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FRACTURE-PLASTIC MATERIAL MODEL FOR CONCRETE, APPLICATION TO ANALYSIS OF POWDER ACTUATED ANCHORS

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Abstract
A constitutive material model is described, which combines smeared crack model with plasticity. The main objective of the model is to simulate concrete crushing under high confinement, concrete cracking and crack closing due to crushing in other material directions.
Key words: Smeared cracks, fracture, plasticity, finite element method

1 Introduction

This paper covers the description of a constitutive material model for concrete and its application to analysis of anchoring problems. The model is based on orthotropic smeared crack model. The method of strain decomposition is used to combine fracture with a plasticity based model for simulation of concrete crushing. Both fracture and plasticity models are developed within the framework of elastic predictor and inelastic corrector. This approach guarantees the solution for all magnitudes of strain increment. From an algorithmic point of view the problem is then transformed into finding an optimal return point on the failure surface. The interaction between fracturing and plastic processes is reflected in the method. A general iterative algorithm is devised to determine the separation of strains into elastic, fracturing and plastic components, such that the stresses in the fracture and plastic models are equivalent.

This model was applied to the analysis of anchoring problems. Powder actuated anchors are inserted into intact concrete by explosion. During this insertion phase high confinement stresses occur around the anchor head. Axi-symmetrical and 3D analyses are used to simulate this behavior using the proposed constitutive model. In the early stages, radial cracks around the anchor head can be observed. These cracks are later closed due to large plastic circumferential strains induced by concrete crushing. After the radial crack closure high confining stresses can build up around the anchor head resisting the pull-out force. Very good agreement is obtained between the experimentally and analytically determined pull-out forces.

2 Material model formulation

The material model formulation is based on the strain decomposition into elastic ε_{ij}^{e}, plastic ε_{ij}^{p} and fracturing ε_{ij}^{f} components, de Borst (1986).

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{p} + \varepsilon_{ij}^{f} \tag{1}$$

The new stress state is then computed by the formula:

$$\sigma_{ij}^{n} = \sigma_{ij}^{n-1} + E_{ijkl}(\Delta\varepsilon_{ij} - \Delta\varepsilon_{ij}^{p} - \Delta\varepsilon_{ij}^{f}) \tag{2}$$

where the increments of plastic strain $\Delta\varepsilon_{ij}^{p}$ and fracturing strain $\Delta\varepsilon_{ij}^{f}$ must be evaluated based on the used material models.

2.1 Rankine-fracturing model for concrete cracking

Rankine criterion is used for concrete cracking

$$F_i^f = \sigma_{ii}^{t'} - f_{ti}' \leq 0 \tag{3}$$

It is assumed that strains and stresses are converted into the material directions, which in case of rotated crack model correspond to the principal directions and in case of fixed crack model are given by the principal directions at the onset of cracking. Therefore, $\sigma_{ii}^{t'}$ identifies the trial stress in the material direction i. Prime symbol denotes quantities in the material directions. The trial stress state is computed by elastic predictor.

$$\sigma_{ij}^{t'} = \sigma_{ij}^{n-1'} + E_{ijkl}\Delta\varepsilon_{kl}' \tag{4}$$

If the trial stress does not satisfy Eqn.(3), the increment of fracturing strain in direction i can be computed using the assumption that the final stress state must satisfy Eqn.(3).

$$F_i^f = \sigma_{ii}^{n'} - f_{t\,i}' = \sigma_{ii}^{t'} - E_{iikl}\Delta\varepsilon_{kl}^{f'} - f_{t\,i}' = 0 \tag{5}$$

This equation can be further simplified under the assumption that the increment of fracturing strain is normal to the failure surface and that always only one failure surface is being checked. For failure surface k the fracturing strain increment has the following form.

$$\Delta\varepsilon_{ij}^{f'} = \lambda \frac{\partial F_k^f}{\partial \sigma_{ij}} = \lambda\ \delta_{ik} \tag{6}$$

After substitution into Eqn.(5) a formula for the fracturing multiplier λ is recovered.

$$\Delta\lambda = \frac{\sigma_{kk}^{t'} - f_{t\,k}'}{E_{kkkk}} = \frac{\sigma_{kk}^{t'} - f_{t\,k}'(w\ )}{E_{kkkk}} \quad \text{and} \quad w = L_t(\hat{\varepsilon}_{kk}^{f'} + \Delta\lambda) \tag{7}$$

This equation must be solved iteratively since for softening materials the value of current tensile strength $f_{t\,k}'(w)$ is a function of crack opening w, which is computed from the total value of fracturing strain $\hat{\varepsilon}_{kk}^{f}$ in direction k, plus the current increment of fracturing strain $\Delta\lambda$ multiplied by the

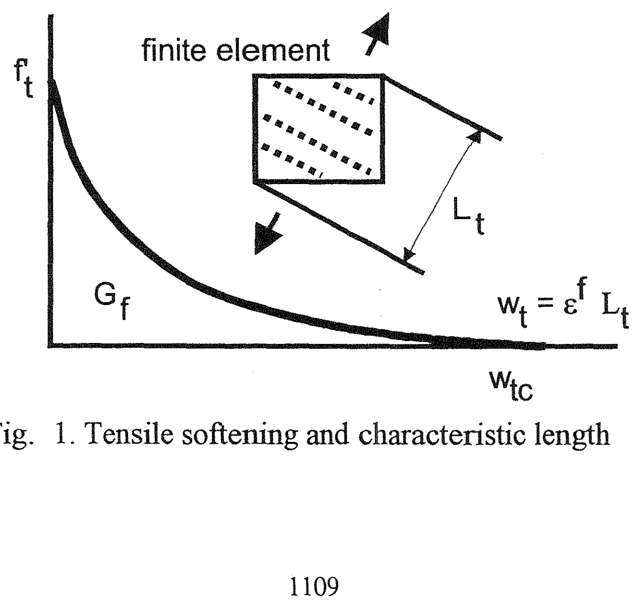

Fig. 1. Tensile softening and characteristic length

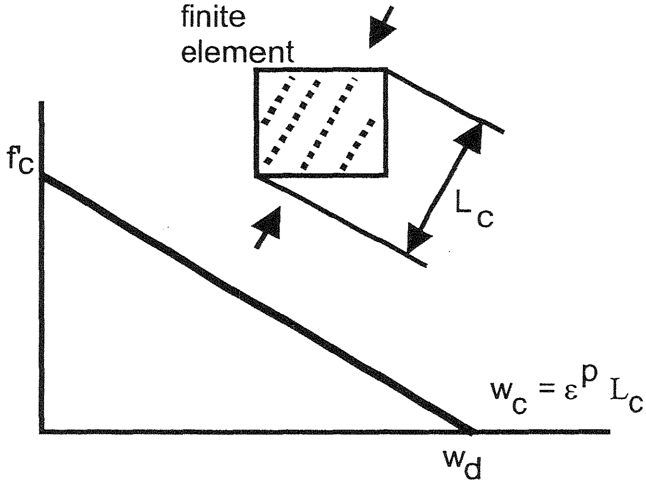

Fig. 2. Compressive softening and compressive characteristic length. Based on experimental observations by van Mier (1986).

characteristic length L_t. Characteristic length represents the width or size of the material volume projected into the direction k. Hordijk's formula, (Hordijk (1991)), is used for the softening function $f'_{t\,k}(w)$. The equation (7) can be solved by recursive substitutions. It is possible to show by expanding $f'_{t\,k}(w)$ into Taylor series that this iteration scheme converges as long as:

$$\left| -\frac{\partial F_k^f(w)}{\partial w} \right| < \frac{E_{kkkk}}{L} \tag{8}$$

This equation is violated for softening materials when snap back is observed in the stress-strain relationship, which can occur if large finite elements are used.

It is important to distinguish between total fracturing strain $\hat{\varepsilon}_{ij}^{f'}$, which corresponds to the maximal fracturing strain reached during the loading process, and current fracturing strain $\varepsilon_{ij}^{f'}$, which can be smaller due to crack closure and is computed using the Eqn.(9) derived in Rots and Blaauwendraad (1989).

$$\varepsilon_{kl}^{f'} = (E_{ijkl} + E_{ijkl}^{cr})^{-1}\, E_{ijkl}\, \varepsilon_{kl}', \text{ and } E_{ijlk}^{cr} \text{ is defined by } \sigma_{ij}' = E_{ijkl}^{cr} \varepsilon_{kl}^{f'} \tag{9}$$

2.2 Plasticity model for concrete crushing

New stress state in the plastic model is computed using the predictor-corrector formula.

$$\sigma_{ij}^{n} = \sigma_{ij}^{n-1} + E_{ijkl}(\Delta\varepsilon_{ij} - \Delta\varepsilon_{ij}^{p}) = \sigma_{ij}^{t} - E_{ijkl}\Delta\varepsilon_{ij}^{p} = \sigma_{ij}^{t} - \sigma_{ij}^{p} \tag{10}$$

The plastic corrector σ_{ij}^{p} is computed directly from the yield function.

$$F^{p}(\sigma_{ij}^{t} - \sigma_{ij}^{p}) = F^{p}(\sigma_{ij}^{t} - \Delta\lambda l_{ij}) = 0 \tag{11}$$

The main problem is then the definition of the return direction l_{ij}, which can be defined as

$$l_{ij} = E_{ijkl}\frac{\partial G^{p}(\sigma_{ij}^{t})}{\partial\sigma_{ij}} \quad \text{then} \quad \Delta\varepsilon_{ij}^{p} = \Delta\lambda\frac{\partial G^{p}(\sigma_{ij}^{t})}{\partial\sigma_{ij}} \tag{12}$$

where $G(\sigma_{ij})$ is the plastic potential function, whose derivative is evaluated at the predictor stress state σ_{ij}^{t} to determine the return direction.

Drucker-Prager failure surface is used with the following formulas for yield function and plastic potential.

$$F^{p}(\sigma_{ij}) = \alpha I_1 + \sqrt{J_2} - k = 0, \quad \text{and} \quad G^{p}(\sigma_{ij}) = \alpha\xi I_1 + \sqrt{J_2} - k \tag{13}$$

where I_1 and J_2 denote first invariant of the stress tensor and second deviatoric invariant respectively. α and k are parameters of the Drucker-Prager surface and ξ determines the return direction of the plastic corrector (i.e. plastic flow direction). If $\xi = 1$ associated plasticity is recovered, if $\xi = 0$ material volume is preserved during crushing, if $\xi > 0$ material volume is increasing , and for $\xi < 0$ volume is decreasing.

2.3 Combination of plasticity and fracture model.

The objective is to combine the above models to a single model such that plasticity is used for concrete crushing and the Rankine fracture model for cracking. This problem can be generally stated as a simultaneous solution of the two following inequalities.

$$F^{p}(\sigma_{ij}^{n-1} + E_{ijkl}(\Delta\varepsilon_{kl} - \Delta\varepsilon_{kl}^{f} - \Delta\varepsilon_{kl}^{p})) \leq 0 \quad \text{solve for } \Delta\varepsilon_{j}^{p} \tag{14}$$

$$F^{f}(\sigma_{ij}^{n-1} + E_{ijkl}(\Delta\varepsilon_{kl} - \Delta\varepsilon_{kl}^{p} - \Delta\varepsilon_{kl}^{f})) \leq 0 \quad \text{solve for } \Delta\varepsilon_{j}^{f} \tag{15}$$

Each inequality depends on the output from the other one, therefore the following iterative scheme is developed.

$$F^p(\sigma_{ij}^{n-1} + E_{ijkl}(\Delta\varepsilon_{kl} - \Delta\varepsilon_{kl}^{f(i-1)} + b\Delta\varepsilon_{kl}^{cor(i-1)} - \Delta\varepsilon_{kl}^{p(i)})) \leq 0 \text{ solve for } \Delta\varepsilon_j^{p(i)}$$

$$F^f(\sigma_{ij}^{n-1} + E_{ijkl}(\Delta\varepsilon_{kl} - \Delta\varepsilon_{kl}^{p(i)} - \Delta\varepsilon_{kl}^{f(i)})) \leq 0 \quad \text{solve for } \Delta\varepsilon_j^{f(i)}$$

$$\Delta\varepsilon_{ij}^{cor(i)} = \Delta\varepsilon_{ij}^{f(i)} - \Delta\varepsilon_{ij}^{f(i-1)} \tag{16}$$

Iterative correction of the strain norm between two subsequent steps can be expressed as

$$\left\|\Delta\varepsilon_{ij}^{cor(i)}\right\| = (1-b)\,\alpha^f \alpha^p \left\|\Delta\varepsilon_{ij}^{cor(i-1)}\right\| \tag{17}$$

where $\alpha^f = \dfrac{\left\|\Delta\varepsilon_{ij}^{p(i)} - \Delta\varepsilon_{ij}^{p(i-1)}\right\|}{\left\|\Delta\varepsilon_{ij}^{f(i)} - \Delta\varepsilon_{ij}^{f(i-1)}\right\|}$, $\alpha^p = \dfrac{\left\|\Delta\varepsilon_{ij}^{cor}\right\|}{\left\|\Delta\varepsilon_{ij}^{p(i)} - \Delta\varepsilon_{ij}^{p(i-1)}\right\|}$

and b is an iteration corrector , which is introduced in order to guarantee convergence. It is to be determined based on the analysis of α^f and α^p, such that the convergence of the iterative scheme can be assured. The necessary condition for the convergence is

$$\left|(1-b)\alpha^f \alpha^p\right| < 1 \tag{18}$$

If b equals 0 iteration algorithm based on recursive substitution is obtained. The convergence can be guaranteed only in two cases: if one of the models is not activated (i.e. implies α^f or $\alpha^p = 0$), or there is no softening in either of the two models. It can be shown that the value of α^f and α^p is directly proportional to the softening rate in each model. Since the softening model is usually a constant for a material model and finite element, it does not change significantly between iterations, and it is possible to select the scalar b such that the inequality (18) is satisfied always at the end of each iteration based on the current values of α^f and α^p. This approach guarantees convergence and its rate depends on material brittleness and finite element size.

Additional constraints are used in the iterative algorithm. If the stress state at the end of the first step violates the Rankine criterion the order of the first two steps is reversed (Fig. 3). Also in reality concrete crushing in one direction has an effect on the cracking in other direction. It is assumed that after the plasticity yield function is violated the tensile strength in all material directions is zero.

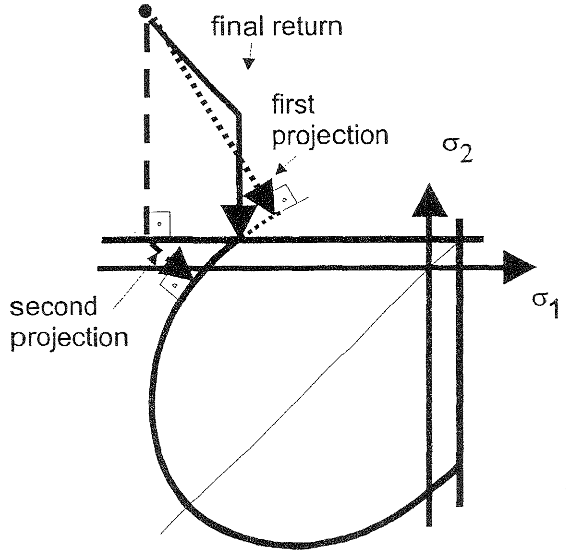

Fig. 3. Schematic description of the iterative process (16). For clarity shown in two dimensions.

On the structural level always secant matrix is used in order to achieve a robust convergence during the strain localization process.

4 Analysis of powder actuated anchors

Powder actuated anchors are inserted into the intact concrete by explosion. Behavior simulation of these anchors was an objective of a cooperative project between Cervenka Consulting and University of Stuttgart. During the insertion phase the friction between steel and concrete causes high temperatures in the steel and partial melting of the material near the outer surface of the anchor. This causes mixing of the two materials: concrete and steel. Therefore, there is no distinct boundary and a very good bond between the two materials. Thus the load carrying capacity of the anchor is mainly given by the strength of the highly damaged concrete surrounding the end of the nail. This was also the objective of the presented analysis: to simulate concrete crushing and cracking around the anchor. The material changes, which occur on the contact between steel and concrete during the fast insertion, are not addressed by this study.

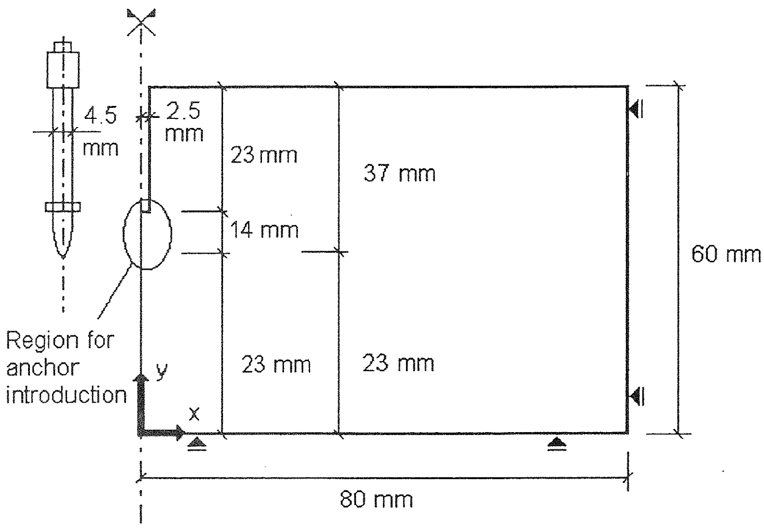

Fig. 4. Geometry of the specimen for testing of powder actuated anchors

4.1 Problem description

Anchors are simulated by axi-symmetrical and 3D analyses. The geometry of the problem is shown in Fig. 4. The anchor is inserted into a pre-drilled hole by explosion.

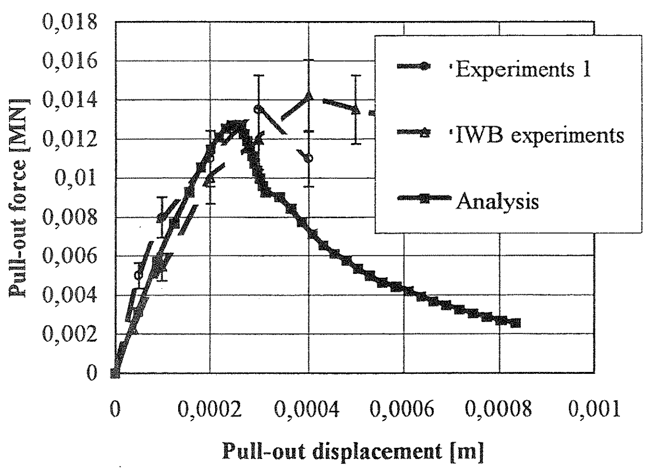

Fig. 5. Comparison of the finite element analysis with experimental data.

This insertion phase was simulated by expanding a hole with zero initial radius by prescribing horizontal displacements at the finite element nodes at the location of the anchor head.

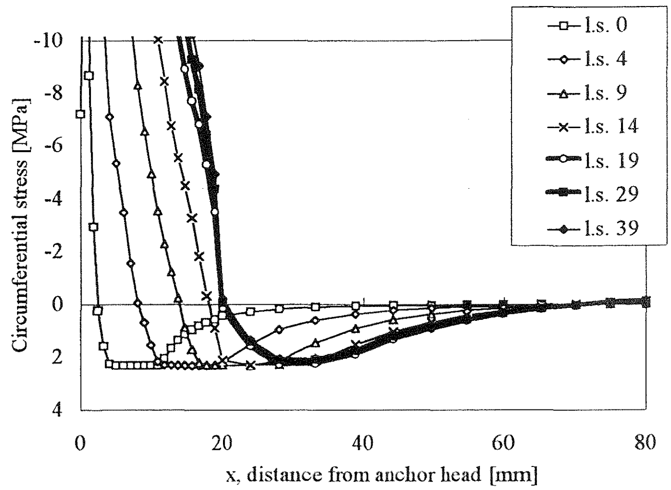

Fig. 6. Evolution of circumferential stresses around the anchor head showing the radial crack closure. Load step 19 is the end of insertion and load step 29 is at the peak pull-out force.

The second phase (i.e. anchor pull-out) was modeled by prescribing vertical displacements along the nodes, which again correspond to the anchor head.

The load-displacement for the pull-out phase are compared with the experimental results in Fig. 5. It should be noted that this excellent agreement is obtained using the standard values of material parameters $f_c^{'} = 255\ \text{MPa}$, $f_t^{'} = 2.32\ \text{MPa}$, $E = 30316\ \text{MPa}$, $\nu = 0.12$, $G_f = 100\ \text{N/m}$ and $w_d = -0.5\ \text{mm}$. The only parameter, for which experimental data or formulas are not available is the parameter ξ (see the Eqn. (13)), which defines the plastic flow. By inverse analysis an optimal value of this parameter is computed to be $\xi = 0$, which physically means that the material volume is preserved during the plastic flow.

Fig. 6 shows the evolution of circumferential stresses at various load steps and Fig. 7 show the distribution of minimal principal stresses at load step 19 (i.e. end of the expansion phase). Maximal compressive stresses in the analysis are around $-350\ \text{MPa}$.

5 Conclusions

Material model for concrete was presented, which includes Rankine fracture model for concrete cracking and Drucker-Prager plasticity model

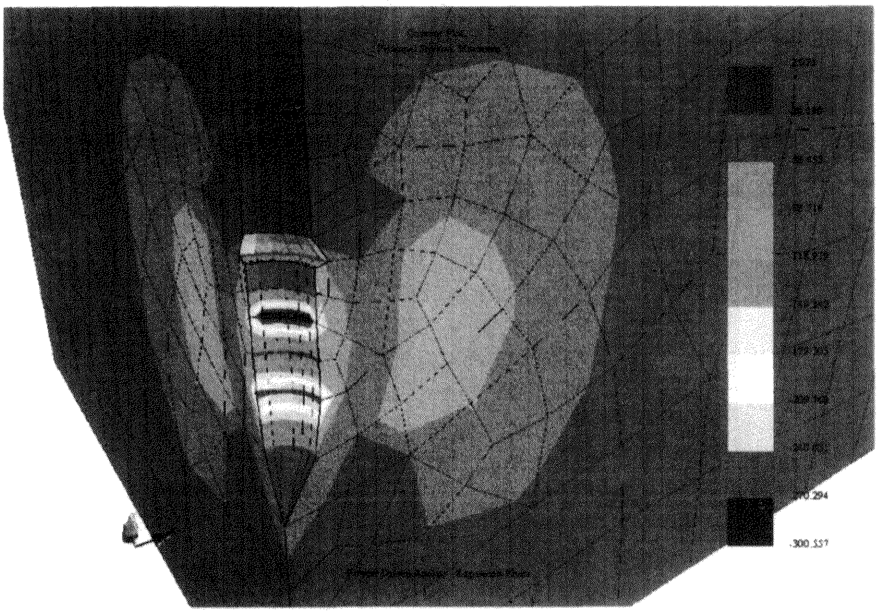

Fig. 7. Minimal principal stresses in the analysis at the end of expansion (i.e. insertion) phase.

for concrete crushing. Both models are formulated separately and a general algorithm is devised to iteratively determine the separation of the strain increment into the fracturing and plastic part. This model was applied to the analysis of powder actuated anchors. It was possible to simulate both qualitatively and quantitatively the behavior of these anchors.

6 References

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