NUMERICAL MODELING AND DETERMINATION OF FRACTURE MECHANICS PARAMETERS. HILLERBORG TYPE MODELS

Manuel Elices and Jaime Planas
Departamento de Ciencia de Materiales, Universidad Politécnica de Madrid, E.T.S.I. Caminos, Ciudad Universitaria, 2840 Madrid, España

Abstract
This contributions outlines the main features of the cohesive cracks as they were conceived by Hillerborg: as a constitutive model able to describe the generation of a progressively fracturing surface and its evolution up to complete failure. After outlining the basic hypotheses, the numerical tools available for structural analysis are examined, including the simplest approximations reducing the problem to effective elastic crack formulations. Then the experimental procedures are summarized that are required to characterize a given material, with particular emphasis on the simplified procedure developed to fit a bilinear softening curve to experimental load-CMOD curves of notched specimens. A few remarks about the various problems that are still open close the contribution.

1 Introduction

Several numerical models for analyzing fracture of concrete and concrete structures are now available and the main purpose of this workshop is to find out how the needed material parameters can be measured from the respective models. In response to this question, this contribution is an outlook on the suitability of Hillerborg type models or, more generally, those based on the cohesive crack concept. In essence, it is a summary of work done by the authors during the last five years with special emphasis on the measurement of parameters associated with the cohesive model.

This paper is structured in the following way; after a brief introduction of the *cohesive crack model* two relevant aspects are considered, numerical implementation and experimental measurement of the parameters. The final comments deal with the predictive capability of this model and with some areas that are worthy of future research.

2 Outline of the cohesive crack model

A review of the relevant properties of this model was done by the authors in some recent papers (Elices and Planas 1989 and Elices, Planas and Guinea 1993 and by Carpinteri 1994).

The first cohesive crack models appeared as an alternative to linear elastic fracture mechanics to eliminate the stress singularity (Barenblatt 1962). Hillerborg generalized the concept of process zone, removing the small size requirement (Hillerborg et al. 1976) and, more recently, the authors have shown that this model may be considered a particular case of continuum mechanics nonlocal formulation (Planas, Elices and Guinea 1993).

A cohesive crack model is characterized by the properties of the bulk material, the crack initiation condition and the crack evolution function. The simplest assumptions are:

1. The bulk material behaviour follows a linear elastic and isotropic stress-strain relationship, with elastic modulus E and Poisson ratio ν. This assumption is not essential but makes computations easy.
2. The crack initiates at a point where the maximum principal stress σ_I at that point reaches the tensile strength f_t. Moreover, the crack forms normal to the direction of the major principal stress.
3. After its formation, the crack opens while transferring stress from one face to another. The stress transferred —the *cohesive stress*— is a function of the crack opening displacement history. For monotonic mode I opening the stress transferred is normal to the crack faces and is a unique function of the crack opening:

$$\sigma = f(w) \tag{1}$$

The function $f(w)$ is called the *softening function*, or softening curve.

For concrete and cementitious materials the softening function is a non-increasing function of the crack opening, as depicted in Fig 1. The main features of this curve are:

1. The tensile strength f_t, which is, by definition, the stress transferred at incipient opening.
2. The initial slope, measured as w_1/f_t. For small cracked specimens and uncracked specimens of any size these two parameters suffice to predict maximum loads (Guinea, Planas and Elices 1994a and Planas, Guinea and Elices 1995).
3. The specific fracture energy G_F, which is the work needed to bring completely apart the two faces of a unit surface of crack, and is represented by the area under the softening curve.
4. The critical crack opening w_c, which is the crack opening for which the stress transferred becomes zero.

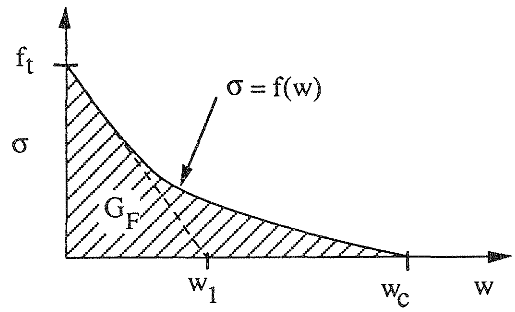

Fig. 1. Softening curve defining the cohesive crack model.

Although the cohesive crack concept is a useful tool for modeling concrete cracking, its implementation is not straightforward and the *equivalent elastic crack concept* emerged as a simplified procedure to evaluate crack growth in cohesive materials. Moreover, the use of crack growth resistance curves (R-Δa) to predict the behaviour of cracked specimens is a well established practice for cementitious materials and can be considered as an equivalent elastic crack approximation of the cohesive model, as shown by the authors (Elices and Planas 1993 and Planas et al. 1993).

The equivalent elastic crack greatly simplifies the computations by shifting from a cohesive material (and hence, non lineal structural analysis) to a linear elastic material (which only requires linear analysis). The price paid for the equivalence is that the linear elastic material has not a constant crack growth resistance (as is G_F for the cohesive model) but a changing value with crack length. This R-Δa curve is not a material property, it depends on the geometry and specimen size. This simplification and associate drawbacks are discussed in length in the quoted references.

3 Numerical aspects

The computational analysis of the fracture of any plain concrete structure is possible from the knowledge of the softening function. However, carrying out the corresponding computations is very difficult and many different expedients have been used.

Three levels of computational approaches are considered: (a) general purpose finite element codes, able, in principle, to analyze any structure subjected to any loading; (b) special computational strategies applicable only to cases where the cohesive crack path is known in advance; and (c) simplified approaches making use of more or less justified strong simplifications.

The first group is very scarce. The most extended approaches actually do not use a cohesive crack but a smeared version of it, similar to a crack band approach (mainly because its FE implementation is simpler). There are many codes in use, but the potential users must be aware of some ill-behaved characteristics when the crack propagates skew to the mesh. The analysis of this problem is outside the scope of this paper, and the reader is referred to the work of Rots (1992) and references therein. An interesting exception to the general trend is that led by Ingraffea at Cornell, who developed several versions of a finite element code with very powerful remeshing capacities, which is able to interactively follow the cohesive crack extension in 2–dimensional problems (for details, see Bittencourt *et al.*, 1992, and references therein).

The second group is essentially formed by special procedures focused on solving symmetric cracking (mode I) which is the kind mainly found in the laboratory. There are two basic approaches in this group. One is to use a commercial finite element package, and situate interface elements with softening (usually of the strut or spring type) along the crack path. This kind of interface elements is available in most good finite element packages. The advantages of this approach are that it provides all the information about the stress, strains and displacement fields, and that it is applicable to any symmetric geometry. The main drawback is that if one wants an accurate computation, i. e., a fine mesh, the time to simulate a single test may be quite long. To get fast results (about 100 times faster) a special purpose program may be developed which is a special version of a boundary elements formulation.

The approach used by the authors is a modification of the influence method developed by Petersson (1981) in which a number of elastic solutions for the geometry used are superimposed to get a nonlinear solution for the crack propagation. Roughly speaking, if one has N elements along the crack path, one has to solve N+1 linear cases from which an "influence matrix" is constructed and from it a crack advance step is reduced to solve a problem with at most N degrees of freedom. This is to be compared with the previous approach where at each increment one has of the order of N^2 degrees of freedom. The authors have modified the Petersson algorithm to improve the performance and to accept any nonlinear softening (Planas and Elices, 1991). A user-friendly program called *Splitting Lab* has been developed by the authors and their collaborators. It runs on Macintosh computers and may simulate a complete fracture test in laboratory in only a few minutes. The program is menu driven and the most widely used specimen geometries and softening functions are selectable from a menu at a single mouse-click. A δ-version is available from the authors.

The last group is that of special methods which reduce a problem of a large number of degrees of freedom to a minimum by using certain special simplifications. One simplification used is to assume that the crack opening profile is linear (Llorca and Elices 1990, Foote *et al.* 1986, Cotterel and Mai 1987). One entire family of approximations in which only two degrees of freedom are retained is based on the *equivalent elastic crack* concept, in which all the fields are approximated by linear elastic fields for a certain *effective crack length.* The importance of this approximation is that it has been used in a completely different framework, and constitutes the link between the classical R-curve approach and the continuum mechanics based approaches, thus providing a global framework for the analysis and comparison of the different models, —in particular two-parameter models of Bazant (1986), Jenq-Shah (1985) or Karihaloo (1989)—, as shown by the authors (Elices and Planas 1993 and Planas et al. 1993).

4. Experimental aspects

Ideally, the tensile strength should be measured in direct tensile tests. However, the usual practice is to make an estimate of the tensile strength using the cylinder splitting strength (Brazilian test), which should give values within 20 percent of the actual f_t.

The fracture energy can be measured using the procedure described in the RILEM draft recommendation (1985). However, it appears that direct application of such procedure delivers measured values of G_F which are size-dependent. The authors have been working extensively on this problem and have shown that a number of spurious energy dissipation sources may be present during one of those tests: hysteresis of the testing arrangement, dissipation in the bulk of the specimen, dissipation by crushing at the supports, and neglect of the final portion of the load-displacement curve. When all these factors are properly taken into account, the size effect of the experimental values is strongly decreased

(Guinea *et al.* 1992, Planas *et al.* 1992a, Elices *et al.* 1992). The observed size-dependence for some concretes still deserves further research.

To get information about the shape of the softening curve without recourse to direct tension tests, Wittmann and co-workers introduced an indirect fitting procedure based on making a parametrization of the softening curve and determining these parameters by best fitting to measured load-deflection curves of stable tests on notched specimens (Roelfstra and Wittmann, 1986; Wittmann *et al.*, 1987, 1988; Mihashi, 1992). To do so they used an unrestricted bilinear function, which has 4 independent parameters. They use an optimization algorithm to determine the four parameters, so that f_t and G_F are obtained as a part of the fitting procedure, not from independent measurements.

In the same spirit, but with a very simple fitting procedure as an objective, the authors have developed a so called general bilinear fitting (GBF) procedure (Guinea, Planas and Elices, 1994b). In the GBF, four parameters completely defining the bilinear softening curve are determined quasi-independently in four sequential operations. Fig 2 illustrates the bilinear curve and the four parameters which are determined:

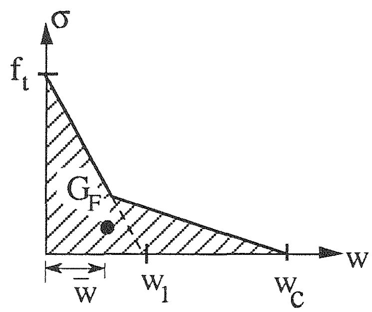

Fig. 2. General bilinear softening curve

1. If direct tension test results are not readily available, the tensile strength f_t is independently estimated as the cylinder splitting strength.
2. The fracture energy G_F is evaluated by the total work of fracture method, using stable tests on notched specimens, with the modifications to the RILEM recommendation previously developed by the authors (Guinea *et al.* 1992, Planas *et al.* 1992a, Elices *et al.* 1992).
3. The abscissa $\bar{w}$ of the center of gravity of the area under the softening curve is obtained by fitting a theoretical expression to the far end of the measured load-deflection (P-u) curve. The only special requisite is that the complete P-u curve must be experimentally obtained, which requires *complete specimen-weight compensation.*

4. The horizontal intercept w_1 of the initial segment of the softening curve is obtained with a semi-graphical procedure which consists essentially in stating that for small size specimens, the peak load predicted by the bilinear softening curve must coincide with that predicted by the linear softening shown as a dashed line in Fig 2.

The softening function can be obtained also from the underlying theory of cohesive cracks, using an indirect J-integral technique, as done by Li et al. (1987). Two precracked specimens with slightly different crack lengths are needed. Another procedure, based again in cohesive crack models, was recently proposed by Kitsutaka et al. (1994). A poly-linear softening function was obtained from the load-displacement values of a three-point bending test of a notched beam jointly with an optimization algorithm.

5. Final comments

Cohesive crack models are efficient predictive models (Hillerborg 1991). Starting from parameters measured on a notched beam and indirect tensile tests one can predict the whole load-displacement curve (or load-CMOD relation) for other geometries —notched or unnotched— and different sizes. Using these procedures the authors made predictions with good agreement with experimental results for notched beams and wedge-splitting specimens of different sizes as well as for unnotched beams (Elices, Planas and Guinea 1993 and Elices, Guinea and Planas 1994).

There are still loose ends to tie up, just to mention some: Computation of crack propagation under mixed mode is not yet satisfactory implemented, the size dependence of some parameters —particularly G_F— for some type of concretes is not yet fully understood and diffuse cracking, or the appearance of a discrete crack system, is not yet predicted in the present formulation. All these topics deserve future research.

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References

Barenblatt, G.J. (1962) The mathematical theory of equilibrium crack in brittle fracture. **Adv. in Applied Mechanics**, 7, 55-125.

Bazant, Z.P., Kim, J.K. and Pfeiffer, P.A. (1986) Nonlinear size effect properties from size effect tests. **Journal of Structural Engineering, ASCE**, 112, 289-307.

Bittencourt, T.N., Ingraffea, A.R. and Llorca, J. (1992) Simulation of arbitrary cohesive crack propagation, in **Fracture Mechanics of Concrete Structures** (ed Z. P. Bazant), Elsevier Applied Science, London, 339-350.

Carpinteri, A. (1994) Cracking of strain-softening materials, in **Static and Dynamic Fracture Mechanics** (eds M.H. Aliabadi et al.), Computational Mechanics Publications, 311-365.

Cotterel, B. and Mai, Y.W. (1987) Crack grwth resistance curve and size effect in the fracture of cement paste. **J. of Materials Science**, 22, 2734-2738.

Elices, M. and Planas, J. (1989) Material models, ch. 3 in **Fracture Mechanics of Concrete Structures** (ed L. Elfgren), Chapman and Hall, London, 16-66.

Elices, M. and Planas, J. (1993) The equivalent elastic crack: 1. Load-Y equivalences. **Int. Journal of Fracture**, 61, 259-272.

Elices, M., Guinea, G.V. and Planas, J. (1992) Measurement of the fracture energy using three-point bend tests: 3. Influence of cutting the P-δ tail. **Materials and Structures**, 25, 327-334.

Elices, M., Planas, J. and Guinea, G.V. (1993) Modeling cracking in rocks and cementitious materials, in **Fracture and Damage of Concrete and Rock** (ed H.P. Rossmanith), E & FN Spon, London, 3-33.

Foote, R.M.L., Mai, Y.W. and Cotterel, B. (1986) Crack growth resistance curves in strain softening materials. **J. Mech. Phys. Solids**, 34, 593-607.

Guinea, G.V., Planas, J. and Elices, M. (1992) Measurement of the fracture energy using three-point bend tests: 1. Influence of experimental procedures. **Materials and Structures**, 25, 212-218.

Guinea, G.V., Planas, J. and Elices, M. (1994a) Correlation between the softening and the size effect curves, in **Size Effect in Concrete**

Structures (eds H. Mihashi, H. Okamura and Z.P. Bazant), E & FN Spon, 233-244.

Guinea, G.V., Planas, J. and Elices, M. (1994b) A general bilinear fit for the softening curve of concrete. **Materials and Structures**, 27, 99-105.

Hillerborg, A. (1991) Application of the fictitious crack model to different types of materials. **Int. Journal of Fracture**, 51, 95-102.

Hillerborg, A., Modéer, M. and Petersson, P.E. (1976) Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements. **Cement and Concrete Research**, 6, 773-782.

Jenq, Y.S. and Shah, S.P. (1985) Two-parameter fracture model for concrete. **Journal of Engineering Mechanics Division, ASCE**, 111, 1227-1241.

Karihaloo, B.L. and Nallathambi, P. (1989) An improved effective crack model for the determination of fracture toughness of concrete. **Cement and Concrete Research**, 19, 603-610.

Kitsutaka, Y., Kamimura, K. and Nakamura, S. (1994) Evaluation of aggregate properties on tension softening behaviour of high-strength concrete, in **High-Performance Concrete ACI SP-149** (ed V.M. Malhotra), ACI, 711-719.

Li, V.C., Chan, C.M. and Leung, C.K.Y. (1987) Experimental determination of the tension softening relations for cementitious composites. **Cement and Concrete Research**, 17, 441-452.

Llorca, J. and Elices, M. (1990) A simplified model to study fracture behaviour in cohesive materials. **Cement and Concrete Research**, 20, 92-102.

Mihashi, H. (1992) Material structure and tension softening properties of concrete, in **Fracture Mechanics of Concrete Structures** (ed Z.P. Bazant), Elsevier, London, 239-250.

Petersson, P.E. (1981) Crack growth and development of fracture zone in plain concrete and similar materials. Div. of Building Mat., Lund Inst. of Technology, Report TVBM-1006.

Planas, J. and Elices, M. (1991) Nonlinear fracture of cohesive materials. **Int. J. of Fracture**, 51, 139-157.

Planas, J. and Elices, M. (1995) Rupture modulus and fracture properties of concrete, in **FRAMCOS-2**, Zurich.

Planas, J., Elices, M. and Guinea G.V. (1992) Measurement of the fracture energy using three-point bend tests: 2. Influence of bulk energy dissipation. **Materials and Structures**, 25, 305-312.

Planas, J., Elices, M. and Guinea, G.V. (1993) Cohesive cracks versus nonlocal models: Closing the gap. **Int. Journal of Fracture**, 63, 173-187.

Planas, J., Elices, M. and Ruiz, G. (1993) The equivalent elastic crack: 2. X-Y equivalences and asymptotic analysis. **Int. Journal of Fracture,** 61, 231-246.

RILEM Draft Recommendation (1985) Determination of fracture energy of mortar and concrete by means of three-point bend test on notched beams. **Materials and Structures**, 18, 285-290.

Roelfstra, P.E. and Wittmann, F.H. (1986) Numerical method to link strain softening with failure of concrete, in **Fracture Toughness and Fracture Energy of Concrete** (ed F.H. Wittmann), Elsevier Science, Amsterdam, 163-175.

Rots, J.G. (1992) Removal of finite elements in strain-softening analysis of tensile fracture, in **Fracture Mechanics of Concrete Structures** (ed Z. P. Bazant), Elsevier Applied Science, London, 330-338.

Wittmann, F.H., Roelfstra, P.E., Mihashi, H., Huang, Y.Y., Zhang, X.H. and Nomura, N. (1987) Influence of age of loading, water-cement ratio, and rate of loading on fracture energy of concrete. **Materials and Structures**, 20, 103-110.