Fracture Mechanics of Concrete Structures
Proceedings FRAMCOS-3
AEDIFICATIO Publishers, D-79104 Freiburg, Germany

CONTINUUM FRACTURE THERMODYNAMICS, ENERGY RELEASE AND OVERALL PROPERTIES BOUNDING OF DAMAGING VISCOELASTIC COMPOSITES WITH SOLIDIFICATION

C. Huet,
Laboratory for Building Materials, Department of Materials Science, Swiss Federal Institute of Technology, Lausanne, Switzerland

Abstract
Further applications and new developments of the general dissipative Fracture Thermodynamics formalism presented by the author in FRAMCOS-2 (Zürich, 1995) for the study of the Non-Linear Fracture Mechanics of concrete seen as a microcracked viscoelastic composite are presented by taking account of the effects of the solidification process due to hydration. The forms of the energy release rate tensors are obtained from an appropriate expression of the viscoelastic complementary energy in terms of the overall shrinkage and creep functions. Some aspects of the problem of bounding the overall properties and elaborating scaling laws for size-effects are discussed.
Key words: Concrete, Composites, Fracture, Thermodynamics, Viscoelasticity, Damage, Solidification.

1 Introduction

Mastering the interactions between viscoelastic effects and cracking or damaging effects is an important issue when trying to assess and warrant the durability of concrete structures. We showed in Huet (1994), (1995), (1996) that Dissipative Continuum Thermodynamics provides, in tensor

form, generalized governing equations and criteria for the crack-growth process that are not limited to the linear elastic case, and thus are well adapted to the study of such interactions. In particular, dissipative and possibly nonlinear mechanisms exhibited by viscoelasticity, climatic changes, chemical aging and the granular composite nature of concrete were shown to be automatically incorporated in the obtained formalism through the use of appropriate internal variables. As a first step, a set of crack-growth criteria were derived in Huet (1996) for the viscoelastic case without aging by using a pseudo-convolutive formalism. This was applied to the assessment and bounding of the overall properties of viscoelastic heterogeneous bodies with damage, and their evaluation in terms of the properties of the constituents in Huet (1997a), with, in Huet (1998a), extension to a general - but unspecified - class of materials with aging. In this paper we apply the results to the class of viscoelastic behaviour with aging based on Bazant's solidification theory, which makes possible to express the Thermodynamic functions in terms of the experimental creep function as recently proposed in Bazant and Huet (1997).

2 Dissipative Fracture Thermodynamics and Governing Equations

We consider a body D_0 in quasistatic conditions with negligible volume forces. For each value t of time, x is the coordinates vector of a material point, ξ its displacement, $\dot{\psi}=\frac{\partial \psi}{\partial t}$ is the time derivative of any quantity ψ which may depend on other variables, (.) the once contracted tensor product; U the total internal energy of the body, P the density of the external surface tractions. We consider the monothermal cases for which the temperature is uniform with value T_0 on the external boundary ∂D_0 of the whole body and for which we introduced the global free energy Φ^ε and the negative Φ^σ of the free enthalpy - or of the Gibbs free energy - of the body as:

$$\Phi^\varepsilon = U - T_0 S \quad ; \quad \Phi^\sigma = \int_D \sigma : \varepsilon \, dV \; - \Phi^\varepsilon \tag{2.1}$$

In Huet (1995, 1996, 1997), the derivations were mainly based on the potential energy Ψ^ε. Here, we make use equivalently of the overall complementary energy Ψ^σ for which, for a dissipative material of any kind, we give the same definition as in the elastic case:

$$\Psi^{\sigma} = \Phi^{\sigma} - \int_{\partial D_{0\xi}} P \cdot \xi^{d} \, d\Sigma = -\Psi^{\varepsilon} \tag{2.2}$$

From this, the overall dissipation D^* in quasi-static conditions at constant or negligible volume force density F^d and uniform external temperature T_0 is obtained from the universal energy and entropy balance equations as:

$$D^* = \dot{\Psi}^{\sigma} - \int_{\partial D_{0\sigma}} \xi \cdot \dot{P}^{d} d\Sigma + \int_{\partial D_{0\xi}} P \cdot \dot{\xi}^{d} d\Sigma \geq 0 \tag{2.3}$$

in which appear the rates of the tractions P^d and displacement ξ^d imposed on the parts $\partial D_{0\sigma}$ and $\partial D_{0\xi}$ of the boundary, respectively. On the other hand, in the micromechanical approach using various tools of materials science and microscopic observations providing informations of microscopic origin for Φ^{ε} and Φ^{σ}, Ψ^{σ} and D^* can be written in explicit form as:

$$\Psi^{\sigma} = \underset{\approx}{\Psi}^{\sigma} \left[\Lambda^{di}(t), T_0(t), \{\chi(x,t)\}, a^k(t) ; \bar{H}(q) \right] \tag{2.4}$$

$$D^* = \underset{\sim}{D}^* \left[q, \dot{q} ; \bar{H}(q) \right] = R_F^k(\ldots, \dot{a}^k, \ldots).\dot{a}^k + \underset{\sim 1}{D}^* \left[q, \dot{q} ; \bar{H}(q) \right] \tag{2.5}$$

Here $q = \{ \Lambda^{di}, T_0, a^k, \{\chi^j(x,t)\}, \{\beta^l(x,t)\} \}$ is the set of involved variables including a finite set of loading parameters $\Lambda^{di}(t)$, the surrounding temperature $T_0(t)$, local advancement degrees χ of physical or chemical reactions at each point of D_0, crack tip geometry parameters a^k and other possible local internal variables β^l corresponding for instance to viscoplastic local behaviour in some places, $\bar{H}(q)$ being the past history accounting for memory effects. The R_F^k's are crack resistive forces that, due to the positive character of the dissipation, should be functions of the rate $\dot{a}^k$ of the crack-growth parameter in addition to the possible other ones. Expressing the additivity property of the dissipation, summation is performed on repeated superscripts k. As shown in Huet (1995), (1996), this results in a set of overall governing equations for the cracked or microcracked dissipative body with constituents having behaviour of any kind. It includes a rate equation for the change of each crack-parameter tensor a^k:

$$R^k(...,\dot{a}^k,...) - \frac{\partial \Psi^\sigma}{\partial a^k} = 0 \qquad (2.6)$$

which generalizes to the - possibly non-linear and time dependent - dissipative case the classical Griffith criterion of the elastic case when an extended energy release rate is defined in tensor form for any virtual variation of the crack parameter a^k as a functional of the history by:

$$G^k(...,a^k,...) = \frac{\partial \Psi^\sigma}{\partial a^k} = -\frac{\partial \Psi^\varepsilon}{\partial a^k} \qquad (2.7)$$

This constitutes an application of the hybrid formalism in which discrete sets of internal variables can be combined with history functionals accounting for the delayed response effects, see Huet (1993), (1995), (1996), (1997) for more details. In Huet (1995), we considered the vicoelastic case without aging for bodies in any boundary conditions. Here we particularize to the case with aging for volume elements in uniform boundary conditions, for which the set of loading parameters Λ^{dm} reduces to a single boundary tensor σ_o which is uniform but may be function of time.

3 Apparent strain and stress in uniform boundary conditions

All materials are heterogeneous and structural analyses are performed by using the concept of an equivalent homogeneous medium, valid under appropriate conditions, Huet (1982). Material properties of the equivalent homogeneous medium are determined by testing specimens of finite size, on which the boundary displacement and traction vector only can be measured, Huet (1984). Even in case of a homogeneous matrix, cracked and microcracked bodies must be considered as heterogeneous ones, for which - like for heterogeneous specimens smaller than the representative volume (EVR), Huet (1990) - the stress and strain must be taken as the apparent ones, defined at any time in terms of the boundary displacement and traction vector at the same time by, respectively:

$$\varepsilon^{app} = \frac{1}{V}\int_{\partial D} sym\ (\xi \times n)\, d\Sigma \quad ; \quad \sigma^{app} = \frac{1}{V}\int_{\partial D} sym\ (P \times x)\, d\Sigma \qquad (3.1)$$

where P is the (prescribed or not) boundary traction vector defined at any time. Using the Gauss theorem, ε^{app} and σ^{app} may also, for impervious interfaces, be expressed in terms of the local variables over the

microstructure of the body as:

$$\varepsilon^{app} = <\varepsilon> + \varepsilon^F \quad ; \quad \sigma^{app} = <\sigma> \tag{3.2}$$

where $<b>$ is the strain volume average defined as the integral of any variable b over the solid part D_s of the body and ε^F the additional apparent strain due to cracking:

$$\varepsilon^F \equiv \frac{1}{V}\sum_k \int_{\Gamma_k} sym([\ \xi\ ]^{+}_{-} \times n^{-})\, d\Gamma_k \tag{3.3}$$

the Γ_k's being the individual microcracks, with normal $n^{-} = -n^{+}$ pointing toward the solid material, for more details see Huet (1997). We consider static uniform boundary condition histories of the kind σ_0, abbreviated to σ_0-SUBC, defined by a prescribed stress vector surface density P^d, applied to the whole boundary ∂D of the body D in the form $P^d(u) = \sigma_0(u).n$ where σ_0 is a given symmetric tensor with dimension of stress. Then one has $\sigma^{app} = \sigma_0$ independent of the material behaviour. Moreover, in the absence of body forces and for unloaded interfaces, one has

$$<\sigma : \dot{\varepsilon}> = \sigma_0 : \dot{\varepsilon}^{app} = \sigma^{app} : \dot{\varepsilon}^{app} \tag{3.4}$$

showing that the extended Hill condition is satisfied in that case. This makes possible to write the universal expressions of the dissipation in terms of the apparent variables only:

$$D^*(t) = \dot{\Phi}_\sigma^{app\,c}(t) - \varepsilon^{app}(t) : \dot{\sigma}_0(t) > 0 \tag{3.5}$$

and the same for the other thermodynamic functions, see Huet (1982), (1984), (1990), (1995b), (1997). Applying the dissipative identity and Mandel (1967) non-duality principle already used in Huet (1993), (1995), (1997) shows that the overall complementary energy is thus endowed with the following potential property for the apparent overall strain in terms of the overall stress:

$$\varepsilon^{app}(t) = \frac{\partial \Phi_\sigma^{app}(t)}{\partial \sigma_0} \tag{3.6}$$

which is a non-trivial result in the heterogeneous case. In mixed boundary conditions, $<\sigma : \dot{\varepsilon}>$ cannot be separated in the product of apparent

variables and Eq. (3.6) does not hold unless the representative volume, which gives overall properties independent of the boundary conditions, is reached. This requires the microcracks remaining small.

4 Apparent stress-strain relationship for a microcracked viscoelastic volume element with aging.

For a heterogeneous and microcracked viscoelastic specimen D_α in σ_0-SUBC, the overall σ_0-apparent creep functions tensor $J^{app\,c}_{\sigma\alpha}$ at constant crack pattern is defined in the case of aging by :

$$\varepsilon^{app}(t) = \varepsilon^{0\,app}(u) + \int_{o^-}^{t} J^{app\,c}_{\sigma\alpha}(t,u) : d\sigma_0(u) \tag{4.1}$$

as in the undamaged case and the external loading parameter is the tensor σ_0. At constant cracked pattern, the overall behaviour is linear and the overall complementary energy per unit volume may be expressed as

$$\Phi^{app}_{\sigma}(t) = \Phi^{*app}_{\chi}(t) + \Phi^{app}_{\sigma M}(t) \tag{4.2}$$

where $\Phi^{*app}_{\chi}(t)$ is that part of the complementary energy due to the physico-chemical phenomena that do not depend on the stress state. In this formula, $\Phi^{app}_{\sigma M}(t)$ corresponds to the external loading. In a very general case of linear viscoelasic behaviour with aging, the latter is given by a Fréchet development in the apparent stress history up to the order two in aging form, Huet (1993b), (1998a):

$$\Phi^{app}_{\sigma M}(t) = \int_{o^-}^{t} f^{0\,app}(t-u,t) : d\sigma_0(u) + \frac{1}{2}\int_{o^-}^{t}\int_{o^-}^{t} f^{app}(t-u,t-v,t) : d\sigma_0(v) : d\sigma_0(u) \tag{4.3}$$

which gives $\dfrac{\partial \Phi^{app}_{M}}{\partial \sigma_0}$ as:

$$\frac{\partial \Phi^{app}_{M}}{\partial \sigma_0} = f^{0\,app}(t,t) + \frac{1}{2}\int_{0^-}^{t} f^{app}(0,t-v,t)d\sigma(v) + \frac{1}{2}\int_{0^-}^{t} f^{app}(t-u,0,t)\,]\,d\sigma(u) \tag{4.4}$$

and the continuation functional $\underset{\approx}{\dot{\Phi}}^{app}_{M;q}$ at constant crack pattern as:

$$\underset{\approx}{\dot{\Phi}}^{app}_{M;q} = \int_{o^-}^{t} \dot{f}^{\mathbf{0}\,app}(t-u\,,t) : d\sigma_0(u) \; + \tag{4.5}$$

$$+\frac{1}{2}\int_{o^-}^{t}\int_{o^-}^{t} \dot{f}^{app}(t-u\,,t-v\,,t) : d\sigma_0(v) : d\sigma_0(u)$$

This gives the apparent creep functions tensor $J^{app}(t,u)$ in the form:

$$J^{app}(t,u) = +\frac{1}{2}\left[\; f^{app\,c}(t-u\,,0,t) + f^{app\,c}(0,t-u,t) \;\right] \tag{4.6}$$

5 Extended energy release rates for a microcracked viscoelastic material with aging

For a given distribution of the cracks or microcracks in the body D_α, each apparent property with damage is a function of the set of internal variable tensors. In particular, one has:

$$f^{app\,c}_{\sigma}(t-u,t-v,t) = \underset{\sim}{f}^{app\,c}_{\sigma}\left(t-u,t-v,t;\, a^1_\alpha\,,\, a^2_\alpha\,,\, a^3_\alpha\,,\, \ldots\,,\, a^k_\alpha\,,\ldots,\, a^{m_\alpha}_\alpha\right) \tag{5.1}$$

From Eq. (2.7), this provides the following expression for the extended energy release rate for the k-th crack for time varying σ_0-SUBC:

$$G^k_\alpha(t) = \frac{\partial \Psi_{\underset{\sim}{\chi}}}{\partial a^k_\alpha} + \int_{o^-}^{t} \frac{\partial \underset{\sim}{f}^{\mathbf{0}\,app\,c}_{\sigma\alpha}}{\partial a^k_\alpha}(t-u\,,t) : d\sigma_0(u) \tag{5.2}$$

$$+\frac{1}{2}\int_{o^-}^{t}\int_{o^-}^{t} \frac{\partial \underset{\sim}{f}^{app\,c}_{\sigma\alpha}}{\partial a^k_\alpha}(t-u\,,t-v\,,t) : d\sigma_0(v) : d\sigma_0(u)$$

which can be substituted in the quasi-static dissipative crack-growth equation (2.6). For vanishing external loading stress σ_0, this reduces to an equation governing the development of microcracking due to the internal physico-chemical processes, the latter being generally accompanied by the progressive development of internal stresses due to incompatible endogenic strains. In a creep experiment, for which σ_0 remains constant in time after a first rapid loading with value σ^0_o applied at time t_0, the

energy release rate $G^k_{\sigma\alpha}(t)$ becomes:

$$G^k_{\sigma\alpha}(t) = \frac{\partial \Psi_{\underset{\sim}{\chi}}}{\partial a^k_\alpha} + \frac{V}{2} \frac{\partial \underset{\sim}{f}^{app\,c}_{\sigma\alpha}(t-t_0, t-t_0, t)}{\partial a^k_\alpha} : \sigma^0_0 : \sigma^0_0 \tag{5.3}$$

In a two steps experiment with amplitudes σ^0_0 and σ^1_0 applied at times t_0 and t_1 respectively, it is given by:

$$\begin{aligned} G^k_{\sigma\alpha}(t) = & \frac{\partial \Psi_{\underset{\approx}{\chi}(t)}}{\partial a^k_\alpha} + \\ & + \frac{V}{2} \frac{\partial \underset{\sim}{f}^{app\,c}_{\sigma\alpha}(t-t_0, t-t_0, t)}{\partial a^k_\alpha} : \sigma^0_0 : \sigma^0_0 + \frac{V}{2} \frac{\partial \underset{\sim}{f}^{app\,c}_{\sigma\alpha}(t-t_1, t-t_1, t)}{\partial a^k_\alpha} : \sigma^1_0 : \sigma^1_0 + \\ & + \frac{V}{2} \left[\frac{\partial \underset{\sim}{f}^{app\,c}_{\sigma\alpha}(t-t_0, t-t_1, t)}{\partial a^k_\alpha} + \frac{\partial \underset{\sim}{f}^{app\,c}_{\sigma\alpha}(t-t_1, t-t_0, t)}{\partial a^k_\alpha} \right] : \sigma^0_0 : \sigma^1_0 \end{aligned} \tag{5.4}$$

which in a creep-recovery experiment gives:

$$\begin{aligned} G^k_{\sigma\alpha}(t) = & \frac{\partial \Psi_{\underset{\approx}{\chi}(t)}}{\partial a^k_\alpha} + \\ & + \frac{V}{2} \left[\frac{\partial \underset{\sim}{f}^{app\,c}_{\sigma\alpha}(t-t_0, t-t_0, t)}{\partial a^k_\alpha} + \frac{V}{2} \frac{\partial \underset{\sim}{f}^{app\,c}_{\sigma\alpha}(t-t_1, t-t_1, t)}{\partial a^k_\alpha} \right. \\ & \left. - \frac{\partial \underset{\sim}{f}^{app\,c}_{\sigma\alpha}(t-t_0, t-t_1, t)}{\partial a^k_\alpha} - \frac{\partial \underset{\sim}{f}^{app\,c}_{\sigma\alpha}(t-t_1, t-t_0, t)}{\partial a^k_\alpha} \right] : \sigma^0_0 : \sigma^0_0 \end{aligned} \tag{5.5}$$

6 Application to solidifying viscoelastic materials

In the general expressions obtained above for ε, ε^0, $J^{app\,c}$ and $G^k_{\sigma\alpha}(t)$, the multivariables kernels above cannot be directly identified from purely mechanical experiments since the measurable variables ε^{app} and $\varepsilon^{\circ app}(t)$ involve values of these kernels for which one of these time arguments is zero, making difficult the separation of energies in the general case. Thus, this needs still further information, that may be of microscopic origin as provided by Integrated Micromechanics Approach, or a priori additional

assumptions that have to be checked afterwards by their consequences. When the complementary energy kernel may be, for some particular class of material, expressed in terms of the creep function only, we speak of a reducing material. The key for making possible the identification from isothermal mechanical experiments is to reduced by 1 the number of arguments involved in the kernels f and f^0 in such a way that they can be expressed in terms of the experimental swelling (negative of the shrinkage) $\underset{\sim}{\varepsilon}^{\circ app\,c}_{\sigma\alpha}(t)$ and experimental creep function $J^{app\,c}(t,u)$. In Huet (1993), we presented a first possible assumption that defines a first class of reducing materials for the relaxation function. In this paper, we particularize to a second class of reducing materials which has been proposed in Bazant and Huet (1997) and was derived from Bazant's solidification theory, Carol and Bazant (1993). We assume that the purely physico-chemical parts of the thermodynamic functions are not affected by the virtual crack-growth rates which of course is only an approximation. The second class of reducing materials is obtained by stating that $f^{0\,app\,c}(t_1,t)$ is reduced as $\varepsilon^{0\,app\,c}(t)$ and $f^{app\,c}$ in the form:

$$f^{app\,c}(t_1,t_2,t)=\frac{1}{2}[\ Y(t_2-t_1)L^{app\,c}(t+t_1,t-t_2)+Y(t_1-t_2)L^{app\,c}(t+t_2,t-t_1)\ ] \quad (6.1)$$

where $Y(t)$ is the Heaviside step function, equal to zero for negative t, and to 1 for positive t. This gives:

$$f^{app\,c}(t-u,t-v,t)=\frac{1}{2}[\ Y(u-v)L^{app\,c}(2t-u,v)+Y(v-u)L^{app\,c}(2t-v,u)\ ] \quad (6.2)$$

and thus, since v is never larger than t in the integration domain:

$$J(t,v)=f^{app\,c}(0,t-v,t)=f^{app\,c}(t-v,0,t)=L^{app\,c}(t,v) \quad (6.3)$$

from which the quadratic term of $\Phi^{app}_{\sigma M}$, reduces to:

$$\Phi^{app\,c}_{\sigma M}(t)=\int_0^t\int_0^u J^{app\,c}(2t-u,v)d\sigma(v)d\sigma(u)=\int_0^t\int_u^t J^{app\,c}(2t-v,u)d\sigma(v)d\sigma(u) \quad (6.4)$$

giving finally the energy release rates in the two equivalent forms:

$$G^k(t) = \frac{\partial \Psi_{\underset{\sim}{\chi}}}{\partial a_\alpha^k} + \frac{\partial \underset{\sim}{\varepsilon}^{0\,app\,c}_{\sigma\alpha}(t)}{\partial a_\alpha^k} : \sigma_0(t)$$
$$+ \int_{o^-}^{t} \int_{o^-}^{u} \frac{\partial \underset{\sim}{J}^{app\,c}_{\sigma\alpha}(2t-u, v)}{\partial a_\alpha^k} : d\sigma_0(v) : d\sigma_0(u) \qquad (6.5)$$

or:

$$G^k(t) = \frac{\partial \Psi_{\underset{\sim}{\chi}}}{\partial a_\alpha^k} + \frac{\partial \underset{\sim}{\varepsilon}^{0\,app\,c}_{\sigma\alpha}(t)}{\partial a_\alpha^k} : \sigma_0(t)$$
$$+ \int_{o^-}^{t} \int_{u}^{t} \frac{\partial \underset{\sim}{J}^{app\,c}_{\sigma\alpha}(2t-v, u)}{\partial a_\alpha^k} : d\sigma_0(v) : d\sigma_0(u) \qquad (6.6)$$

It can be remarked that when a single creep function is involved in an isostatic problem, the correspondence principle applies, and the partial derivative affects a geometrical coefficient only. Then, $G^k(t)$ in creep is simply proportional to the scalar creep function. For the recovery test considered in Section 5, $G^k(t)$ may become negative as is the case in the non-aging case as shown in Guidoum et al. (1998). This mean that during the crack-growth, more energy is absorbed from the loading machine than restituted to it by elasticity. Illustrative examples of numerical computations of the time evolution of $G(t)$ in the nonaging case using a viscoelastic finite element software for the micromechanics of granular composites developed in our laboratory by Guidoum et al. (1993) are shown on Figures 1 and 2.

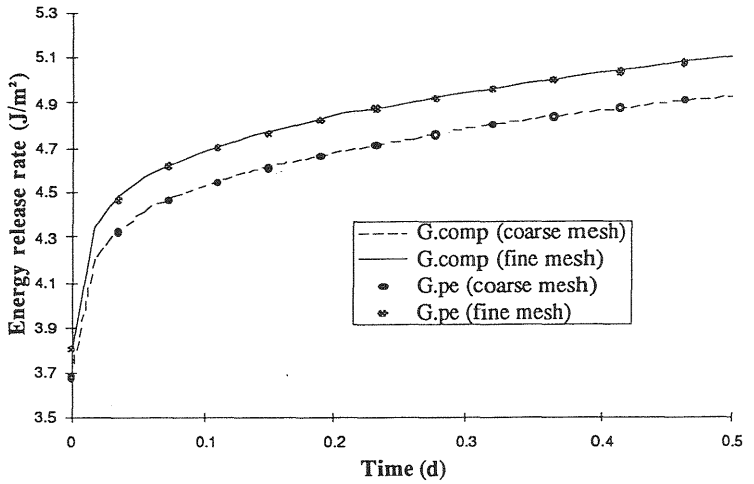

Fig. 1. Finite element compution of the energy-release rate evolution of a notched beam during creep: comparison of various modes of meshing and computation.

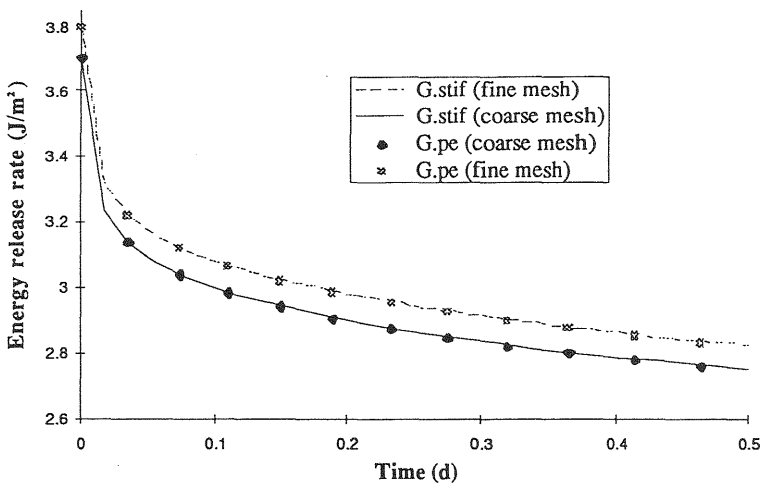

Fig. 2. Finite element compution of the energy-release rate evolution of the same beam in relaxation (same comparison)

Two methods have been compared: an internal one directly computing the elastic energy from the internal variables in each element, and an external one using the formulae we presented in FRAMCOS 2, Huet (1995). Two kinds of finite elements were also used at the crack tips, standard ones and quarter-point singular ones. Very good agreement has been observed between the two methods when singular finite elements are used, confirming the physical significance of the derived criterion and evolution equations.

7 Bounds on overall properties

In the elastic case, it was shown in Hazanov and Huet (1994) that the overall properties for the mixed boundary conditions are bracketed by the ones obtained in σ_0-SUBC on the one hand, in ε_0-KUBC (kinematic uniform boundary conditions with imposed displacement $\xi^d = \varepsilon_0.x$ on the other hand, ε_0 being a constant tensor with dimension of strain. Moreover, two-sided hierarchies for size-effects have been obtained in Huet (1990) for the elastic case without cracks and extended to the elastic case with cracks in Huet (1997). This has been applied to the analysis of heterogeneous elastic structures by Ostoja-Starzewski (1993), (1994) - using two-sided bounds - and by Zohdi et al. (1996), using domain decomposition methods. The bounds and hierarchies obtained in Huet (1990), Hazanov and Huet (1994) have been verified by numerical experiments on numerical models of granular composites in Guidoum (1994), Huet et al. (1995) both for the 2D and 3D cases. They have been verified experimentally in Amieur et al. (1995), Hazanov and Amieur (1995). For viscoelastic bodies, the problem is more difficult. Bounds have been derived for the non-aging viscoelastic case without damage in Huet (1995) and with damage in Huet (1997). The comparison with numerical simulations in the 1D, 2D and 3D case have shown good agreement despite the fact that further consideration has shown the derivation still needs improvements. Moreover, the obtained bounds appear as very good approximation, confirming the observations made long ago by Schapery (1974). Another formulation, which applies to the case studied here, has been given in Huet (1998b) for the case with aging. In all cases, the derivation makes use of associated elastic or thermoelastic problems to which classical minimum theorems of elasticity apply. When they give bounds for the viscoelastic problems, the hierarchies obtained in Huet (1990) apply to the viscoelastic solution.

Acknowledgments

Financial supports from the Swiss National Science Foundation, the Swiss Federal Office for Water Economy and the Swiss Federal Institutes of Technology Council are gratefully acknowledged. We thank also our coworkers A. Guidoum and C. Cecot for performing the numerical simulations.

References

Amieur, M., Hazanov, S., Huet, C., Numerical and experimental assessment of the size and boundary conditions effects for the overall properties of granular composite bodies smaller than the representative volume. In D. F. Parker and A.H. England (Eds), **IUTAM symp. on anisotropy, inhomogeneity and nonlinearity in solid mechanics**, Kluwer Academic Publishers, Netherlands, 1995, 149-154.

Bazant Z.P. and Huet, C. (1997) Thermodynamic Functions for Aging Viscoelasticity: Integral Form Without Internal Variables (submitted for publication).

Carol, I. and Bazant, Z.P. (1993) Viscoelasticity with aging caused by solidification of non-aging constituent. ASCE **Journal of Engineering Mechanics**,119 (11), 2252--2269.

Guidoum A. (1994) Simulation numérique 3D des comportements des bétons en tant que composites granulaires. **Doctoral Thesis no 1310**, Ecole polytechnique fédérale, Lausanne, Suisse.

Guidoum, A., Navi, P. and Huet, C., (1993b), Numerical evaluation of microstructural effects on the shrinkage properties of concrete through a three-dimensional model of cohesive granular material. In **Creep and Shrinkage of concrete** (eds Z. P. Bazant and I. Carol) , Spon, London, 1993, 127-132.

Guidoum, A., Huet C., Bengougam A., Cecot C. (1998) 2D and 3D Finite-Element micromechanical computation of the long term development of internal stresses and deterioration in random viscoelastic granular composites with chemical solidification, climate sensitivity and microcracks, In E. Oñate and S.R. Idelsohn (Eds) **Computational Mechanics: New Trends and Applications**, Proceedings of the Buenos Aires WCCM, CIMNE, Barcelona (submitted).

Hazanov, S., Amieur, M., On Overall Properties of Elastic Heterogeneous Bodies Smaller than the Representative Volume, **International Journal of Engineering Science**, vol. 33, n° 9, 1995, 1289-1301.

Huet, C. (1990) Application of variational concepts to size effects in elastic heterogeneous bodies. **Journal of the Mechanics and Physics of Solids**, Vol. 38, 6, 813-841.

Huet, C. (1993a) **Micromechanics of Concrete and Cementitious Materials.** Presses Polytechniques et Universitaires Romandes, Lausanne, Switzerland.

Huet, C. (1993b) Some basic tools and pending problems in the development of constitutive equations for the delayed behaviour of Concrete, in **Creep and**

Shrinkage of Concrete (eds Z.P. Bazant and I. Carol), Spon, London, 189-200.

Huet, C. (1994a) Some Aspects of the Application of Fracture Mechanics to Concrete Dams, in **Dam Fracture & Damage** (eds E. Bourdarot, J. Mazars and V. Saouma), Balkema, Rotterdam, 79-89.

Huet, C. (1996) A continuum thermodynamics approach for size effects in the failure of concrete type materials and structures, in **Size-scale Effects in the Failure Mechanisms of Materials and Structures** (ed. A. Carpinteri), IUTAM Symposium, Torino, October 1994, Spon, London (in press).

Huet, C. (1995a) A Continuum Thermodynamics Approach for Studying Microstructural Effects on the Non-Linear Fracture Behaviour of Concrete Seen as a Multicracked Granular Composite Material. In **Fracture Mechanics of Concrete and Concrete Structures** (ed. F.H. Wittmann), Aedificatio, Freiburg, vol. 2, 1089-1108.

Huet, C. (1995b) Hybrid continuum thermodynamics framework and numerical simulations examples for the delayed micromechanical behaviour of heterogeneous materials with chemical, climatic and defects sensitivity. In **Engineering Mechanics Today** (eds Q. S. Nguyen and V. D. Nguyen), University of Hanoi, 170-184.

Huet, C. (1995c) Bounds for the overall properties of viscoelastic heterogeneous and composite materials. **Archives of Mechanics**, 47, 6, 1125-1155.

Huet, C. (1996a) Recent advances in the long term deformation and deterioration behaviour of structural materials and components through the integrated micromechanics and thermodynamics of solids approach. In **Advances in building materials** (ed. A. Gerdes), Aedificatio, Freiburg, 161-196.

Huet, C. (1997) An Integrated Micromechanics and Statistical Continuum Mechanics Approach for studying the Fracture Behaviour of Microcracked Heterogeneous Materials with delayed Response. **Engineering Fracture Mechanics**, special issue on Statistical Fracture Mechanics (in press).

Huet (1998a) Dissipative continuum and fracture thermodynamics, energy release and hierarchical bounds for overall properties of damaging viscoelastic composites with dissolution, solidification and microcracks, In E. Oñate and S.R. Idelsohn (Eds) **Computational Mechanics: New Trends and Applications**, Proceedings of the Buenos Aires WCCM, CIMNE, Barcelona (submitted, 1998).

Huet (1998b) Size and Boundary condition effects in heterogeneous and composite bodies with standing or growing defects". In Z.P. Bazant (Ed.) **Symposium on Size Effects and Scaling Laws,** US Nat. Cong. Appl. Mech., Gainesville (submitted).

Huet, C., Guidoum, A. and Navi, P. (1995) A 3D micromechanical model for numerical analysis and prediction of long term deterioration in concrete. In **Concrete under Severe Conditions: Environment and loading** (vol. 2) (Eds. K. Sakai, N. Banthia, O.E. Gjorv), Spon, London, 1458-1467.

Ostoja-Starzewski, M. (1993), Micromechanics as a basis of stochastic finite elements and differences-an overview, **Applied Mechanics Review**, Special Issue: Mechanics Pan America , 46, Part 2, S136-S147.

Ostoja-Starzewski, M., (1994) Micromechanics as a basis of continuum random field, **Applied Mechanics Review, Special Issue: Micromechanics of random media**, 47, 1, part 2, S221-S230.

Ostoja-Starzewski, M., (1997) Size and boundary conditions effects in elasticity and damage mechanics of random composites. In J.W. Ju, J.L. Chaboche and G. Voyadjis, (Eds.), Damage Mechanics, **Stud. Appl. Mech**. (in press).

Schapery, R. A. (1974) Viscoelastic behaviour and analysis of composite materials. In G.P. Sendecky (Ed.) **Mechanics of composite materials**, Academic Press, N.Y.

Staverman A.J. and Schwarzl, P. (1952) Non-equilibrium Thermodynamics of Visco-elastic Behaviour, **Proc. Acad. Sc.**, The Netherlands, 55, 486-492.

Wang, J., Navi, P. and Huet, C.(1992) Numerical Study of Granule Influences on the Crack Propagation in Concrete. In Z.P. Bazant (Ed.), **Fracture Mechanics of Concrete Structures**, Elsevier, 1992, 373-378.

J. Wang, P. Navi, C. Huet, "Numerical analysis of crack propagation in concrete under tension specimens considered as a 2D multicracked granular composite", in **Materials and Structures**, 30, pp.11-21 (1997).

Zohdi, T.I., Oden J. T. & Rodin G. J. (1996), Hierarchical modeling of heterogeneous bodies. **Comput. Methods Appl. Mech. Engrg**. 138, 273-298.