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STRENGTH OF A CONCRETE ELEMENT UNDER THE ACTION OF CONCENTRATED TENSILE OR SHEAR FORCE

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Abstract

Global strength of a concrete element under the action of a concentrated tensile or shear force, which is applied at a given location on the inside or on element surface, is considered. Methods of fracture mechanics are used for strength evaluation and mathematical expressions are derived, taking into consideration size effects. Published test data have been examined and compared with the proposed formulae. It has been shown that in the considered cases three possible types of fracture may develop: quasi-brittle, pseudo-plastic and an intermediate type. The fracture brittleness degree is determined by the values of the ratio between problem characteristic size and concrete characteristic size.

1 Introduction

The problem under discussion is very common in practice; for instance, in the case of concrete anchorage calculation. A great deal of research has been done in this field (ACI 355. 1R-91 1991; CEB 1991; Rehm et al. 1992; Shapiro and Yagust 1980; Yagust 1982a and others). A classification of observed modes of failure shows different types of concrete fracture: concrete cone failure, bursting failure, splitting failure and pull-out of anchor due to crushing under local compression action.

Concrete cone failure has been studied most. It has been found that the strength of a concrete element depends on many factors, such as the strength characteristics of the concrete and the location and line of action of the applied force (shear force is acting on the element surface, tensile force is acting on the inside of the element). The published literature recommends linear fracture mechanics (LEFM) as an appropriate tool that may be applied in the considered cases. However, some of the existing formulae need improvement, while other formulae may be used properly.

In the present research an advanced approach for strength calculation in the case of concrete cone failure has been developed which is based on test data processing, and takes into account size effects. Size effects depend on a fracture brittleness degree and on a characteristic size of a problem. The characteristic size of a problem is determined by the relative position of the force location and the element dimensions.

2 Strength evaluation

2.1 Spatial concrete element

It is evident that linear fracture mechanics is by no means appropriate in all cases (Barenblatt 1962; Irwin 1958; McClintock and Irwin 1965 and others). It is also well known that the limits of the use of LEFM are determined by the ratio between geometry parameter of the problem and concrete characteristic size d, which is the length of the zone of non-elastic concrete deformation ahead of the crack tip (the zone of microcrack formation or the fracture process zone - FPZ) (Hillerborg et al. 1976; Modeer 1979; Yagust 1982a, 1982b, 1983 and others). d-dimension can be approximately evaluated, for example, using G.I. Barenblatt's formula (Barenblatt 1962):

$$d = \pi/8(K_{Ic}/f_{ct})^2 \qquad (1)$$

In addition to that, the considered limits also depend on the type of the problem (Yagust 1982a, 1982b, 1983). These factors, when they are not considered, lead to distortion of the actual size effect influence on strength.

Figs. 1,2 show experiment data processing of 3-dimensional elements. Tests where there are no local cracks except for the major crack

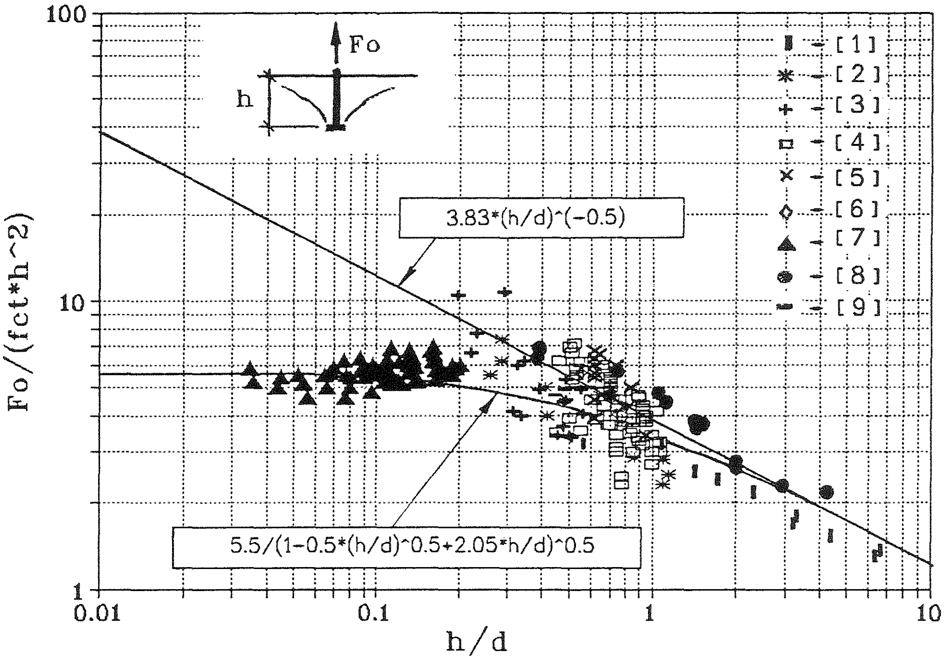

Fig. 1. Experimental results and approximation of size effect on ratio $F_o/(f_{ct}\cdot h^2)$ for 3-dimensional elements:
[1] Eligehausen and Sawade (1989); [2] Holmjanski (1968); [3] Kononov (1962); [4] Lukojanov (1964); [5] Nizhnikovski (1964); [6] Sattler (1962); [7] Skramtaev and Wolf (1939); [8] Shapiro and Yagust (1980); [9] Tchujko (1972)

near the force point of action have been considered. The values of the relative fracture load $F_o/(f_{ct}\cdot h^2)$ and $Q_o/(f_{ct}\cdot c^2)$ complies with linear fracture mechanics for quasi-brittle fracture only for elements with relatively large h/d or c/d respectively. On the other hand, the values of relative fracture load for pseudo-plastic failure are only correct for small values of h/d or c/d. There is a zone in which an intermediate type of fracture develops. Calculations based on use of non-linear fracture mechanics (NLFM) for any parameter values $o<h/d<\infty$ or $o<c/d<\infty$ may yield insufficiently accurate results. The root dependence which had been proposed by Z.P. Bažant (1984) for account of the size effect is simpler and precise enough, if it utilizes the d-dimension instead of the maximum aggregate size. Under these conditions limiting cases for h/d, $c/d\rightarrow o$ and h/d, $c/d\rightarrow\infty$ are exact, while intermediate values can be approximated to the test data using

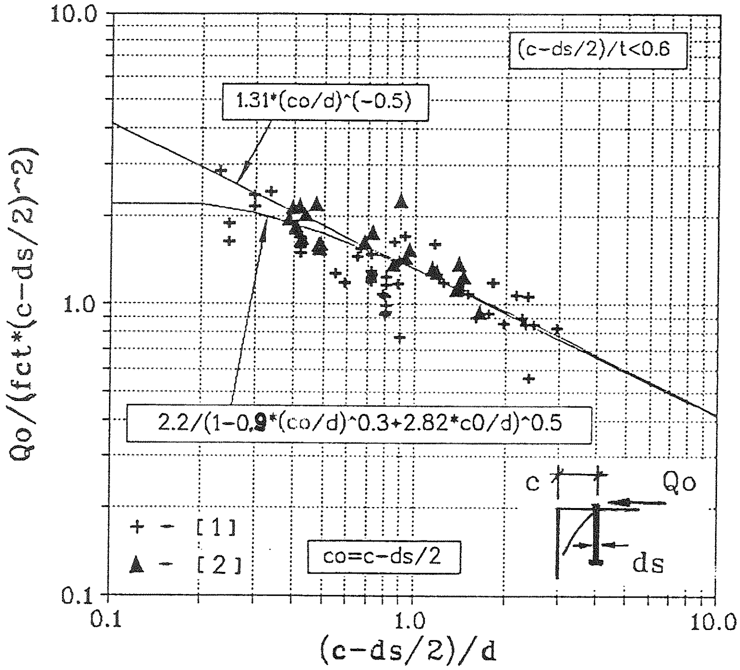

Fig. 2. Experimental results and approximation of size effect on ratio $Q_o/(f_{ct}\cdot(c-d_s/2)^2)$ for 3-dimensional elements:
[1] Fuchs (1992); [2] Ueda et al. (1991)

additional terms under the root. The root value cannot be less than 1. On this basis, the following approximate formulae have been developed:

$$F_o = 5.5\cdot f_{ct}\cdot h^2/\sqrt{1-0.5(h/d)^{0.5}+2.05\cdot h/d} \tag{2}$$

$$Q_o = 2.2\cdot f_{ct}\cdot c^2/\sqrt{1-0.9(c/d)^{0.3}+2.82\cdot c/d} \tag{3}$$

LEFM is used for h/d, c/d≥0.6.

The examples shown in Figs. 1,2 demonstrate the sufficiently accurate estimation of the concrete element strength.

2.2 Use of K_{Ic}

Use of linear fracture mechanics assumes introduction of K_{Ic} for concrete (or f_{ct} and d) into the appropriate formulae. The use of $\sqrt{f_c}$ instead of K_{Ic}, which had been adopted in a series of works, is not exact, as it may introduce errors sometimes up to 50%.

2.3 Plane concrete element

Several important cases of concentrated force action on elements of limited sizes have not yet been treated. Figs. 3, 4 are based on test data processing for plane elements, and shows as an example the influence of h/d or c/d on relative fracture load $F_o/(f_{ct}\,th)$ and $Q_o\,(f_{ct}\,tc)$ respectively for a concentrated force which is located at the center of the plane element (tension) or on the surface of the element (shear). All the earlier comments (section 2.1) are valid for this case as well. The derived approximate formulae are:

$$F_o = 1.4 \cdot f_{ct} \cdot h \cdot t / \sqrt{1 - 0.3(h/d)^{0.5} + 0.49 \cdot h/d} \tag{4}$$

$$Q_o = 0.73 \cdot f_{ct} \cdot c \cdot t / \sqrt{1 - 0.72(c/d)^{0.2} + 0.64 \cdot c/d} \tag{5}$$

In this case the LEFM applies for h/d, c/d ≥ 2-3.

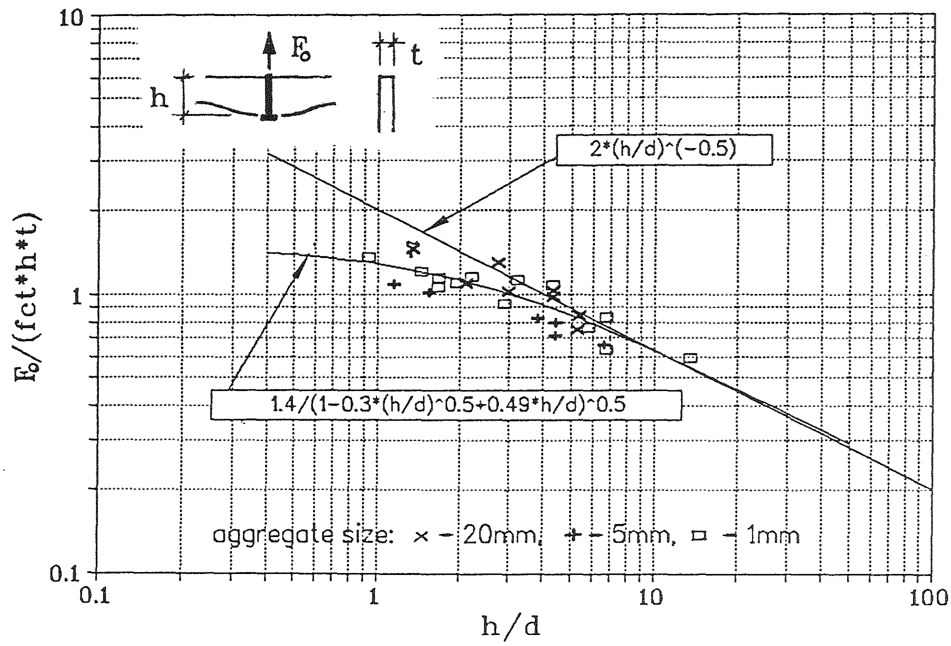

Fig. 3. Experimental results and approximation of size effect on ratio $F_o/(f_{ct} \cdot t \cdot h)$ for plane elements (Shapiro and Yagust 1980)

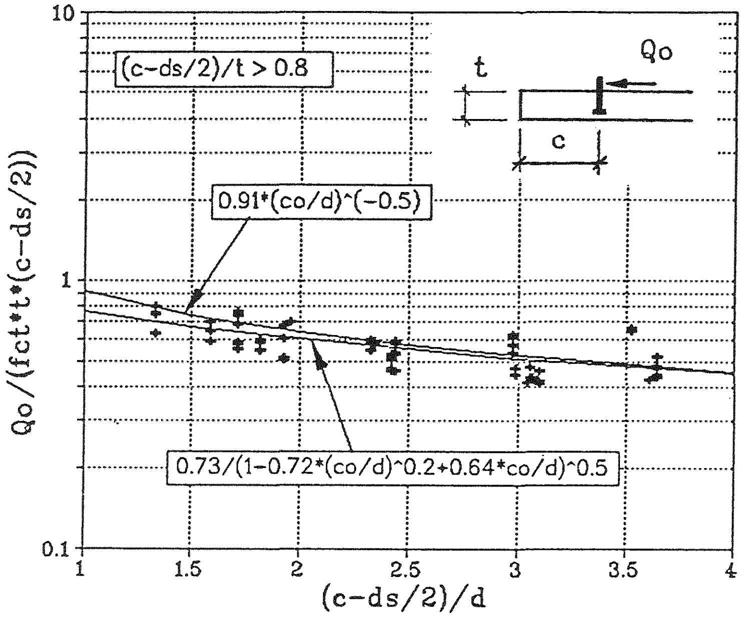

Fig. 4. Experimental results and approximation of size effect on ratio $Q_o/f_{ct} \cdot t \cdot (c-d_s/2))$ for plane elements (Fuchs 1992)

3 Conclusions

Study of the problem under consideration shows that in spite of the pronounced progress further work is required: more complete classification of the cases met in practice is needed, as well as further research of various kinds of fracture, more precise account of the size effect in strength formulae, etc. This work presents a modest step towards a more precise account of the size effect, and is supported by calculation examples for spatial and plane elements.

References

ACI 355. 1R-91(1991) State of the art report on anchorage to concrete. Reported by Com. 355, 71.

Bažant, Z.P. (1984) Size effect of blunt fracture: concrete, rock, metal. **J. of Eng. Mech.**, 110, 4, 518-535.

Barenblatt, G.I. (1962) The mathematical theory of equilibrium cracks in brittle fracture. **Advances in Appl. Mech.**, 7, 55-129.

CEB (1991) Fastenings to reinforced concrete and masonry structures. **Bulletin d'formation**, 206, 486.

Eligehausen, R. and Sawade, G. (1989) A fracture mechanics based description of the pull-out behavior of headed studs embedded in concrete, in **Fracture Mechanics of Concrete Structures. From Theory to Applications** (ed L. Elfgren), Chapman & Hall, London, 281-299.

Fuchs, W. (1992) Fastening behavior under shear load in uncracked concrete. **Deutscher Ausschuss für Stahlbeton**, 425 (in German).

Hillerborg, A., Modeer, M. and Petersson, P.E. (1976) Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements. **Cem. & Concr. Res.**, 6, 773-782.

Holmjanski, M.M. (1968) **The Laying Details of Precast Reinforced Concrete Structures**. Stroiizdat, Moscow (in Russian).

Irwin, G.R. (1958) **Fracture I. Handbuch der Physik VI**. Springer, 558-590.

Kononov, I.A. (1962) The embedment depth determination, in **Vibration Application in Building** (ed. I.J. Petrov), Gosstroiizdat, Moscow, 31-45 (in Russian).

Lukojanov, U.N. (1964) The experimental study of behaviour concrete in anchor fastenings, in **Design and Construction of Industrial Buildings and Constructions** (ed V.G. Desjatov), Stroiizdat, Moscow, 18-27 (in Russian).

McClintock, F.A. and Irwin, G.R. (1965) Plasticity aspects of fracture mechanics. **ASTM STP** 381, 84-113.

Modeer, M. (1979) A fracture mechanics approach to failure analysis of concrete materials. Report TVBM-1001, Div. of Building Materials, Lund Inst. of Technology, Sweden.

Nizhnikovski, G.S. (1964) The new type of anchor bolt joining. **Express Information**, 149, Orgenergostroi, Moscow (in Russian).

Rehm, G., Eligehausen, R. and Mallee, R. (1992) Befestigungstechnik. **Beton-Kalender,** 597-715 (in German).

Sattler, K. (1962) Betractung über neuere Verdübelung in Verbundbau. **Der Bauingenieur**, 37, 1, 1-8 (in German).

Shapiro, G. and Yagust, V. (1980) Strength of plane concrete element under concentrated load, in **Investigations of Bearing Concrete and Reinforced Concrete Structures of Multistory Precast Buildings** (eds. G.N. Lvov and U.M. Strugazky), MNIITEP, Moscow, 74-111 (in Russian).

Skramtaev, B.G. and Wolf, I.V. (1939) **Control of the concrete strength.** Gosstroiizdat, Moscow (in Russian).

Tchujko, P.A. (1972) The study of concrete strength by method of cone failure and shear failure, in **The Interbranch Problems of Building. Home Experience** (ed D.A. Korshunov), CINIS, Moscow (in Russian).

Ueda, T., Stitmannaithum, B. and Matupayont, S. (1991) Experimental investigation on shear strength of bolt anchorage group. **ACI Structural Journal**, 88,3, 292-300.

Yagust, V. (1982a) Resistance to development of crack in concrete structures taking into account the influence of material macrostructure, D.Sc. thesis, NIIZHB, Moscow (in Russian).

Yagust, V. (1982b) On limits of application of linear fracture mechanics to concrete. **Beton & Zhelezobeton**, Moscow, 6, 25-26 (in Russian).

Yagust, V. (1983) Application of the model of Leonov-Panasjuk-Dugdale for evaluation of development of crack in concrete structures, in **Strength Investigations of Bearing Structures of Multistory Precast Buildings** (eds G.N. Lvov and U.M. Strugazky), Moscow, 66-84 (in Russian).