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# **3D FINITE ELEMENT ANALYSIS OF OVER-REINFORCED BEAMS**

J. Ožbolt, Y.-J. Li, and R. Eligehausen, Institut für Werkstoffe im Bauwesen, University of Stuttgart, Germany

#### Abstract

In the present paper the numerical results for the over-reinforced beams of two different sizes and made of three different concrete qualities, as proposed by RILEM TC 148SSC, are shown. The analysis is performed by the three-dimensional finite element code *MASA3* which is based on the microplane material model for concrete.

Key words: Beams, compression failure, microplane model, numerical analysis, size effect.

#### **1** Introduction

The round robin analysis: "Modeling of Over-reinforced Concrete Beams" has been recently proposed by the RILEM TC 148SSC. The aim of the study is to see whether the current modeling techniques in combination with the standard procedure for the identification of the macroscopical properties of concrete  $(f_i, f_c, G_F)$  are able to correctly predict the behavior of over-reinforced concrete beams. In the present paper a complete numerical results for cases proposed by RILEM are reported. The report contains the results for a small over-reinforced beams made of three different concrete types and the results for a large over-reinforced beams made of normal strength concrete.

#### 2 Geometry, spatial discretization and material properties

Two different geometries of the beams loaded in four-point bending are considered: (1) small beam with the span of l = 3600 mm, the height h = 200 mm and the width b = 100 mm and (2) large beam with l = 7200 mm, h = 400 mm and b = 200. All beams are over-reinforced with the reinforcement ratio of approximately 7.3%. Three different materials are used namely, NSC (normal strength concrete), HSC (high strength concrete) and FRC (fiber reinforced concrete). The study includes the analysis results for the small beam using NSC, HSC and FRC and the results for the large beam made of NSC. The geometry of the beams are shown in Figs. 1 and 2.



Fig. 1 Geometry and reinforcement of the small beam





Fig. 2 Geometry and reinforcement of the large beam



Fig. 3 The 3D finite element mesh - one fourth of the beam

The numerical analysis is carried out by the use of the special purpose three-dimensional FE code *MASA3* that is based on the mixedconstrained formulation of the microplane model for concrete (Ožbolt et al., 1997). The 3D eight node solid FE elements with eight integration points are used for the modeling of concrete. The steel bars (longitudinal reinforcement and hoops) are modeled by as a truss elements. The geometry, including hoops, is as proposed by RILEM. The finite element mesh used in all case studies was the same (see Fig. 3). To avoid mesh sensitivity the crack band approach (Bažant and Oh, 1983) was employed. In the model the symmetry of the beams was exploited and therefore only one quarter of the beam was modeled.

The properties of three different concrete materials used in the study are summarized in Table 1. In Fig. 4 are plotted the corresponding constitutive laws for uniaxial compression. The reinforcement steel is assumed to be ideally elasto-plastic material. For ribbed steel bars the material properties are taken as: Young's modulus  $E_s = 222000$  MPa, Poisson's ratio and the yield stress  $\sigma_y = 650$  MPa. For the stranded wires the following properties were used:  $E_s = 195000$  MPa and  $\sigma_y = 1650$ MPa.

Table 1	l.	Summary of macroscopical concrete properties used in the						
analysis								

Concrete	NSC	HSC	FRC
$E_c$ [MPa]	22000	55000	50000
v	0.18	0.18	0.18
$f_t$ [MPa]	1.89	4.83	7.14
$f_c$ [MPa]	22.75	118	114
G <sub>F</sub> [N/m]	92	125	3780



Fig. 4 Constitutive laws for uniaxial compression: NSC, HSC and FRC

The load was applied by the control of displacement at the mid-line of the loading plate. This plate is modeled by the 3D solid elements. It is with the concrete beam surface connected by the truss elements which can not take up the shear (friction) force between loading plate and concrete surface. For the plate the linear elastic material with:  $E_s = 210000$  MPa and v = 0.3 is adopted. The boundary conditions are the same as proposed by RILEM.

### **3** Numerical results

Peak loads for all cases considered are summarized in Table 2. Fig. 5 shows the calculated load-displacement curves for small beams made of NSC, HSC and FRC. The load represents the beam reaction (half of the total applied load). The displacement is measured at the loading point. All three beams failed by crashing of concrete in the beam compressive zone. The typical failure modes, in terms of the maximal principal strains, are shown in Figs. 6a-c. Note that the beam made of FRC shows higher peak resistance (although the uniaxial compressive strength is slightly lower) h similar peak load but more ductile response than the beam made of HSC.



Fig. 5 Load-displacement curves for small beams made of NSC, HSC and FRC

Table 2. Summary of the predicted peak loads (reaction i.e. one half of the total load)

Concrete	small beam	large beam
NSC [kN]	14.7	56.2
HSC [kN]	50.2	-
FRC [kN]	58.8	-



Fig. 6 Failure modes in terms of the maximal principle strains at post peak for: a) NSC, b) HSC and c) FRC

Load-displacement curves for small and large beams made of NSC are plotted and compared in Fig. 7. As can be seen the peak resistance of larger beam compared to the small one is almost proportional to the increase of the size. There is no significant size effect on the peak load i.e. the relative decrease of the peak load is only about 5%  $(4P_{u,smal} / P_{u,lar} = 1.05)$ . Failure of the large beam (see Fig. 8) is again due to the crushing of concrete in the concrete compressive zone.



Fig. 7 Calculated load-displacement curves for small and large beams made of NSC



Fig. 8 Failure modes of large NSC beam in terms of maximal principle strains at post-peak

It is interesting to pointed out that after the peak resistance was reached we observed an lower yield limit for all beams except for that made of HSC. This due to the yielding of reinforcement of the cross-section at the mid-span whose depth is reduced due to the failure of the compressed part of the whole (intact) cross-section.

# **4** Conclusions

- Numerical analysis shows that all beams fail due to the concrete crashing in the compressive part of the beam.
- The response of the over-reinforced beam made of FRC seems to be more ductile than the response of the same size HSC-beam. Furthermore, the response of HSC beam is much more brittle than that obtained for the beam made of NSC.
- There is no significant size effect on the peak load for NSC beams, however, the post-peak response of larger beams is more brittle i.e. the size effect on the post-peak response tends to be stronger than the size effect on the peak load.

## **5** References

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