2}(1-D_c)\left[\lambda_0 \mathrm{Trace}\,(\underline{\underline{\varepsilon}}-\underline{\underline{\varepsilon}}_{ft})\,\underline{\underline{I}} + 2\mu_0(\underline{\underline{\varepsilon}}-\underline{\underline{\varepsilon}}_{ft})\right] : (\underline{\underline{\varepsilon}}-\underline{\underline{\varepsilon}}_{ft})$$

$$-\frac{1}{2}\begin{bmatrix} \lambda_0 \mathrm{Trace}\,(\underline{\underline{D}}_t - D_c\underline{\underline{I}})^{1/2}(\underline{\underline{\varepsilon}}-\underline{\underline{\varepsilon}}_{ft})(\underline{\underline{D}}_t - D_c\underline{\underline{I}})^{1/2}\,\underline{\underline{I}} \\ + 2\mu_0(\underline{\underline{D}}_t - D_c\underline{\underline{I}})^{1/2}(\underline{\underline{\varepsilon}}-\underline{\underline{\varepsilon}}_{ft})(\underline{\underline{D}}_t - D_c\underline{\underline{I}})^{1/2}\end{bmatrix}^{+} : (\underline{\underline{\varepsilon}}-\underline{\underline{\varepsilon}}_{ft}) \quad (10)$$

Using the Euler theorem, derivation of this potential versus the observable state variable $(\underline{\underline{\varepsilon}}-\underline{\underline{\varepsilon}}_{ft})$ gives the associated variable $\underline{\underline{\sigma}}$. Euler theorem states that if $\rho(X)$ is a homogeneous function on X ($\rho(\lambda X)=\lambda\,\rho(X)$) then $X\frac{\partial \rho(X)}{\partial X}$ is equal to X. Here terms inside brackets in relation (9) are homogeneous function of $(\underline{\underline{\varepsilon}}-\underline{\underline{\varepsilon}}_{ft})$ and the theorem can be applied.

Clausius Duhem inequality is not easy to verify when the principal axes of the load rotate. This problem is investigated.

3.3 Evolution laws of the irreversible strain $\underline{\underline{\varepsilon}}_{ft}$

As for the two scalars damage model, the strain tensor increment $\dot{\underline{\underline{\varepsilon}}}_{ft}$ is set independent on tensile damage variables and it is dependent on irreversible compressive damage variables. $\dot{\underline{\underline{\varepsilon}}}_{ft}$ has the same principal axes than $(\underline{\underline{\varepsilon}}-\underline{\underline{\varepsilon}}_{ft})$ tensor and its principal values are obtained using the following relation:

$$\dot{\underline{\underline{\varepsilon}}}_{ft} = \begin{pmatrix} -\varepsilon_{fc} & \alpha & \alpha \\ \alpha & -\varepsilon_{fc} & \alpha \\ \alpha & \alpha & -\varepsilon_{fc} \end{pmatrix} \begin{pmatrix} \dot{D}_{c1}/(1-D_{c1}) \\ \dot{D}_{c2}/(1-D_{c2}) \\ \dot{D}_{c3}/(1-D_{c3}) \end{pmatrix} \quad (11)$$

D_{ci} are irreversible variables related, as shown later, to the negative principal compressive strain $\langle \varepsilon_i \rangle^-$.

ε_{fc} is a scalar value that defines the fictitious strain state at the focus point for unloading compressive path.

α is a material dependent scalar value. This parameter can be chosen in order to get a given apparent Poisson ratio ν_p at the peak stress under uniaxial compressive stress.

If - ε_p is the strain associated to the peak stress, α can be written as follow:

$$\alpha = \nu_0\varepsilon_{fc} + \varepsilon_p(\nu_p - \nu_0)\frac{1-D_{cp}}{D_{cp}} \quad (12)$$

ν_0 is the elastic Poisson ratio.

ν_p is the apparent Poisson ratio at the maximum compressive stress.

D_{cp} is the compressive damage value at the peak stress.

If ν_p is equal to ν_0 the apparent Poisson ratio keep a constant value. When ν_p is greater than ν_0 the apparent Poisson ratio increases with the load during an uniaxial compressive test.

ν_p can be set equal to 0.5 for concrete material. This parameter is useful to control volume change under meanly compressive loads.

For biaxial plane stress loading, the expression of ε_{fc} is modified in order to get constant focus point coordinates:

$$\varepsilon_{fc} = \varepsilon_{fc0} + \alpha \left\langle \frac{\Sigma \tilde{\sigma}_i^-}{\mathrm{Max}\left(\tilde{\sigma}_i^-\right)} - 1 \right\rangle^+ \qquad (13)$$

ε_{fc0} is a positive scalar equal to the focus point strain component for a one dimensional plane stress compressive test.

$\tilde{\sigma}_i$ is the principal effective stress component given by:

$$\underline{\underline{\tilde{\sigma}}} = \lambda_0 \mathrm{Trace}\,(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}_{ft})\,\underline{\underline{I}} + 2\mu_0\,(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}_{ft}) \qquad (14)$$

3.4 Evolution laws for damage variables

3.4.1 Compressive damage variables:

As shown in relation (11), $\dot{\underline{\underline{\varepsilon}}}_{ft}$ is related to $\dot{D}_{ci}$, i= 1 to 3. Evolution of D_{ci} is obtained using a modified evolution law proposed by J. Mazars (1984).

D_{ci} is given from the next relation:

$$1 - D_{ci} = \frac{1}{\tilde{\varepsilon}_{ci} + \varepsilon_{fc}} \left[-\frac{\varepsilon_0^c}{\varepsilon_p - \varepsilon_0^c} + \frac{\varepsilon_p\,\tilde{\varepsilon}_{ci}}{\varepsilon_p - \varepsilon_0^c} \exp^{\frac{\langle \tilde{\varepsilon}_{ci} - \varepsilon_0^c \rangle^+}{\varepsilon_p}} + \varepsilon_{fc} \right] \qquad (15)$$

ε_0^c is a compressive strain threshold. $\tilde{\varepsilon}_{ci}$ is an equivalent compressive strain defined by $\tilde{\varepsilon}_{ci} = \sqrt{\langle \varepsilon_i \rangle_-^2}$, $\langle \varepsilon_i \rangle_-$ is the negative principal strain.

The compressive damage variable D_c is obtained using $\tilde{\varepsilon}_c$ in relation (15) with:

$$\tilde{\varepsilon}_c = \sqrt{\Sigma\, \langle \varepsilon_i \rangle_-^2} \qquad (16)$$

Remarks: Compacting phenomena under high compressive stress could be simulated by increasing ε_{fc} versus the negative part of the volumetric strain ε_v. This aspect is investigated.

3.4.2 Tensile damage variables:

Evolution law of the second order tensor $\underline{\underline{D}}_t$ is obtained using the general relation:

$$\underline{\underline{D}}_t^n = \Re\left(\underline{\underline{D}}_t^o, \Delta\underline{\underline{D}}_t\right) \tag{17}$$

$\underline{\underline{D}}_t^n$ refers to the updated value or the new tensile damage tensor and $\underline{\underline{D}}_t^o$ refers to the old value. $\Delta\underline{\underline{D}}_t$ represents the tensile damage tensor increment.

The expression of the $\Re$ function is not yet established when the principal load axes rotate. This function has to verify the two following conditions:

- The new principal values of the updated tensile damage tensor should be greater or equal to the previous one.
- These principal values should be less or equal to 1.

For fixed load axes expression (17) takes the simple form:

$$\underline{\underline{D}}_t^n = \underline{\underline{D}}_t^0 + \Delta\underline{\underline{D}}_t \tag{18}$$

$\Delta\underline{\underline{D}}_t$ has the same principal axes than the effective stress tensor $\underline{\underline{\tilde{\sigma}}}$.

Principal values of $\Delta\underline{\underline{D}}_t$ are obtained using a modified J. Mazars law:

$$\begin{aligned} &\Delta d_{ti} = (d_{ti})^n - (d_{ti})^0 \quad && \text{if} \quad (d_{ti})^n \rangle (d_{ti})^0 \\ &\Delta d_{ti} = 0 && \text{if} \quad (d_{ti})^n \leq (d_{ti})^0 \end{aligned} \tag{19}$$

The expression of d_{ti} is related to principal effective stress components and to principal closure stress components. d_{ti} is given by:

$$1 - d_{ti} = \frac{1}{\tilde{\sigma}_{ti}^{+} - \sigma_{fti}^{-}}\left[\sigma_0^c\,(1 - A_t) + A_t\,\tilde{\sigma}_{ti}^{+} \exp^{-B_t\langle \tilde{\sigma}_{ti}^{+} - \sigma_{fti}^{-} \rangle_+ / E_0} - \sigma_{fti}^{-}\right]$$

$$\text{and } d_{ti} \geq (D_c)^{0.4} \tag{20}$$

$\tilde{\sigma}_{ti}^{+}$ is the positive part of the principal effective stress component.

σ_{fti}^{-} is the negative part of the principal closure stress component.

σ_0^c is the compressive stress damage threshold.

A_t and B_t are the Mazars coefficients.

3.5 Response of the anisotropic and unilateral explicit damage model

Figure 2 gives the model response for a compressive and tensile test in plane stress conditions. On the left side the $\sigma_1 - \varepsilon_1$ relation is plotted. The closure stress is set to zero. On the right side the $\sigma_1 - \varepsilon_2$ relation is given.

Figure 3 shows the $\sigma_1 - \varepsilon_v$ curve for the same loading.

The volume is negative before the stress peak, is equal to zero at the peak stress, and becomes positive after the peak.

This curve corresponds to an apparent Poisson ratio at peak stress equal to 0.5.

Figure 4 gives the evolution of damage variables versus ε_1 strain.

Large dashed line correspond to Dc.

Continuous line refers to transverse tensile damage variables d_{t2} and d_{t3} ($d_{t2} = d_{t3}$).

Small dashed line is relative to longitudinal tensile damage variable d_{t1}.

During the compressive phase Dc increases and d_{t1}, d_{t2}, and d_{t3} are equal to $D_c^{0.4}$.

During unloading all damage variables remain constant except d_{t1} that increases again when the longitudinal stress becomes positive.

Figure 5 shows the maximum tensile or compressive stress obtained in plane stress conditions. Dashed line gives the initial damage threshold.

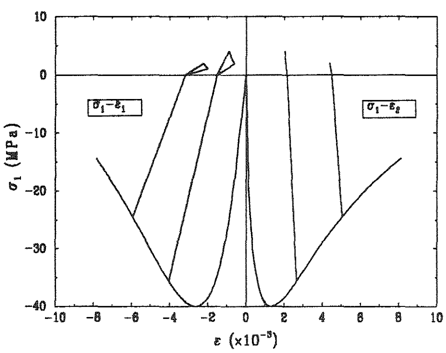

Figure 2. Stress strain relation under tensile or compressive loads

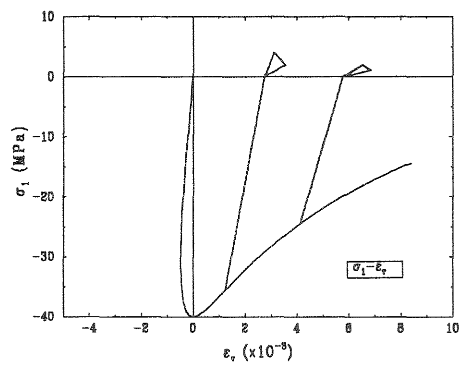

Figure 3. Stress-volumetric strain under tensile or compressive loads

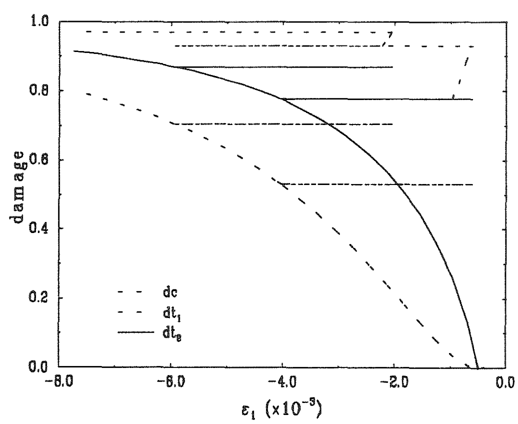

Figure 4. Damage variables versus longitudinal strain (tension or compression)

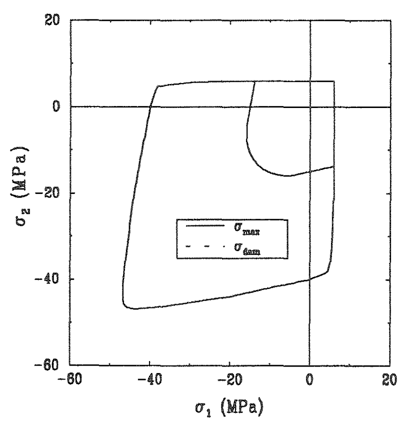

Figure 5. Maximum stress under plane stress conditions

4 Conclusions

Explicit damage models has been developed to model behaviour of brittle materials like concrete and rocks.

The first model is a two scalars damage model that describes crack closure effect and permanent strains. The compressive damage variable is obtained from the principal strains. The inelastic closure strain that defines the strain state between tension and compression is also related to the compressive damage variable and to the principal strains. Finally tensile damage variable can be obtained directly without any iterative process.

The second anisotropic and unilateral model is itself an extension of the previous one. In this model, a second order tensor is used to described the anisotropic material stiffness. Inelastic anisotropic strains are also introduced in the constitutive relations. Under meanly compressive loads volume change due to damage effects can be controlled accurately. This general damage model, still under development, will be completely finished when the difficult problem of rotating load axes will be solved and when compacting effects under high confined compressive stress will be included.

5 References

Laborderie, C. (1991) Phénomènes unilatéraux dans un matériau endommageable: modélisation et application à l'analyse de structures en béton. Thesis, Laboratoire de Mécanique et Technologie de Cachan, Cachan, France.

Mazars, J. (1984) Application de la mécanique de l'endommagement au comportement non linéaire et à la rupture du béton de structure. Thesis, Laboratoire de Mécanique et Technologie de Cachan, Cachan, France.

Pontiroli, C. (1995) Comportement au souffle de structures en béton armé. Thesis, Laboratoire de Mécanique et Technologie de Cachan / Centre d'Etudes de Gramat (to be published), Gramat, France.

Ramtani, S. (1990) Contribution à la modélisation du comportement multiaxial du béton endommagé avec description du caractère unilatéral. Thesis, Laboratoire de Mécanique et Technologie de Cachan, Cachan, France.