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SIZE EFFECT IN COMPOSITE BEAMS WITH DEFORMABLE CONNECTIONS

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Abstract
The paper analyzes the failure of composite steel-concrete beams with stud shear connectors. Due to cracking damage in concrete, the studs fail in a brittle manner, exhibiting a post-peak decline of shear force with increasing slip. The load-deflection diagram of a composite beam in which the stud failures propagate along the steel-concrete interface is analyzed and the size effect determined. A satisfactory agreement with the limited test data available in the literature is demonstrated. The numerically calculated size effect is explained by energy analysis, which indicates that, in the usual case that the studs are not scaled, there is a reverse size effect for small sizes followed by a gradual transition to the usual asymptotic size effect of LEFM type for large sizes. In the case of perfect geometric scaling, in which the size of the studs and the steel-concrete interface area per stud are increased with the beam size, there is a compound size effect, in which the size effect in the failure of individual studs is superposed on the size effect due to the propagation of connection failure along the beam, resulting for very large sizes into a size hyper-effect that is stronger than that in LEFM.
Key words: Beam, composite, concrete, failure, fracture, size effect, scaling, studs.

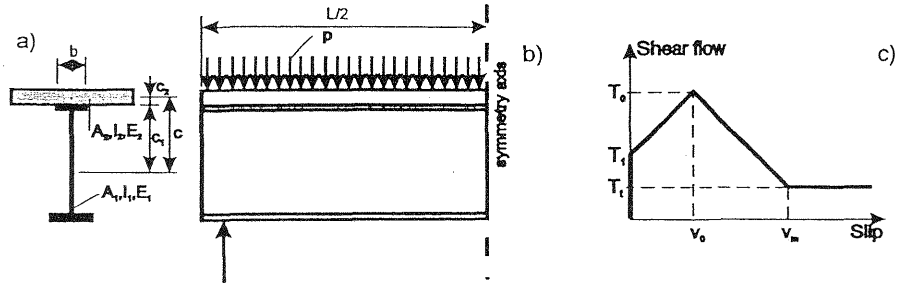

Fig. 1. a) Cross-section of the beam, b) Elevation of the half span, c) Approximate shear flow versus slip diagram of the stud connectors.

1 Introduction

Composite beams consisting of a steel beam and a concrete slab (Fig. 1a, 1b) are often used as floors of buildings or as bridge structures. The steel beam and concrete slab are typically connected by a large number of connectors, usually welded studs. These connectors need a certain slip between steel and concrete in order to get fully activated and develop their maximum shear force. There is no yield plateau, i.e., the shear force decreases after the peak as damage develops. Those connectors at which a large slip occurs carry a lower force than others at which the slip corresponds to the maximum force of the individual connector. This causes a size-dependent redistribution of forces among the individual connectors.

The aim of the paper is to summarize the results of a previous study (Bažant and Vítek 1994) of the response of steel studs and give a preliminary report on the analysis of the size effect on the failure behavior of composite beams.

2 Force-slip diagram of studs

Analyzing a number of experimental observations (Oehlers, 1989; Wright and Francis, 1990; Eligehausen and Ožbolt, 1990; Rehm et al., 1992; Eligehausen et al., 1992; Eligehausen and Zhao, 1993), an approximate energy-based model for the failure and post-peak behavior of studs in composite beams has been developed by Bažant and Vítek, (1994). This led to the approximate piecewise linear diagram of the shear flow in connection versus its slip, as shown in Fig. 1c. For the first (virgin) loading, an initially vertical segment represents an increase of shear force without any slip. In the case of repeated loading, the initial perfect bond (represented by shear force development without any slip) does not exist any more and the diagram begins with a finite slope. However, this is not the case when a repeated load is superposed on a significant dead load.

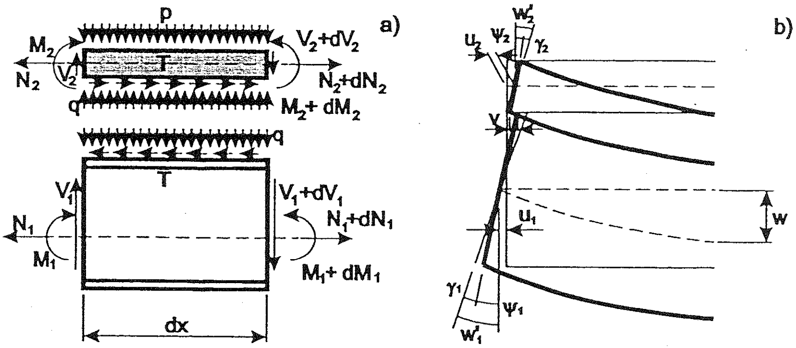

Fig. 2. Internal forces and displacements of the composite beam element

The second linear segment is a simplification of a rising curve leading to the shear flow peak. The first two linear segments of the diagram have been experimentally calibrated by the data of Eligehausen and Zhao (1993). The third linear segment, approximately describing post-peak softening, has been predicted strictly on the basis of energy arguments (Bažant and Vítek, 1994). The last, fourth linear segment was assumed to describe frictional slip at constant stress equal to the residual shear strength. Only very limited and incomplete test data to verify the post-peak response of the connection have been found.

3 Load deflection curve

The failure of a composite beam may be caused by failure of the steel beam, the concrete slab, or the connection. We are interested in the last cause of failure, and so the behavior of steel may be assumed to be within the linear elastic range. As an approximation, we assume the same for concrete. Considering the beam to be sufficiently slender, we can describe it by one-dimensional theory of bending, with the shear strains neglected.

Fig. 2 shows an element of the beam. The steel part is subjected to internal forces M_1, N_1, V_1, and the concrete part to internal forces M_2, N_2, V_2. A vertical distributed load p is applied on the concrete slab. The interaction of the beam and slab is represented by shear flow T and by normal force q distributed along the interface. The total bending moment, normal force and shear force transmitted by the whole cross section are denoted as M, N, and V. The deformations are characterized by deflection w, which is common to both parts of the cross section, slip v between steel and concrete, axial displacements u_1 in the steel beam centroid and u_2 in the concrete slab centroid, and cross section rotations ψ and γ of beam and slab (Fig. 2).

Consider now a uniformly loaded, simply supported, symmetric beam. We assume sytmmetric response up to the peak load. At the midspan, the slip vanishes due to the symmetry. The half span may be divided into four intervals, in each of which the shear flow and the slip correspond to one of the four linear segments of the diagram in Fig. 1c, described by a linear equation, $T = T_v v + T_1$ where $T_v = dT/dv$ =constant = slope of the second segment of the diagram, and T_1 = constant. The spatial coordinates x_1, x_2 and x_3 of the dividing points between these intervals are unknown, cannot be explicitly calculated in advance, and vary as load increases depending on the behavior of the whole beam. They may be calculated by an iterative solution of the beam.

The solution of the system of differential equations can be reduced to one linear ordinary second-order differential equation with constant coefficients: $v'' - \alpha T c = -[c/(R_1+R_2)]v$ where α is a coefficient depending on the stiffness of the beam, $\alpha = [(Z_1 + Z_2)(R_1 + R_2) + Z_1 Z_2 c^2]/[Z_1 Z_2 c(R_1 + R_2)]$ Here $Z_i = E_i A_i \quad (i = 1, 2)$ = axial stiffnesses and $R_i = E_i I_i \quad (i = 1, 2)$ = bending stiffnesses of the concrete and steel parts of the cross section, and T is shown in Fig. 1c. The coefficients for each linear segment of the force-slip diagram of the stud are different, and so one needs to solve a different equation for each of four corresponding intervals of the beam. The boundary conditions at the ends of each interval located at x_1, x_2 and x_3 ensure the compatibility of intervals of the the beam.

The four beams tested by Wright and Francis (1990) have been analyzed. The beams were 8 m long, consisting of a rolled steel I beam 312.5 mm deep and a concrete slab with profiled steel sheeting, 115 mm thick and 2.5 m wide. The four beams differed in the stiffness of connection. Stud connectors were welded in one row at equal spacing along the beam. Beam No. 1 had 7 studs (of diameter 19.0 mm, length 100 mm and tensile strength 450 MPa), beam No. 2 had 5 studs, beam No. 3 had 4 studs, and beam No. 4 had only 3 studs per half span. Concentrated loads were applied in four pairs, approximating a uniformly distributed load. The beams were first loaded suddenly (about 50% of the ultimate load) and then the load was raised monotonically to the ultimate load. The post-peak response could not be measured. The connection failed first, and the concrete slab cracked and failed subsequently. The slip between the slab and the steel beam has been also measured. Fig. 3 shows comparison of the calculated load-deflection diagram with the test data, which is found to be acceptable.

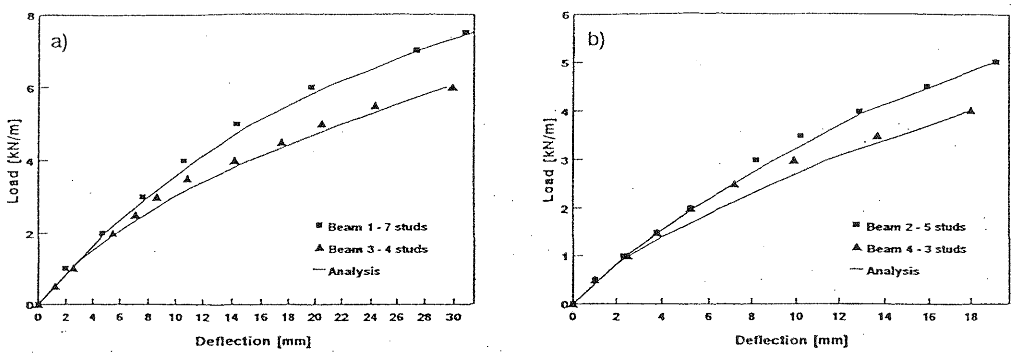

Fig. 3. Comparison of measured and calculated load deflection curves

4 Energetic analysis of compound size effect

Quasibrittle structures typically exhibit a size effect, which is understood as a dependence of the nominal strength of the structure σ_N on its size D (chosen characterisic dimension) when geometrically similar structures are compared. In the case of similarity in three dimensions, which is the case here, $\sigma_N = P_{max}/D^2$ where P_{max} = the maximum of load, load resultant, or load parameter. In our case of distributed load p, we may define $\sigma_N = p_{max}/D$ and take D = depth of the beam. If the failure criteria are expressed exclusively in terms of stresses and strains, as in plasticity, σ_N is always independent of D, i.e. there is no deterministic size effect (Bažant, 1984, 1994; Bažant and Chen 1997; Bažant and Planas 1997).

The size effect in the composite beam is an example of a compound size effect which represents the superposition of two size effects: (1) The size effect in the failure of a structure as a whole (macrolevel), and (2) another size effect in the failure of a substructure—the individual connectors (mezzolevel). We will now try to calculate the compound size effect by analyzing the energy release. We consider again our example of simply supported composite beam, but with arbitrary distributed or concentrated loading. The bending moment and shear force in the composite beam may always be expressed in the form $M(x) = PDq(\xi)$ and $V(x) = Pq'(\xi)$ where P = load parameter, x = coordinate measured from the left end of the beam, $\xi = x/D$, $q(\xi)$ = some size-independent dimensionless function, $q'(\xi) = dq(\xi)/d\xi$.

First assume, for the sake of simplicity, that there are two regions of length a (Fig. 4 a) in which the studs are totally failed. In the sense of continuum smearing of the studs, these regions may regarded as two symmetric sharp interface cracks. The studs ahead of each crack are assumed not to slip at all.

The concrete and steel ahead of the crack tip act as a composite

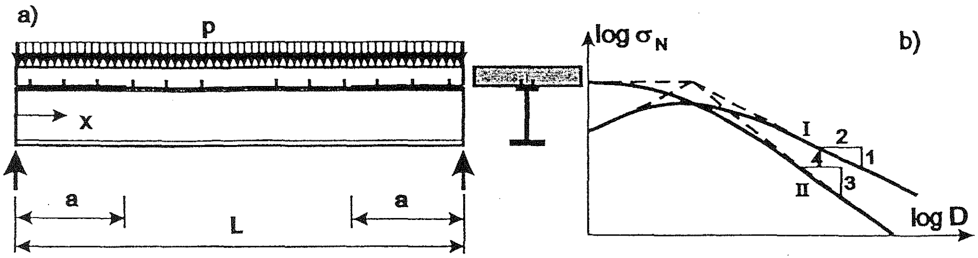

Fig. 4. a) Composite beam with crack-like propagating segments of failed studs, b) Size effect curves obtained by energy release analysis

beam of bending stiffness $R = R_1 + R_2 + A_1c_1^2 + A_2c_2^2$ where $A_1, A_2 =$ cross section areas of steel beam and concrete slab, and $c_1, c_2 =$ the eccentricities of their centroids from the centroid of the transformed cross section (Fig. 1 a).

Behind the crack tip, the steel and concrete behave as two separate beams forced to deflect equally (we assume the slab not to lift above the steel beam). Therefore, $M = M_1 + M_2$, and since the curvatures of the steel beam and the slab are equal, their bending moments (for $x < a$) are $M_1 = MR_1/(R_1 + R_2)$ and $M_2 = MR_2/(R_1 + R_2)$, respectively, and the bending energies per unit length of beam are $M_1^2/2R_1$ and $M_2^2/2R_2$. Summing the last two values, the total bending energy of both parts per unit length of the cracked region of the beam is found to be $M^2/2(R_1 + R_2)$. Likewise, the shear forces in steel and concrete behind the tip are $V_1 = VS_1/(S_1 + S_2)$ and $V_2 = VS_2/(S_1 + S_2)$, and the total shear shear force $V = V_1 + V_2$. The shear energy behind the tip is $V^2/2(S_1 + S_2)$, while in front of the tip it is $V^2/2S$ where $S =$ shear stiffness of the composite cross section.

The complementary energy of the left half of the cracked beam is:

$$\Pi^* = \int_0^a \left(\frac{[PDq(\xi)]^2}{2(R_1 + R_2)} + \frac{[Pq'(\xi)]^2}{2(S_1 + S_2)} \right) dx + \int_a^{L/2} \left(\frac{[PDq(\xi)]^2}{2R} + \frac{[Pq'(\xi)]^2}{2S} \right) dx \tag{1}$$

where $a =$ length of the cracks. By differentiation, the energy release rate of the beam:

$$\left[\frac{\partial \Pi^*}{\partial a}\right]_{P=\text{const.}} = \frac{[PDq(\alpha)]^2}{2E_sD^4\hat{R}} + \frac{[Pq'(\alpha)]^2}{2E_sD^2\hat{S}} = b\,\mathcal{G}_{cr} \tag{2}$$

where $(\hat{R})^{-1} = E_sD^4[(R_1 + R_2)^{-1} - R^{-1}]$ and $(\hat{S})^{-1} = E_sD^2[(S_1 + S_2)^{-1} - S^{-1}]$, which are positive size-independent dimensionless parameters of the cross-section geometry. We may now set $b\mathcal{G}_{cr} =$

$G_{stud}n/s$ = energy dissipated by stud failures per unit length of beam, where b = width of the steel beam, G_{stud} = energy required to fail one stud, s = longitudinal spacing of studs, and n = number of rows of studs across the width of the interface strip. From (2) we may calculate the nominal stress:

$$\sigma_N = \frac{P}{D^2} = \frac{1}{D}\sqrt{\frac{2E_sG_{stud}n}{\{[q(\alpha)]^2\hat{R}^{-1} + [q'(\alpha)]^2\hat{S}^{-1}\}s}} \tag{3}$$

Let us now assume that the cracks at failure of composite beams of different sizes are geometrically similar, i.e. $a/D = \alpha$ = constant (relative crack length at failure). From experience with various types of fracture, this is often a good assumption for a significant range of sizes.

The stud failures are in reality spread over a certain finite length, which we will denote as $2c_0$. The behavior may be approximated by an effective (equivalent) sharp LEFM crack in the steel-concrete interface, having length $a = \alpha_0 D + c_0$ where $\alpha_0 = a_0/D$ and a_0 represents the length with totally failed studs. By analogy with the size of the fracture process zone in quasibrittle materials, we may consider c_0 and α_0 to be approximately constant, i.e., size independent. Now, substituting $\alpha = \alpha_0 + (c_0/D)$ into (3) we may conveniently introduce Taylor series expansions $q(\alpha) = q(\alpha_0) + q'(\alpha_0)c_0/D + ... = q_0 + q_1(c_0/D) + q_2(c_0/D)^2 + ...$ and $q'(\alpha) = q'(\alpha_0) + q''(\alpha_0)c_0/D + ... = q_1 + q_2(c_0/D) + ...$ where $q_0, q_1, q_2, ...$ = constants. Thus we obtain

$$\sigma_N = \frac{1}{D}\sqrt{\frac{2(n/s)E_sG_{\text{stud}}}{[q_0 + q_1(c_0/D) + ...]^2\hat{R}^{-1} + [q_1 + q_2(c_0/D) + ...]^2\hat{S}^{-1}}} \tag{4}$$

We will now consider two basic geometrical types of scaling:

Type I. The composite beam is scaled in proportion to D while the connection characteristics per unit area of steel-concrete interface remain constant (as would be the case for a bonded interface with a crack). In that case, $n/D, s$ and G_{stud} are constant, and so are D/n and the transverse spacing of stud rows, b/n. Then, if the series expansions are truncated after the first (linear) term, (4) can be rearranged to the form of the classical size effect law (I in Fig.4b):

$$\sigma_N = \frac{\sigma_N^0}{\sqrt{1 + (D/D_0)}} \tag{5}$$

in which

$$D_0 = c_0\frac{A_1}{A_2}, \qquad \sigma_N^0 = \sqrt{\frac{2}{s}\left(\frac{n}{D}\right)\frac{E_sG_{\text{stud}}}{A_1c_0}} \tag{6}$$

$$A_0 = q_0^2\hat{R}^{-1} + q_1^2\hat{S}^{-1}, \qquad A_1 = 2q_1(q_0\hat{R}^{-1} + q_2\hat{S}^{-1}) \tag{7}$$

Asymptotically, for very large D, (5) indicates that $\sigma_N \propto D^{-1/2}$, which is the scaling of linear elastic fracture mechanics (LEFM).

Type II. Not only the composite beam but also the connectors and their spacing are geometrically scaled. In this case, representing the complete geometric scaling of the entire structure including the substructure of studs (mezzolevel), one must take into account the effect of stud diameter d on the nominal strength of the stud. Since d is now proportional to D, this may be done by expressing the energy required for failure of the studs per unit length of the beam as follows:

$$G_{\text{stud}} = \sigma_{N\text{stud}} D^2 \delta_{\text{stud}} \tag{8}$$

Here δ_{stud} is the effective slip displacement of studs, which characterizes the energy dissipated and may be considered as constant for various stud sizes, while the nominal strength of the stud is subjected to size effect:

$$\sigma_{N\text{stud}} = \frac{\sigma_{s0}}{1 + (d/d_{s0})} = \frac{\sigma_{s0}}{1 + (D/D_{s0})} \tag{9}$$

where d_{s0}, D_{s0} = constants. Using (8) and (9), (4) can be rearranged to the form:

$$\sigma_N = \sqrt{\sigma_{s0}\sigma_{b0}} \left(1 + \frac{D}{D_0}\right)^{-1/2} \left(1 + \frac{D}{D_{s0}}\right)^{-1/4} \tag{10}$$

in which D_0 and σ_{b0} are constants;

$$D_0 = c_0 \frac{A_1}{a_0}, \qquad \sigma_{b0} = \frac{2n}{A_1 c_0} \left(\frac{D}{s}\right) E_s \delta_{\text{stud}} \tag{11}$$

For very large sizes D, this expression leads to the asymptotic size effect (II in Fig. 4 b):

$$\sigma_N \propto \frac{1}{D^{3/4}} \tag{12}$$

It is at first surprising that this size effect, which may be called the size hyper-effect, is stronger than the LEFM size effect $1/\sqrt{D}$. The reason is that this is a compound size effect. The size effect due to failure of the beam as a whole (macrolevel) is amplified by the size effect in the failure of individual studs (mezzolevel). Obviously, from the viewpoint of the size effect, when the beam size is increased it is better not to increase the size of the studs if possible. This is the normal practice anyway, but is not feasible when the beam size is enlarged very much.

5 Concluding Comments

Composite beams with deformable connection (often represented by welded studs) must have adequate safety margin against the failure of the connection, which means that the corresponding ultimate load capacity must be realistically calculated. A simplified analysis based on the beam theory is presented. It shows a reasonable agreement with experimental results.

Because the connection failure is brittle rather than plastic, the nominal strength of beam corresponding to connection failure depends on the size of the beam. Two types of similarity are considered. For Type I, the studs and their spacing are of the same size but their number is increased in proportion to the flange width or beam size. The classical size effect law for quasibrittle failure is followed (although for small sizes the numerical results show a mild reverse size effect, which results from variable stiffness of connectors at beams of different sizes, while their slip at failure is size independent).

For Type II, the number of studs remains constant at all beam sizes but the stud size increases in proportion to the beam size. In that case there is a compound size effect which is stronger than predicted by the classical size effect law or by LEFM.

Real beams follow neither Type I nor Type II scaling. The depth of the steel beams used in practice varies in the range from 0.3 m for building floors to approximately 3 m for bridges. The slab thickness can vary in the range from 0.15 m to 0.5 m, which is much less than the depth of the steel beam. The stud sizes used are roughly proportional to the slab thickness (diameters 12 mm—32 mm). Type II scaling does not represent the practice too well. Type I scaling is closer to reality. Anyhow, both scaling types represent limit cases. The beams used in practice will exhibit an intermediate size effect between these two limit cases. In any case, this means that the size effect may be stronger than for non-composite beams.

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