Fracture Mechanics of Concrete Structures
Proceedings FRAMCOS-3
AEDIFICATIO Publishers, D-79104 Freiburg, Germany

DELAMINATION FAILURE IN CONCRETE BEAMS RETROFITTED WITH A BONDED PLATE

C. K. Y. Leung
Department of Civil and Structural Engineering, Hong Kong University of Science & Technology, Clear Water Bay, Kowloon, Hong Kong

Abstract
Concrete beams can be retrofitted by bonding a steel or composite plate to its bottom. The failure of such members is usually due to the delamination of the plate from the beam. Delamination can occur either at the end of the reinforcing plate or at the location of a crack in the concrete beam. This paper will focus on the analysis of delamination initiated at the mouth of a flexural crack. When loading is applied, a flexural crack tends to open at the bottom of the beam, inducing high shear stress which may result in delamination. The increment of interfacial shear stress with applied moment is affected by many material and geometric parameters. In this study, the bridging stress in the bonded plate and the maximum interfacial shear stress (τ_{max}) are first related to the crack mouth opening. A fracture mechanics based analysis is then carried out to obtain the crack mouth opening and τ_{max} for a given applied moment. Through a systematic parametric analysis, the effect of various parameters on τ_{max}, which reflects the likelihood of delamination, can be deduced. It is believed that the present analysis can provide insight into the prevention of delamination failure in retrofitted concrete beams.
Key Words: Delamination, Repair/Retrofit, Failure, Interface

1 Introduction

After years in service, concrete structures may be damaged and are therefore in need of repair. In some cases, to accommodate for a load carrying capacity higher than the original design value, members need to be retrofitted. For concrete beams, a common repair/retrofitting technique is to bond a plate to the bottom of the beam. Initially, steel plates were employed. Recently, attention has been directed towards the use of fiber reinforced plastic (FRP) plates, which offer higher strength/weight and improved durability over their steel counterparts. When beams with bonded plates are loaded in flexure, final failure is often associated with delamination of the plate from the beam (Saadatmanesh and Ehsani, 1991, Meier, 1992). Early theoretical studies (Wei, Saadatmanesh and Ehsani, 1991, Triantafillou and Plevris, 1992) have, however, focused on analyzing failure associated with concrete crushing or plate rupturing. Recently, Taljsten (1997) and Malek et al (1998) have developed analytical solutions for the shear and normal stresses along the interface between the bonded plate and the concrete beam. By considering the interfacial stresses at the end of the plate, it is possible to (i) identify situations when delamination is likely to occur, and (ii) propose a criterion for the onset of delamination failure.

In Taljsten (1997) and Malek et al (1998), one important assumption is that delamination will always start at the end of the bonded plate. In concrete beams, flexural and flexural/shear cracks are commonly found on the tensile side. Under loading, these cracks tend to open and may also induce interfacial stresses which can initiate delamination. This argument is supported by a recent experimental study by Hearing and Buyukozturk (1997). In their experiment, shear cracks were introduced to a reinforced concrete beam before a composite plate was bonded to its bottom. Lines of silver paint were applied across the composite plate near its end and the current through each line was monitored. When delamination reaches a given line, the resulting separation will break the circuit. The beam was loaded to failure, and the loss of current was found to occur earlier at lines further away from the end of the plate. This result clearly indicates that delamination can be initiated at cracks at a distance from the end of the plate. The analysis of such a failure mode is the major objective of the present work.

2 Statement of the Problem

The delamination of bonded plate at a crack in the concrete beam is governed by the interfacial stresses at the vicinity of the crack. The determination of such stresses is not a trivial problem. The interfacial stresses are functions of crack opening (w) and applied moment (M). The relation between M and w depends on the crack size (a). For known properties of the concrete, plate and adhesive, a, w and M are related to one another through non-linear equations that can be derived from fracture mechanics. In the general case, these equations are very difficult to derive.

As a first attempt to this problem, we consider the simple case illustrated in Fig.1. The retrofitted beam, which is assumed to contain one single crack, is put under pure bending (Fig.1). In the analysis, we will only focus on the interfacial shear stress, which is believed to be the major cause of plate delamination. A model is first developed to relate the bridging stress in the plate to the crack opening. Then, equations will be set up to relate M, a and w. By solving these equations, the variation of maximum interfacial shear stress (τ_{max}) with the applied moment can be obtained. Parametric studies are then carried out to identify the combinations of material parameters that will produce high values of τ_{max}, and hence likely to cause delamination failure. Despite the simplistic nature of the model, the analysis can provide useful qualitative guidelines for the prevention of delamination at concrete cracks.

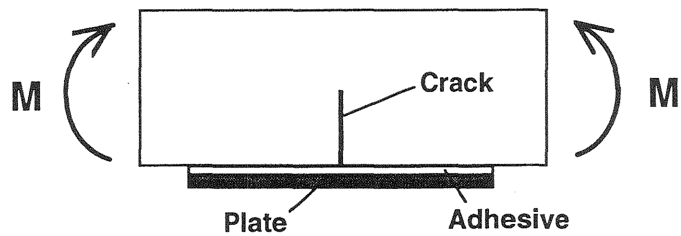

Fig. 1 The simplified problem to be analyzed

3 Derivation of Bridging Stress in the Plate

Consider the retrofitted beam in Fig.1. When the beam bends, loading is transmitted to the plate through shearing of the adhesive. The interfacial shear stress (τ) is related to the longitudinal stress (σ_p) in the plate, as well as the longitudinal displacements at the plate (u_p) and at the bottom of the beam (u_b) through:

$$\tau = t\, d\sigma_p/dx = G\,(u_p - u_b)/h \qquad (1)$$

where t is the thickness of the plate, h the thickness of the adhesive and G the adhesive shear modulus. To satisfy equilibrium, σ_p and σ_b (the longitudinal stress at the bottom of the beam) are related to the applied moment (M) by:

$$\sigma_b = 6M/(BD^2) - 4\rho\sigma_p \qquad (2)$$

B and D are the width and depth of the beam, and $\rho = A_p/BD$, where A_p is the cross sectional area of the plate. In the derivation of (2), we have assumed the beam to be rectangular, and have not considered the steel reinforcement in the calculation of section modulus. Differentiating (1) and substituting (2), the following governing equation is obtained:

$$d^2\sigma_p/dx^2 - \alpha^2\sigma_p = - 6MG/(htE_cBD^2) \tag{3}$$

where $\alpha^2 = [G/(htE_p)]\ [1 + 4\rho E_p/E_c]$, and E_p, E_c are respectively the Young's modulus of the plate and concrete. Using the boundary conditions, $\sigma_p = 0$ at the end of the plate, and $\sigma_p = \sigma_r$ at the crack, eqn(3) can be solved to obtain σ_p as a function of x. With σ_p, eqn(1) gives $\tau(x)$ and, specifically, τ_{max} at the bottom of the crack. The crack mouth opening (w) is given by:

$$w = 2\ (u_p - u_b)_{\text{at the crack}} = 2h\tau_{max}/G \tag{4}$$

Using (4), the following relation between v and σ_r can be derived:

$$\sigma_r = [6ME_p/(E_cBD^2)]\ [1-1/\cosh(\alpha L)]\ /\ [1 + 4\rho E_p/E_c] + Gw\tanh(\alpha L)/(2\alpha ht) \tag{5}$$

L is the plate length on one side of the crack. For a given crack opening w, $F_r = A_p\sigma_r$ is the bridging force provided by the bonded plate.

4. Computation of Maximum Interfacial Shear vs Moment

When a retrofitted reinforced concrete beam is under bending, the resistance to crack opening consists of three different components. The first component comes from the post-peak response of concrete, the second from the bridging force in the reinforcing steel, and the third from the bridging force in the bonded plate. The compatibility equation for crack mouth opening is of the following form:

$$C_M(a)M - \int_0^a [C_{conc}(y,a)\sigma_{conc}(w_y)]dy - C_s(a)F_s(w_s) - C_r(a)F_r(w) = w \tag{6}$$

In eqn(6), the C_i's are compliance factors relating the crack mouth opening to the corresponding moment, stress or force. w_s is the crack opening at the level of the steel reinforcement and w_y is the opening at a distance y from the bottom of the crack. To simplify the computation, it is assumed that the crack profile is linear. Both w_s and w_y can then be related to w through a simple relation. σ_{conc} is the crack bridging stress provided by concrete in its softening regime. A linear reduction of σ_{conc} with crack opening is assumed. The relation between F_r and w is given by eqn(5). The variation of F_s with w_s is taken to be (Kaar and Mattock, 1963):

$$f_s = F_s/A_s = 11876.5\ w_s / A^{1/4} \qquad (7)$$

In eqn(7), the steel stress f_s is in MPa and w_s is in mm. A_s is the steel area and A is the effective area of concrete taken by each steel bar. Eqn(7) is only valid for relatively large crack openings. Note that the steel bar carries a certain stress f_{sc} when concrete just starts to crack. After cracking, the steel bar should not be unloaded. If $f_s < f_{sc}$, the steel stress is taken to be f_{sc}.

In eqn(6), both the crack mouth opening and crack sizes are unknowns. Another equation is required, and it is provided by the superposition of stress intensity factors at the crack tip. When a crack is in equilibrium at size a, the sum of stress intensities at the tip must be zero. The equation is of the following form:

$$k_M(a)M - \int_0^a [k_{conc}(y,a)\sigma_{conc}(w_y)]dy - k_s(a)F_s(w_s) - k_r(a)F_r(w) = 0 \qquad (8)$$

In eqn(8), the k_i's are stress intensity factors for a unit value of the corresponding moment, stress or force. They can be derived using weight functions for an edge cracked beam given by Wu and Carlsson (1991). Knowing the k_i's, the C_i's can be derived from the relation between the stress intensity factor and compliance change. A special note should be made about the determination of $k_r(a)$. The bridging force provided by the bonded plate is not acting directly on the crack mouth but is transmitted to the bottom of the beam as distributed shear stresses. To find the stress intensity factor due to the shear stresses, the following simplification is made. The shear stresses is assumed to act as a single shear force, located at the centroid of the distribution. The stress intensity factor is first calculated by assuming the shear force to act at the crack mouth. The obtained value is then corrected by a factor derived by Rooke and Jones (1978) to account for the difference between a shear force at the crack and one away from it.

After deriving the expressions for all the k_i's and C_i's, eqn(6) and (8) are solved in an iterative manner. An initial crack size is first assumed. The crack mouth is then opened to a certain extent. With both a and w, M can be obtained from eqn(6). M, a and w are then put into eqn(8) to see if it is satisfied. If not, the crack mouth opening is further increased and M is re-calculated. The calculation is repeated until convergence is obtained for a given crack size. The maximum interfacial shear stress (τ_{max}) is then calculated from w using eqn(4). By repeating the computation for different crack sizes, the variation of τ_{max} with M can be obtained.

5. Results and Discussions

Based on the above theoretical framework, a computer program is developed and the variation of τ_{max} with M is obtained for various combinations of design parameters. Several sets of simulated results will be presented here and discussed. A more thorough parametric study is still under way and will be reported in a future paper.

In all the simulations, unless otherwise specified, the following parameters are employed. For the beam, B = 200mm, D = 500mm, For the bonded plate, L = 2B, t = 2.5mm, ρ = 0.002, E_p = 170GPa. For concrete, E_c = 26GPa, f_t = 2.96MPa, fracture energy G_F = 0.0647N/mm. For steel, A_s = 0.01BD, E_s = 200GPa, cover = 0.1D, yield strength = 345MPa. For the adhesive, G = 1GPa and h = 1.0mm. The simulation results are plotted in terms of maximum shear stress (τ_{max}) vs normalized moment, with BD^2 being the normalizing factor.

The first set of simulations consider the effects of varying plate thickness and adhesive thickness. The results are shown in Fig.2. The solid line is for the standard case with parameters given in the above paragraph. The other two cases are for (i) a reduced thickness but the same cross-sectional area for the plate, and (ii) an increased thickness of the adhesive. With a reduced plate thickness, the width of the plate has to be increased to keep the sectional area constant. In other words, there is a larger contact area between the plate and the adhesive. For a given moment, τ_{max} is found to be reduced. In other words, with a larger contact area, the likelihood of delamination is decreased. When a thicker layer of adhesive is employed, τ_{max} will also be reduced. At a normalized moment of 4, the reduction is about 30% compared to the standard case.

Fig.3 shows the results from the second set of simulations, which considers the effect of size. The behavior of a larger beam (400mm x 1000mm) and a small one (100mm x 250mm) are analyzed to compare with the standard case. In the analysis, ρ and A_s/BD are kept constant. The plate thickness is scaled with the beam size so all three cases are geometrically similar. The adhesive thickness, however, is kept constant. In practical applications, the designer may change the dimensions of the plate. However, since the adhesive is applied by standard techniques, the thickness should be fairly uniform regardless of the member size. The simulation results clearly indicate a strong effect of size on delamination behavior. Over a wide range of applied moment, τ_{max} for the large beam is about twice that for the small beam. Two explanations for this observation

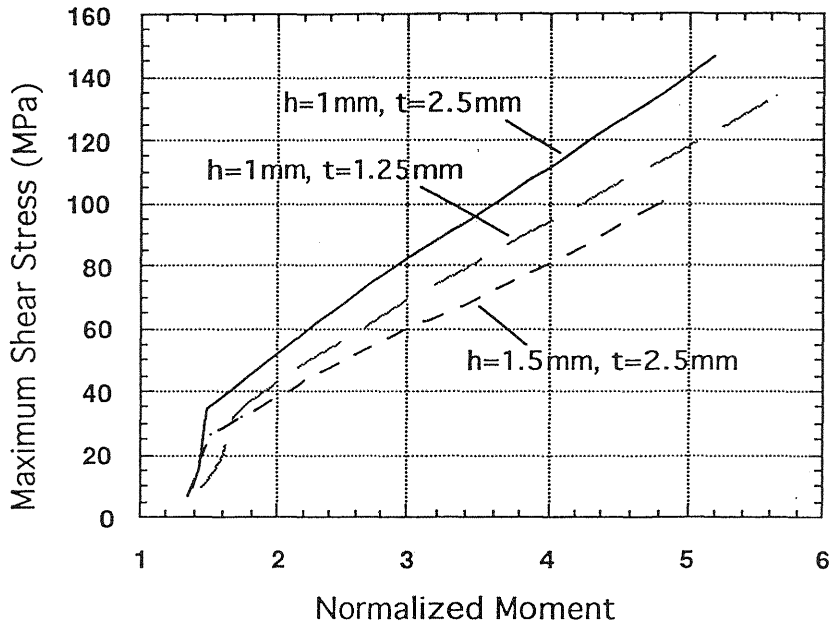

Fig. 2 Effect of Plate and Adhesive Thickness on τ_{max}

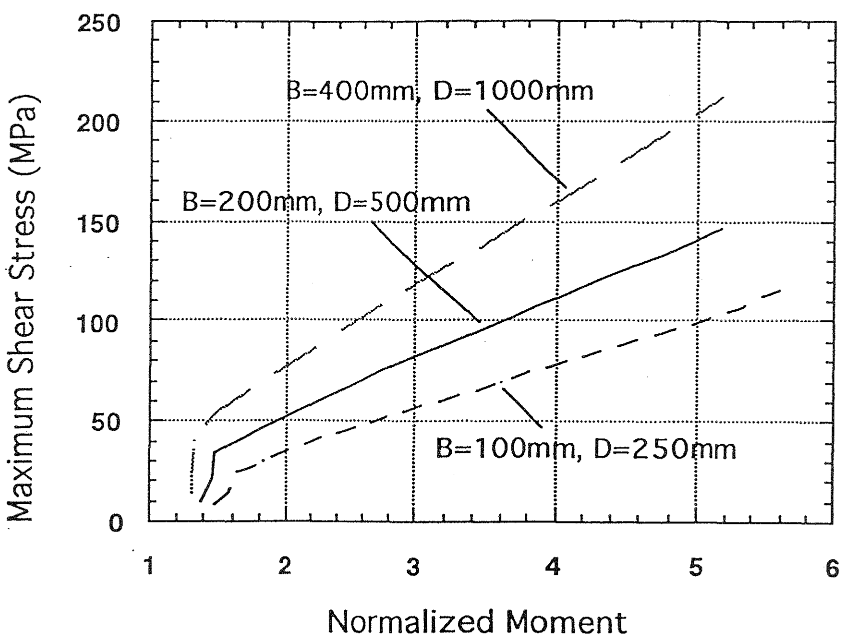

Fig. 3 Effect of Specimen Size on τ_{max}

can be proposed. First, for a larger beam, the bridging stress contributed by the concrete will decrease more rapidly. This has to be compensated by a higher stress in the bonded plate, which will also lead to higher interfacial shear stresses. Also, we have seen from our first set of analysis that τ_{max} decreases when the adhesive thickness increases. By keeping h constant, the relative adhesive thickness is higher for the small beam and thus will lead to a lower value of τ_{max}.

In the third set of simulation, we consider the effect of plate stiffness. In one case, the plate modulus is reduced from 170GPa to 80GPa. The plate size is kept the same. In another case, E_p is reduced to 25% of its original value, but ρ is also quadrupled to keep A_pE_p constant. Note that in this simulation, the plate thickness is also quadrupled to keep the same contact area. The results in Fig.4 show that a reduction in E_p increases the value of τ_{max}. This is because a larger strain in the plate (and hence a higher crack opening) is required to give the same bridging force. The low crack bridging effectiveness of the 80GPa plate is reflected in the shape of the curve. In the regime where the moment is low, τ_{max} is found to increase with decreasing M. This is due to the drop of M with increasing crack size, which is characteristic of reinforced concrete members of high brittleness (Bosco and Carpinteri, 1992). At the high moment regime, the

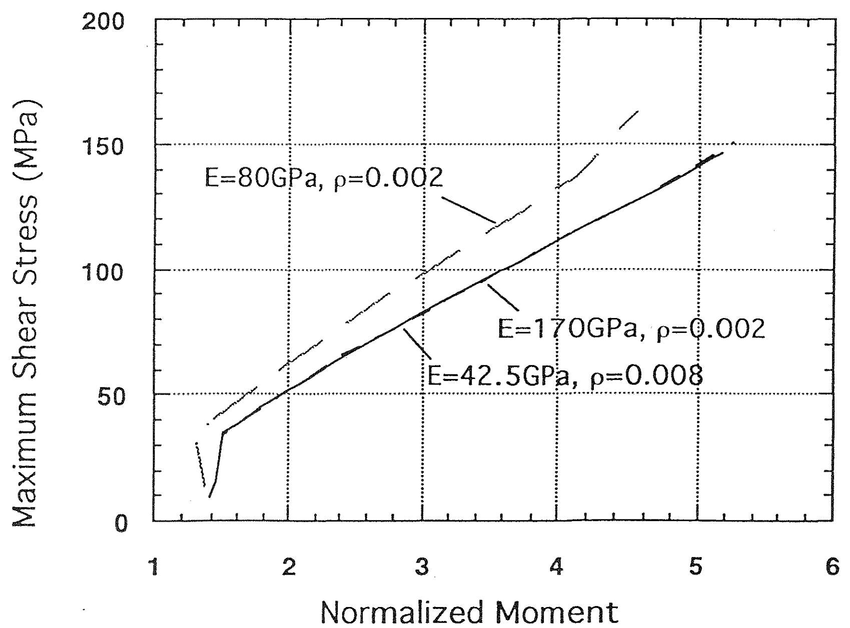

Fig. 4 Effect of Plate Stiffness on τ_{max}

tilting up of the curve is because the crack has opened so much that the steel reinforcing bar has reached its yield point. The reduction in plate modulus can be compensated by an increase in plate size. As shown in Fig.4, for two cases with the same A_pE_p, the curves are essentially identical, provided the contact area between the plate and the adhesive is the same.

Before concluding the paper, some remarks should be made about the magnitude of the maximum shear stress. One can see on the plots that τ_{max} often exceeds the strength of concrete, so how can these stresses be carried? To address this question, it should be noted that the interfacial shear stresses increases in an exponential manner when the crack is approached. The very high stresses therefore only exist over a very small region. The failure in such a small region is unlikely to be governed by concrete strength measured from much larger specimens. Also, it is quite likely that inelastic behavior will commence at the interface near the concrete crack. The local high stress can hence be relieved. If the inelastic zone is small in size, one can assume elastic behavior along the interface and still obtain accurate results. Otherwise, the inelastic process has to be modeled explicitly in the analysis. This is a critical issue in the development of quantitative delamination criterion, and further investigation is certainly required.

6. Conclusions

In this paper, a theoretical framework is developed to analyze the delamination of bonded plate from a concrete beam at the location of a flexural crack in the beam. To demonstrate the applicability of the theory, simulations are carried out to study the variation of maximum interfacial shear stress with applied moment for various combinations of material and geometrical parameters. The results indicate that delamination is favored by large member size, low adhesive thickness, low plate stiffness and small contact area between plate and adhesive. Despite all the simplifying assumptions made in the model, the results do provide useful qualitative information for the prevention of delamination failure.

7 References

Bosco, C. and Carpinteri, A. (1992) Fracture Mechanics Evaluation of Minimum Reinforcement in Concrete Structures, in **Applications of Fracture Mechanics to Reinforced Concrete** (ed. A. Carpinteri), Elsevier Applied Science, London, 231-265.