Fracture Mechanics of Concrete Structures
Proceedings FRAMCOS-3
Aedificatio Publishers, D-79104 Freiburg, Germany

DETERMINATION OF THE DOUBLE-K FRACTURE PARAMETERS IN STANDARD THREE-POINT BENDING NOTCHED BEAMS

S. Xu and H.W. Reinhardt
Institute of Construction Materials, University of Stuttgart, Stuttgart, Germany

Abstract
In this paper, an attempt is made to determine the double-K fracture parameters K_{Ic}^{ini} and K_{Ic}^{un} using three-point bending tests. Based on the linear asymptotic superposition assumption proposed by the authors the critical effective crack length a_c is analytically evaluated by inserting the critical crack mouth opening displacement $CMOD_c$ and the maximum load P_{max}, i.e. the secant compliance c_s, into the formula of LEFM. Using the analytical solution of a fictitious crack with cohesive forces in an infinite strip model the double-K fracture parameters K_{Ic}^{ini} and K_{Ic}^{un} as well as the critical crack tip opening displacement $CTOD_c$ were analytically determined. The experimental evidence showed that the double-K fracture parameters K_{Ic}^{ini} and K_{Ic}^{un} are size-independent and can be considered as the material parameters to describe the material properties of cracking initiation and unstable fracture in concrete structures. The testing methods required for the determination of the K_{Ic}^{ini} and K_{Ic}^{un} is quite simple. A closed-loop testing system is not necessary.
Key words: Double-K fracture parameters, standard tests, cracking initiation, unstable fracture, cohesive force, fictitious crack model.

1 Introduction

Based on several fracture models which include the fictitious crack model (FCM) by Hillerborg et al. (1976), the size effect model(SEM) by Bazant and Kazemi (1990), the two-parameter fracture model (TPFM) by Jenq and Shah(1985) and the effective crack model (ECM) by Karihaloo and Nallathambi (1986) and Swartz and Refai (1987), the corresponding methods to measure the fracture parameters, for examples, the fracture energy G_F introduced in the FCM, the critical energy release rate G_f and the critical effective crack extension c_f for the infinite specimen in the SEM, and the critical stress intensity factor K_{Ic}^s and the critical crack tip opening displacement $CTOD_c$ in the TPFM, were recommended by RILEM in 1985 and in 1990 respectively.

To describe the different stages of crack propagation including crack initiation, stable crack propagation and unstable fracture a double-K fracture criterion for crack propagation in the quasi-brittle material like concrete was proposed by Xu and Reinhardt (1997a). In this paper an attempt is made to combine the cohesive force on the fictitious crack with the double-K crack propagation criterion based on stress intensity factor. As the result, a practical measuring method is proposed using standard three-point bending beams.

2 Evaluation of the Effective Crack Length

In order to evaluate the effective crack length a linear asymptotic superposition assumption was proposed by Xu and Reinhardt (1997b) which takes the nonlinear part of the P-CMOD curve into account. According to the assumption, the effective crack length a_c can be determined on the basis of linear elastic fracture mechanics (see Tada's Stress Analysis of Cracks Handbook, 1985).

$$CMOD = \frac{6PSa}{D^2BE} V_1(\alpha) \tag{1}$$

for S/D = 4, the function $V_1(\alpha)$ is given as follows:

$$V_I(\alpha) = 0.76 - 2.28\alpha + 3.87\alpha^2 - 2.04\alpha^3 + \frac{0.66}{(1-\alpha)^2} \tag{2}$$

where $\alpha = (a + H_0)/(D+H_0)$, P = Load, S = specimen loading span, D = beam depth, B = beam width, H_0 = thickness of clip gauge holder.

Young's modulus E can be calculated from the measured initial compliance C_i through a method proposed by Jenq and Shah (1985) using the equation (3):

$$E = \frac{6 \quad S a_0 V_1(\alpha_0)}{C_i B D} = \frac{24}{C_i B} \frac{a_0}{D} V_1(\alpha_0) \tag{3}$$

where $\alpha_0 = (a_0 + H_0)/(D + H_0)$ and a_0 = initial crack length

On the other hand, according to the testing results of Karihaloo and Nallathambi (1991), Young´s modulus E from standard compressive cylinder tests can be used to predict the average length of the critical effective crack of a group of specimens.

3 The approaches to determine double-K parameters K^{ini}_{Ic} and K^{un}_{Ic}

Concrete as a quasi-brittle softening material shows three different situations of crack propagation: crack initiation, stable crack propagation and unstable fracture. The proposed double-K fracture parameters K^{ini}_{Ic} and K^{un}_{Ic} can be applied to these such three different situations. The criterion for unstable fracture is defined as the critical stress intensity factor K^{un}_{Ic}. So, for a three-point bending notched beam, the K^{un}_{Ic} can be evaluated by inserting the maximum load P_{max} and the critical effective crack length a_c into the following expression (Tada, 1985):

$$K_I = \frac{3PS}{2D^2 B}\sqrt{a}\, F_1\left(\frac{a}{D}\right) \tag{4}$$

where the geometry factor F_1 (a/D) depends on the ratio of span to depth of the beam and which is for S = 4D given as follows:

$$F_1\left(\frac{a}{D}\right) = \frac{1.99 - \left(\frac{a}{D}\right)\left(1 - \frac{a}{D}\right)\left[2.15 - 3.93\frac{a}{D} + 2.7\left(\frac{a}{D}\right)^2\right]}{\left(1 + 2\frac{a}{D}\right)\left(1 - \frac{a}{D}\right)^{3/2}} \tag{5}$$

Theoretically speaking, the initiation toughness K^{ini}_{Ic} is defined as the initial cracking stress intensity factor created at the initial crack tip by the initial cracking load P_i. However, in practical experiments, to distinguish a sole initial cracking load P_i by various investigation approaches is not easy. In ordinary tests, it is not convenient too. Therefore, another approach to determine the initiation toughness K^{ini}_{Ic} is presented.

The initiation toughness K^{ini}_{Ic}, in fact, is the inherent toughness of a material. It implies that a crack does not propagate when the stress intensity factor at the initial crack tip is less than the inherent toughness, i.e., the initiation toughness K^{ini}_{Ic}.

Due to the steady crack propagation, the toughness of a loaded body increases from the value of K^{ini}_{Ic} to the one of K^{un}_{Ic}. The contribution due to cohesive forces is called K^{c}_{Ic}. This leads to:

$$K^{ini}_{Ic} + K^{c}_{Ic} = K^{un}_{Ic} \tag{6}$$

For the critical situation at which the maximum external load is reached and the crack tip opening displacement arrives at its critical value $CTOD_c$, in three-point bending tests, the cohesive toughness K^{c}_{Ic} can be calculated by using a Green`s function as follows:

$$K^{c}_{Ic} = \int_{a_0}^{a_c} \frac{2\sigma(\frac{x}{a_c})}{\sqrt{\pi a_c}} F_2(\frac{x}{a_c}, \frac{a_c}{D}) dx \tag{7}$$

Let $\frac{a_c}{D} = V, \frac{x}{a_c} = U, \quad dx = a_c dU$,

then, when $x = a_c$, $U = 1$; $x = a_0$, $U = a_0/a_c$,

So, eq. (7) can be rewritten as

$$K^{c}_{Ic} = \int_{a_0/a_c}^{1} 2\sqrt{\frac{a_c}{\pi}} \sigma(U) F_2(U,V) dU \tag{8}$$

correspondingly, where

$$F_2(U,V) = \frac{3.52(1-U)}{(1-V)^{3/2}} - \frac{4.35-5.28U}{(1-V)^{1/2}} + \left\{ \frac{1.30-0.30U^{3/2}}{(1-U^2)^{1/2}} + 0.83 - 1.76U \right\} \cdot \{1-(1-U)V\} \tag{9}$$

The corresponding cohesive force distribution $\sigma(x/a_c)$ on the fictitious crack zone shown in equation (8) can be expressed as follows:

$$\sigma(x) = \sigma_s(CTOD_c) + \frac{x-a_o}{a_c-a_o}[f_t - \sigma_s(CTOD_c)] \qquad 0 \le CTOD \le CTOD_c or a_o \le x \le a_c \tag{10}$$

At the integral boundary of eq. (8), the integration has a singularity. Numerical results of the integral can be gained by using Gauss-Chebyshev guadrature.

The $\sigma_s(CTOD_c)$ value in equation (10) can be determined by an expression proposed by Reinhardt et al. (1986) as follows:

$$\frac{\sigma}{f_t} = \left\{1 + (c_1 \frac{w}{w_0})^3\right\}\exp(-c_2 \frac{w}{w_0}) - \frac{w}{w_0}(1+c_1^3)\exp(-c_2) \tag{11}$$

where the coefficient c_1, c_2 are constants and the w_0 is the maximum crack opening width at the stress to be zero. Of course, for the aim of simplicity, a bilinear σ-w relation can be used too.

Once, the $CMOD_c$ is measured in the tests, the $CTOD_c$ can be evaluated by the following expression (see Jenq and Shah (1985)):

$$COD(x) = CMOD_c\left\{\left(1-\frac{x}{a}\right)^2 + \left(1.081 - 1.149\frac{a}{D}\right)\left[\frac{x}{a} - \left(\frac{x}{a}\right)^2\right]\right\}^{1/2} \tag{12}$$

The detailed determination procedure contains the following steps:

1. According to the initial compliance C_i taken from the linear segment of the P-CMOD curve Young's modulus E is calculated using eq. (3),.or E measured from compressive cylinder tests can be used.
2. P_{max}, $CMOD_c$ and E are inserted into eq. (1) to calculate the critical effective crack length a_c.
3. Submitting P_{max} and a_c into eq. (4), K^{un}_{Ic} can be obtained.
4. Using $CMOD_c$ and a_c $CTOD_c$ is evaluated by eq. (12). Then, inserting $CTOD_c$ into eq. (12), $\sigma_s(CTOD_c)$ in eq. (11) can be gained.
5. Carrying out a numerical scheme, the integral value K^{c}_{Ic} of eq. (7) is received.
6. Finally, K^{un}_{Ic} and K^{c}_{Ic} are inserted into eq. (6) and the initiation toughness K^{ini}_{Ic} can be got too .

4 Experimental Validation

The validation of the proposed method for determining the double-K fracture parameters is carried out on the data of Refai and Swartz (1987). It must be noted that in both series B and C, before a beam was tested to failure the beams were precracked using strain control. The initial precrack lengths of ten specimens in series B and fourteen specimens in series C were directly measured using a dye-penetrant technique. Then the authors carried out a satisfactory regression to gain the regression expressions of the compliance calibration curve and the maximum load calibration curve. As a result, the lengths of initial precracks a_i of other specimens were evaluated by the regression expression.

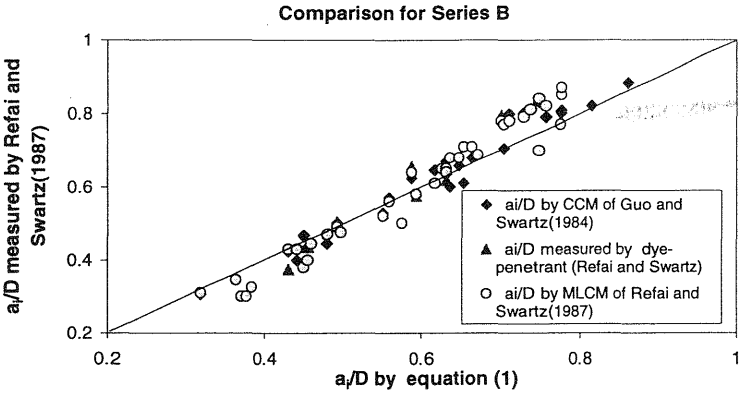
In our evaluation the lengths of the initial precracked cracks a_i, the initial compliance C_i for each beam was carefully determined according to the P-CMOD curves presented in the report of Refai and Swartz (1987). Using the measured values of the initial compliance C_i the lengths of the initial precracked cracks a_i can be calculated according to eq. (1). The related lengths a_i/D of the initial precracks to the depth evaluated by eq. (1) and the corresponding values of a_i/D obtained by the dye-penetrant technique and regression expressions in the report of Refai and Swartz (1987) are compared and plotted in Fig. 1 (a) and (b) for series B and C separately. It can be seen that the values of a_i/D evaluated by eq. (1) are in good agreement with those gained by the approaches used by Refai and Swartz (1987). The mean error is less than 2% compared with those measured by the dye-penetrant technique and is less than 1% compared with those evaluated by the maximum load calibration method and less than 7% compared with those evaluated by the compliance calibration method by Refai and Swartz (1987) respectively for both series B and C. The corresponding coefficients of variation are less than 8%, 9% and 7%, respectively.

Now, the above-mentioned procedures to determine the double-K fracture parameters K^{ini}_{Ic} and K^{un}_{Ic} will be used. In the calculation, for convenience, the σ-w relation shown in eq. (6) and corresponding $c_1 = 3$, $c_2 = 7$ and $w_0 = 160$ μm were used. All values evaluated are presented in Table 1 and Table 2 for series B and C, respectively.

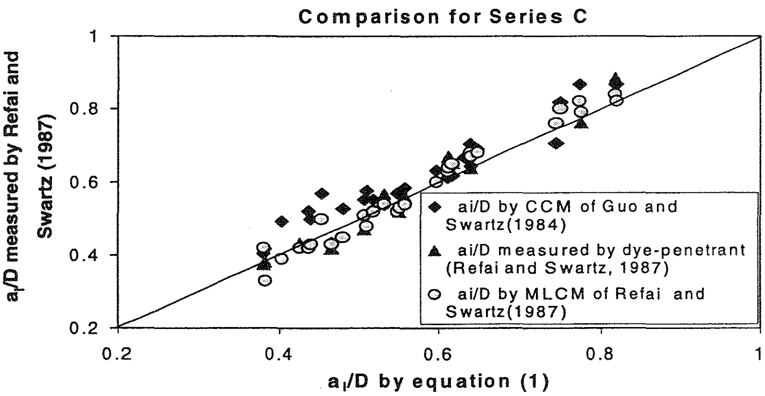
The average values of K^{ini}_{Ic} and K^{un}_{Ic} for series B are 0. 843 MPa $m^{1/2}$ and 1.538 MPa $m^{1/2}$. These values for series C are 0.778 MPa $m^{1/2}$ and 1.651 MPa $m^{1/2}$. The coefficients of variation of the measured values for series B are 0.227 and 0.125. These values for series C are 0.201 and 0.116. This means that in the region of the tested specimen sizes, the evaluated values of K^{un}_{Ic} and K^{ini}_{Ic} are size-independent.

Table 1. The results of double-K fracture parameters K^{ini}_{Ic} and K^{un}_{Ic} determined for series B (S x D x B = 762 x 203 x 76 mm, H_0 =3.2 mm, f_c = 53.1 MPa, E = 38.4 GPa)

Nos.of specs.	C_i x10^{-6} (mm/N)	a_i/D	P_{max} (N)	$CMOD_c$ (μm)	$CTOD_c$ (μm)	a_c/D	K_{Ic}^{c} ($MPam^{1/2}$)	K_{Ic}^{ini} ($MPam^{1/2}$)	K_{Ic}^{un} ($MPam^{1/2}$)
B1	6.289	0.383	5523	45.9	12.67	0.436	0.566	0.795	1.361
B3	8.575	0.442	4365	51.1	12.98	0.499	0.599	0.709	1.308
B4	4.489	0.319	5612	43.4	17.89	0.422	0.822	0.507	1.33
B5	16.489	0.558	3207	89.5	21.07	0.641	0.75	0.883	1.643
B7	29.189	0.648	2249	89.4	12.79	0.69	0.536	0.922	1.457
B8	26.942	0.636	2227	80	11.55	0.677	0.525	0.828	1.354
B9	44.903	0.706	1537	92.8	10.46	0.742	0.503	0.843	1.346
B10	30.534	0.654	2004	77.4	9.59	0.687	0.468	0.818	1.286
B11	82.623	0.775	980	116.6	10.36	0.809	0.519	0.902	1.422
B12	82.623	0.775	891	120.6	12.19	0.82	0.625	0.805	1.43
B13	126.63	0.815	579	130.2	10.97	0.858	0.657	0.757	1.415
B15	6.061	0.376	5033	43.1	13.23	0.442	0.641	0.622	1.264
B16	4.265	0.309	5790	44	18.89	0.419	0.849	0.51	1.358
B17	5.612	0.362	5166	53.3	21.01	0.476	0.872	0.563	1.436
B18	10.46	0.478	4053	65.5	17.54	0.555	0.702	0.766	1.468
B19	11.493	0.495	3919	58	11.83	0.54	0.527	0.822	1.349
B20	9.423	0.459	4187	61.4	17.3	0.538	0.714	0.761	1.43
B21	25.146	0.626	2450	86.6	14	0.675	0.576	0.894	1.47
B22	26.044	0.631	2450	107.4	20.27	0.703	0.714	0.971	1.686
B24	32.869	0.665	1982	86.6	11.26	0.703	0.51	0.865	1.375
B25	23.799	0.617	2784	94	15.71	0.668	0.577	1.031	1.608
B26	82.14	0.774	1559	329.9	43.05	0.854	0.856	2.363	3.219
B27	68.998	0.756	1203	130.5	14.05	0.801	0.594	1.003	1.597
B28	54.76	0.73	1403	141.5	19.5	0.794	0.727	1.019	1.746
B29	63.887	0.748	1069	171.1	25.32	0.833	0.953	0.932	1.885
B30	58.411	0.738	1292	145.5	19.93	0.804	0.758	0.99	1.748
B31	8.979	0.45	4855	82.8	27.49	0.564	0.861	0.945	1.806
B32	35.025	0.673	2138	101.5	13.16	0.713	0.515	1.044	1.559
B33	5.836	0.369	4832	64	27.49	0.521	1.047	0.502	1.549
B34	18.249	0.575	2494	90.3	22.46	0.678	0.895	0.62	1.515
B35	63.887	0.748	891	107.8	13.92	0.811	0.777	0.546	1.323
B36	8.081	0.431	4409	62.8	20.81	0.533	0.827	0.652	1.479
B37	9.203	0.455	4676	73.8	22.73	0.551	0.784	0.88	1.664
B38	26.044	0.631	2539	100	17.17	0.689	0.627	1	1.627
B39	19.758	0.588	2539	87.8	19.48	0.672	0.782	0.716	1.497
B40	11.134	0.49	3830	80.8	24.44	0.599	0.868	0.762	1.63
B41	47.459	0.713	1514	158.5	25.99	0.798	0.87	1.05	1.919
B42	43.808	0.703	1515	184.6	34.54	0.811	1.056	1.074	2.13
B43	242.77	0.863	579	182	7.23	0.878	0.353	1.441	1.794
B44	20.479	0.594	3073	95.5	18.49	0.657	0.641	1.035	1.677
B45	15.713	0.55	3563	80	16.43	0.608	0.606	0.974	1.581
Mean					17.355		0.695	0.843	1.538
S.D.					5.86		0.161	0.191	0.193
C.V.					0.338		0.232	0.227	0.125



(a)



(b)

Fig.1. The comparison between initial crack lengths evaluated by eq.(1) and those measured: (a) for series B; and (b) for series C beams.

It can be also observed that the coefficients of variation of the measured values K^{ini}_{Ic} and K^{un}_{Ic} are in the same range as those for normal strength parameters of concrete like f_t, f_c. For example, the coefficients of variation of the measured values of f_t, f_c for the series B cylinder data are 0.057 and 0.229.

But, the mean values of $CTOD_c$ measured in the two series beams seem size-dependent. For the series B the mean value of $CTOD_c$ is 17.4 μm and for series C it is 22.7 μm.

Table 2. The results of K^{ini}_{Ic} and K^{un}_{Ic} determined for series C.(S x D x B = 1143 x 305 x 76 mm, H_0=3.2 mm, f_c= 54.4 MPa, E = 39.3 GPa)

Nos.of specs.	C_i x10 $(mm/N)^{-6}$	a_i/D	P_{max} (N)	$CMOD_c$ (μm)	$CTOD_c$ (μm)	a_c/D	K_{Ic}^{c} (MPa $m^{1/2}$)	K_{Ic}^{ini} (MPa $m^{1/2}$)	K_{Ic}^{un} (MPa $m^{1/2}$)
C1	6.737	0.403	6547	100.4	40.33	0.552	1.229	0.706	1.935
C2	8.081	0.437	6057	106.8	39.16	0.575	1.179	0.773	1.952
C3	8.216	0.44	5879	96	33.45	0.562	1.093	0.721	1.814
C4	10.101	0.478	5612	98.8	29.16	0.575	0.949	0.865	1.814
C5	14.815	0.546	4543	100.8	22.07	0.612	0.779	0.932	1.711
C6	12.391	0.515	4543	108.5	31.18	0.623	1.046	0.744	1.791
C7	20.202	0.597	3385	107	21.13	0.664	0.814	0.81	1.625
C8	25.14	0.631	3207	120.4	20.43	0.687	0.742	0.984	1.726
C9	26.308	0.638	2450	124	25.06	0.725	1.006	0.639	1.645
C10	28.286	0.648	2494	130.9	24.98	0.729	0.956	0.755	1.711
C11	60.236	0.745	1514	185	25.19	0.814	0.947	1.022	1.969
C12	62.974	0.75	1269	142.3	17.47	0.807	0.856	0.75	1.606
C13	77.577	0.772	846	168.9	22.32	0.852	1.199	0.557	1.756
C14	131.42	0.82	868	198.2	15.76	0.861	0.739	1.232	1.971
C15	8.797	0.453	4899	99.4	36.02	0.598	1.259	0.481	1.739
C16	11.862	0.507	5077	120.3	35.93	0.622	1.069	0.908	1.977
C17	15.713	0.556	4276	98.4	20.47	0.617	0.756	0.896	1.653
C19	15.056	0.549	4498	101.6	22.12	0.615	0.78	0.934	1.714
C20	11.672	0.504	4676	96.4	26.95	0.6	0.969	0.712	1.681
C21	9.428	0.465	5879	82.8	22.13	0.537	0.8	0.862	1.662
C22	6.017	0.381	7660	68.4	21.95	0.455	0.803	0.849	1.653
C23	5.963	0.379	6013	64	23.89	0.488	1.024	0.422	1.446
C24	7.634	0.426	6124	64.4	17.16	0.485	0.729	0.733	1.462
C25	127.77	0.818	668	135	10.1	0.853	0.693	0.81	1.503
C26	13.468	0.529	4276	76	14.8	0.576	0.661	0.752	1.413
C27	22.761	0.616	2895	87.2	13.3	0.657	0.64	0.727	1.367
C28	26.487	0.639	2628	97.6	15	0.686	0.693	0.739	1.432
C29	22.222	0.612	2984	86	12.84	0.651	0.617	0.749	1.366
C30	22.082	0.611	3118	92.5	14.65	0.655	0.654	0.794	1.449
C31	79.403	0.774	1203	119.1	8.47	0.796	0.483	0.93	1.413
C32	5.163	0.352	7571	72.2	29.12	0.468	1.032	0.662	1.695
C33	15.238	0.551	3697	99.3	24.59	0.641	0.961	0.629	1.59
C34	45.634	0.712	1514	95.5	10.9	0.75	0.652	0.605	1.257
Mean					22.669		0.873	0.778	1.651
S.D.					8.235		0.196	0.156	0.192
C.V.					0.363		0.225	0.201	0.116

5 Conclusions

An experimental method is proposed to determine double-K fracture parameters K^{ini}_{Ic} and K^{un}_{Ic} on single size three-point bending notched beams. The initial compliance C_i and the secant compliance C_s is measured without unloading and reloading procedures. It only needs a monotonically increasing load on a beam until the maximum load is reached and the measurement of the P-CMOD curve in the ascending branch. In common materials and structures laboratories, this testing method can be performed even without a closed-loop testing system. The experimental evidence shows that the double-K fracture parameters K^{ini}_{Ic} and K^{un}_{Ic} are size-independent and can be used to describe the material features of cracking initiation and unstable fracture of concrete structures.

6 References

Bazant, Z.P. and Kazemi, M. T. (1990) Determination of fracture energy, process zone length and brittleness number from size effect, with application to rock and concrete. **Int. J. Fracture, 44**, 111-131.

Hillerborg, A. Modeer, M. and Petersson, P.E., (1976), Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements. **Cem. & Concr. Res. 6**, 773-782.

Jenq, Y.S. and Shah, S.P.(1985) Two parameter fracture model for concrete. **J. Engin. Mech. (ASCE), 111**, 1227-1241

Nallathambi, P. and Karihaloo, B.L. (1986) Determination of specimen-size independent fracture toughness of plain concrete. **Magz. of Concr. Res., 38**, 67-76

Karihaloo, B.L. and Nallathambi, P. (1991) Notched beam best: mode I fracture toughness, in **Fracture Mechanics Test Methods for Concrete**(eds S.P. Shah and A. Carpinteri), Chapman & Hall, London, 1-86.

Refai, T.M.E. and Swartz, S.E. (1987) Fracture behavior of concrete beams in three-point bending considering the influence of size effects. Report No.190, Engin. Exper. Station, Kansas State University.

Reinhardt, H.W. et al. (1986) Tensile tests and failure analysis of concrete. **J. Struct. Engin., 112**, 2462-2477.

Tada, H., Paris, P.C. and R. Irwin, G. (1985) **The Stress Analysis of Cracks Handbook**. Paris Productions Incorporated, Missouri, USA.

Xu, S. and Reinhardt, H.W. (1997a) Determination of double-K criterion for crack propagation in quasi-brittle materials, Part 1: Experimental investigation of crack propagation, **Int. J. Fracture**, submitted for publication.

Xu, S. and Reinhardt, H.W. (1997b) Crack extension resistance and fracture properties of quasi-brittle materials like concrete based on the complete process of fracture, **Int. J. Fracture**, submitted for publication.