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Fatigue analysis of fiber reinforced concrete overlaid/underlaid beams

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ABSTRACT: Fatigue life of concrete beams that are overlaid or underlaid with FRC is analyzed with a fracture mechanics based model. The problems analyzed are the cases for concrete beams overlaid or underlaid with hooked-end steel fiber reinforced concrete. For both of overlaid and underlaid beams, three cases of beam heights, three values of fiber volume fraction, and the five levels of the FRC thickness ratio are examined. In the case of overlay, resulting fatigue crack growth plots and theoretical S-N diagrams show that neither small dosage of fibers nor thin FRC repair layer is effective to suppress the fatigue crack growth. On the other hand, in the case of underlay, fatigue crack growth plots show the suppression of fatigue crack growth in the early stage, and theoretical S-N diagrams show improvements in fatigue strength. Overall, overlay and underlay exhibit different characteristics in the role of fatigue crack tip shielding.

1 INTRODUCTION

Overlay or underlay is a repair / retrofit solution often applied to a slab like structure. A typical example is a bridge deck slab. Currently, a large number of existing bridge deck slabs are reported to be damaged, and the number of aged slabs that are in need for a repair / retrofit is predicted to increase for the coming decades. Bridge deck slabs are a fatigue intensive structural member which is subjected to repeating heavy traffic loads during their service life. The failure of bridge deck slabs starts with the development of dense cracks in a grid pattern, followed by a final punching shear crack. Overlay or underlay repair / retrofit is often applied to such damaged bridge deck slabs. For example, steel plates or carbon fiber sheets can be put on the bottom surface of a bridge deck slab, or fiber reinforced concrete (FRC) can be overlaid or underlaid on a bridge deck slab. FRC overlay repair method has been proven effective to prolong the life of a bridge deck slab, and the design / construction guideline is already available for use (Express Highway Research Foundation of Japan 1995).

However, the current FRC overlay design relies on time-consuming real-scale experiments which usually adopt a moving wheel machine to test various design cases. Although moving wheel tests certify a tested design case, the failure mechanisms are rather qualitative, and not all design variables have been examined. This is due to the fact that the effects of FRCs on slab fatigue failure are not represented by a micromechanical model.

Therefore, it is necessary to develop an analytical model that is based on the fatigue failure mechanism of bridge deck slabs in order to develop a more economical design for FRC overlay repair method. In addition to economical FRC overlay design, the establishment of FRC underlay design is needed. Underlay design has an advantage over overlay design, since the traffic does not need to be closed during the repair construction.

This paper is a preliminary study which treats the fatigue problems of beams overlaid / underlaid with FRCs. The study is planned to be extended to the fatigue life analysis of bridge deck slabs.

Fatigue life of concrete beams that are overlaid or underlaid with FRCs is analyzed with a fracture mechanics based model. The model accounts for the effect of cycle-dependent fiber bridging on fatigue crack propagation, which is the governing mechanism of fatigue fracture in a quasi brittle material such as FRCs.

The model computes the fatigue crack growth with the presence of degrading crack bridging stress, and, therefore, fatigue life can be obtained. In addition, the model is expressed in terms of microstructural parameters, which enables to simulate fatigue crack development for an FRC with given microstructural parameters. Throughout this study, the model is utilized to analyze the fatigue crack propagation of FRC overlaid/underlaid beams.

2 FRACTURE MACHANICS BASED FATIGUE MODEL

The model used in this study is based on fracture mechanics, and it can treat a crack problem where fiber bridging tractions exist under either monotonic or cyclic fatigue loading (Li & Matsumoto 1998, Matsumoto & Li 1999). Fiber bridging traction is modeled based on micromechanics of fiber bridging, namely it is expressed as a function of microstructural parameters including fiber length, fiber diameter, matrix fracture toughness, and interfacial frictional bond strength. Model descriptions for monotonic and cyclic fatigue analysis follow below.

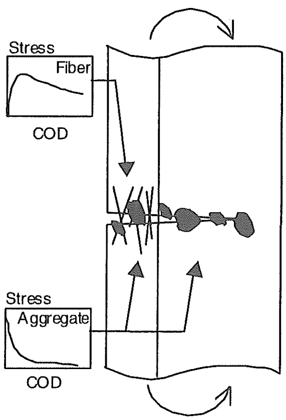

Figure 1. FRC underlaid beam under monotonic loading.

2.1.1 *Monotonic analysis*

Extension of a bridged crack under monotonic loading can be treated by using the monotonic bridging law, $\sigma_b(\delta)$, which is the relation between crack bridging stress and crack opening displacement (Figure 1).

Crack extension takes place when fracture conditions is met:

$$K_{tip} = K_c \tag{1}$$

where K_{tip} = crack tip stress intensity factor and K_c = matrix fracture toughness. K_{tip} is the net stress intensity factor which accounts for the crack tip shielding experienced at the crack tip, and it can be decomposed into two parts:

$$K_{tip} = K_a + K_b \tag{2}$$

where K_a = stress intensity factor due to external applied loading and K_b = stress intensity factor due to crack bridging. Here, K_a is a function of applied flexural stress, beam dimension, and crack configuration, while K_b is a function of the monotonic bridging law.

Crack bridging in the current overlay / underlay problem is exerted by both aggregates and fibers in the new repair layer and only by the former in the old existing layer. Therefore, the monotonic bridging law, $\sigma_b(\delta)$, in the new layer is given by the superposition of crack bridging stress - crack opening displacement relation due to fibers and aggregates:

$$\sigma_b = \sigma_f + \sigma_m \tag{3}$$

where σ_f = fiber bridging stress and σ_m = aggregate bridging stress, while it is given only by the relation due to aggregates:

$$\sigma_b = \sigma_m \tag{4}$$

in the old layer. Detail expressions of the monotonic bridging laws can be found in Matsumoto & Li (1999).

Monotonic analysis leads to the relation between crack propagation stress and crack length. This is done by the successive computation of the fracture condition for a given crack length which varies between the initial flaw size and the beam depth. From this relation, the maximum stress sustainable throughout the crack propagation can be obtained, and this is the modulus of rupture of the material analyzed.

Monotonic analysis is used not only for the computation of the modulus of rupture, but also for the setup of initial conditions of fatigue crack growth and the judgement of final fast fracture condition in cyclic fatigue analysis.

2.2 *Cyclic fatigue analysis*

Under cyclic fatigue loading, fatigue crack growth is mainly governed by three factors: matrix fatigue crack growth law, crack bridging, and cycle-dependent degradation of crack bridging. The fracture mechanics based model treats the fatigue crack propagation with these three factors considered. Brief explanations are given below.

2.2.1 *Matrix fatigue crack growth law*

Matrix fatigue crack growth law in this study adopts a Paris law type equation. Paris law assumes that crack growth rate, $\mathrm{d}a/\mathrm{d}N$, is governed by crack tip stress intensity factor amplitude, ΔK_{tip}:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C(\Delta K_{tip})^n \tag{5}$$

where a = crack length, N = number of cycles, C = Paris constant, and n = Paris constant. Fatigue failure can be defined by final fast fracture subsequent to subcritical fatigue crack growth. Fatigue life is the number of cycles to failure, N_f, and it can be obtained by

$$N_f = \int_{a_i}^{a_f} \frac{1}{C(\Delta K_{tip})^n} \mathrm{d}a \tag{6}$$

where a_i = initial crack length and a_f = critical crack length. The critical crack length is determined by the final fast fracture condition, which is simply given by

$$K_{max} = K_c \tag{7}$$

where K_{max} = crack tip stress intensity factor at maximum load level and K_c = matrix fracture toughness.

Therefore, fatigue life can be computed via Paris law, if ΔK_{tip} and K_{max} are obtained for an overlay / underlay problem with a given FRC. ΔK_{tip} can be determined using a similar procedure to K_{tip}. Namely, the crack tip stress intensity factor amplitude of FRCs can be decomposed into two parts:

$$\Delta K_{tip} = \Delta K_a + \Delta K_b \tag{8}$$

where ΔK_a = stress intensity factor amplitude due to external applied loading and ΔK_b = stress intensity factor amplitude due to crack bridging.

2.2.2 *Cyclic bridging law*

Crack bridging in the current overlay / underlay problem is exerted by both aggregates and fibers in the new repair layer and only by the former in the old existing layer. Therefore, the cyclic bridging law, $\Delta\sigma_b(\Delta\delta)$, in the new layer is given by the superposition of crack bridging stress-crack opening displacement relation due to fibers and aggregates under cyclic loading:

$$\Delta\sigma_b = \Delta\sigma_f + \Delta\sigma_m \tag{9}$$

where $\Delta\sigma_f$ = fiber bridging stress change under cyclic loading and $\Delta\sigma_m$ = aggregate bridging stress change under cyclic loading, while, in the old layer, it is given only by the relation due to aggregates under cyclic loading:

$$\Delta\sigma_b = \Delta\sigma_m \,. \tag{10}$$

Cyclic bridging law due to fibers, $\Delta\sigma_f(\Delta\delta)$, has been derived based on the micromechanics of fiber bridging under cyclic loading, and details on the expression of $\Delta\sigma_f(\Delta\delta)$ together with $\Delta\sigma_m(\Delta\delta)$ can be found elsewhere (Matsumoto & Li 1999).

2.2.3 *Cycle-dependent degradation of crack bridging*

Cycle-dependent degradation of crack bridging is a governing factor of subcritical fatigue crack growth of fiber composites. Possible degradation sources are two kinds: interfacial bond degradation and fatigue rupture. In the current problem, only degradation of fiber- and aggregate-matrix interface bond is considered, and no fiber or aggregate fatigue rupture is taken into account.

Based on observations, a simple bilinear degradation function for fiber bridging is assumed (Zhang 1998). As for the degradation of fiber bridging, the fiber-matrix interfacial frictional bond strength, τ, is assumed to decrease as follows:

$$\frac{\tau}{\tau_i} = \max \text{ of } \begin{cases} 1.0 + D_1 \sum_{i=1}^{N} \Delta\delta_i(x) \\ B + D_2 \sum_{i=1}^{N} \Delta\delta_i(x) \end{cases} \tag{11}$$

where τ_i = initial bond strength, D_1 = degradation coefficient for the early trend (negative for degradation), $\Delta\delta_i(x)$ = crack opening displacement change at i-th cycle, B = intercept for the long trend, and D_2 = degradation coefficient for the long trend (negative for degradation). Here, the interfacial bond degradation is measured with

$$\sum_{i=1}^{N} \Delta\delta_i(x) = \text{accumulated crack opening displacement change at } x \tag{12}$$

where x is the position on the crack surface measured from the crack mouth. The role of this parameter is to measure both the number of cycles and the crack opening displacement change experienced at each point on the bridged crack surface. A bridged fatigue crack suffers from its damage in a non uniform manner with respect to the location and the number of cycles. The most severely damaged portion is near the crack mouth where the crack surface has been created in the early stage of fatigue loading and the number of cycles experienced is the largest. By contrast, the least damaged portion is near the crack tip where the crack surface has just been created and the number of cycles is the smallest.

Similarly, as for the degradation of aggregate bridging, the current degraded aggregate bridging stress, σ_m, is assumed to decrease from the original undegraded aggregate bridging stress, $\sigma_{m\,o}$, with the

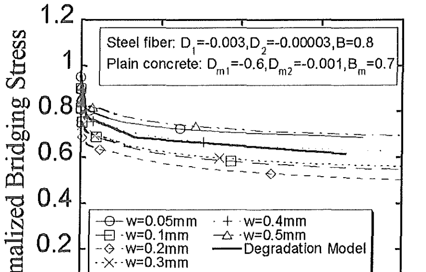

Figure 2. Crack bridging degradation with different initial opening displacements, w.

accumulated crack opening displacement change, namely

$$\frac{\sigma_m}{\sigma_{m\,o}} = \max \text{ of } \begin{cases} 1.0 + D_{m1} \sum_{i=1}^{N} \Delta\delta_i(x) \\ B_m + D_{m2} \sum_{i=1}^{N} \Delta\delta_i(x) \end{cases} \qquad (13)$$

where D_{m1} = degradation coefficient for the early trend (negative for degradation), B_m = intercept for the long trend, and D_{m2} = degradation coefficient for the long trend (negative for degradation). These degradation coefficients are calibrated with the experiments done by Zhang (1998) and shown in Figure 2.

2.2.4 *Governing equation*

The above mentioned monotonic and cyclic fatigue analysis requires to compute K_{tip} and both of K_{tip} and ΔK_{tip} respectively, and numerical scheme is required for the computation. Details of the numerical scheme are not mentioned in this paper and can be found in Cox & Marshall (1991).

3 ANALYSIS OF FRC OVERLAID / UNDERLAID BEAMS

The problems analyzed are the cases for plain concrete beams overlaid or underlaid with hooked-end steel fiber reinforced concrete. For both of overlaid and underlaid beams, three cases of beam height (0.1, 0.2, and 0.3 m) are examined. In addition, fiber volume fraction is varied at 1, 2, and 3 %. The ratio of FRC layer thickness to beam height is varied at 0.1, 0.33, 0.5, 0.66, and 0.9. Cyclic fatigue flexural load is applied to each beam at four maximum load levels (60, 70, 80, and 90 % in relative to the ultimate loading capacity) and R = 0.2 in order to perform fatigue crack growth computations. Fatigue life is defined by the number of cycles at which final unstable fracture takes place subsequent to stable fatigue crack growth, and it is represented as a theoretical S-N diagram. The material constants needed for the model computations are summarized in Tables 1 - 4.

3.1 *Monotonic analysis*

Results of monotonic analysis for overlay and underlay are shown in Figures 3 and 4 respectively. Only the cases of beam height = 0.1 m and V_f = 3 % are shown. Both figures describe the change in the relation between crack propagation stress and crack length, as FRC thickness ratio increases from 0.1 to 0.9.

Figure 3 shows the rise of crack propagation stress in the later stage of crack extension. This is due to the fiber bridging action when the crack goes into the overlay. However, the fiber bridging action does not contribute to the flexural strength increase, until the FRC thickness ratio reaches 0.66.

Table 1. Paris constants (Baluch et al. 1987).

C	n
9.03×10^{-6}	3.12

Table 2. Matrix parameters (Zhang 1998).

Elastic modulus	Tensile strength	Fracture toughness
GPa	MPa	MPa $m^{0.5}$
35	5.4	0.5

Table 3. Fiber parameters (Zhang 1998).

Fiber length	Elastic modulus	Fiber diameter
mm	GPa	μm
30	210	500

Table 4. Interface parameters (Li & Stang 1997, Li et al. 1993).

Initial bond strength
MPa
6.0

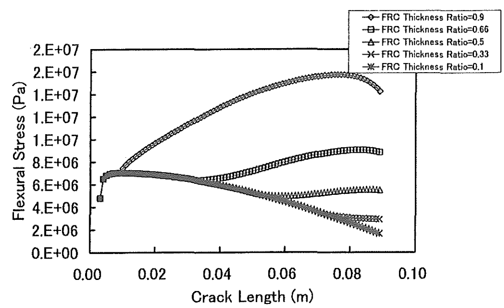

Figure 3. Crack propagation in a beam overlaid with 3 % FRC for varied FRC thickness ratios.

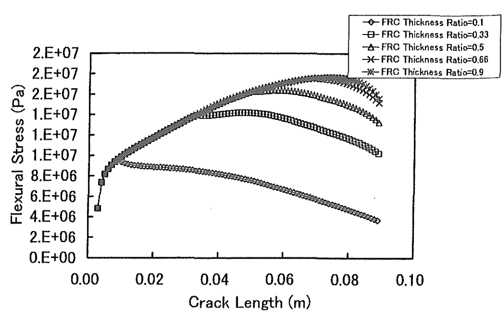

Figure 4. Crack propagation in a beam underlaid with 3 % FRC for varied FRC thickness ratios.

By contrast, Figure 4 shows the rise of crack propagation stress from the early stage of crack extension. The fiber bridging action in the underlay contributes to the flexural strength increase, since it effectively plays the role of crack tip shielding.

If the fiber volume fraction is less than 3 %, the contrast becomes even more clear. Overlay design without sufficient fiber volume fraction does not exhibit the flexural strength increase, if the FRC thickness ratio is below 0.9. On the other hand, underlay design can achieve the flexural strength increase even with a fiber volume fraction below 3 %.

3.2 *Cyclic fatigue analysis*

Fatigue crack growth curves for overlaid beams with varied fiber volume fractions (1, 2, and 3 %) are shown in Figures 5 - 7. Only the cases of beam height = 0.1 m and FRC thickness ratio = 0.66 are shown. The horizontal line in each figure describes the boundary of the new and old layers.

Although FRCs occupy more than half of the beam height in these cases, FRC with V_f = 1 % shows fatigue crack growth curves that have no difference from those of plain concrete. At all load levels, crack growth is always accelerated. This means that fiber bridging action is not sufficient for retarding the crack growth, even if the crack goes into the FRC overlay.

By contrast, FRCs with V_f = 2 and 3 % exhibit different crack growth behaviors. Crack growth is initially accelerated, but it is decelerated after the crack goes into the FRC overlay. Especially, FRC with V_f = 3 % shows strong deceleration so that slow growth rate can be achieved in the overlay. This strong deceleration can be explained with the crack propagation under monotonic loading in Figure 3 where two peaks are observed in the propagation curve. The first one is attributed to the aggregate bridging in the old layer, while the second one to the fiber bridging in the new layer. The two peaks dictate the behavior of fatigue crack growth of the overlay / underlay system. If the modulus of rupture is determined by the first peak, fatigue crack growth is accelerated throughout. On the other hand, if the modulus of rupture is determined by the second peak, fiber bridging action retards fatigue crack growth sufficiently.

Theoretical S - N diagrams can be constructed via fatigue crack growth computations. Figure 8 is an S - N diagram for overlaid beams with varied FRC thickness ratios and fiber volume fractions for beam height = 0.1 m. As can be expected from the fatigue crack growth curves, fatigue life can be improved as the FRC thickness ratio increases. However, the improvement is not sensitive to the FRC thickness ratio increase. This is again explained by the crack propagation curve under monotonic loading. Namely, fatigue life can not be improved, until the modulus of rupture is achieved by the second peak due to fiber bridging action. On the other hand, Figure 9 shows an S – N diagram for underlaid beams with varied FRC thickness ratios and V_f = 3 % for beam height = 0.1 m. The fatigue life improvement is sensitive to the FRC thickness ratio, and it seems to reach a certain limit state.

Theoretical S – N diagrams are summarized in another form in Figures 10. This figure shows the normalized fatigue energy against various thickness ratios of overlay and underlay systems of t = 0.1 m. Fatigue energy is defined here by the area below an S – N curve of a given system, and it is normalized

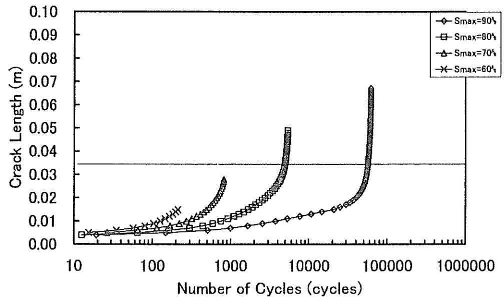

Figure 5. Fatigue crack growth in a beam overlaid with 1 % FRC (beam height = 0.1 m and FRC thickness ratio = 0.66).

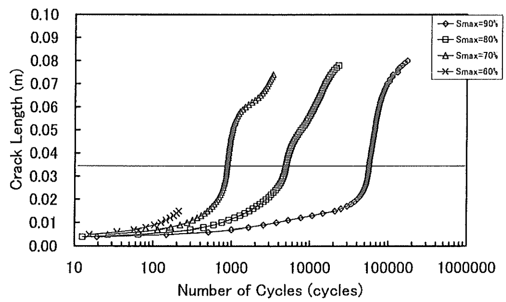

Figure 6. Fatigue crack growth in a beam overlaid with 2 % FRC (beam height = 0.1 m and FRC thickness ratio = 0.66).

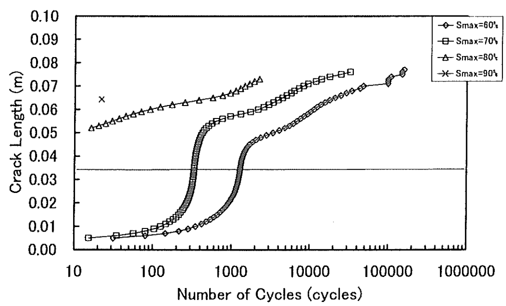

Figure 7. Fatigue crack growth in a beam overlaid with 3 % FRC (beam height = 0.1 m and FRC thickness ratio = 0.66).

with the value of plain concrete, e.g. FRC thickness ratio = 0.

The normalized fatigue energy clearly shows the different trends of fatigue life improvement between overlay and underlay system. Overlay does not exhibit fatigue energy improvement until FRC thickness ratio is above 0.5, and this is not changed by fiber volume fraction increase. By contrast, underlay improves fatigue energy even at a lower FRC thickness ratio, and it reaches a certain limit state around 0.5. Fiber volume fraction changes the fatigue energy improvement even at a lower FRC thickness ratio.

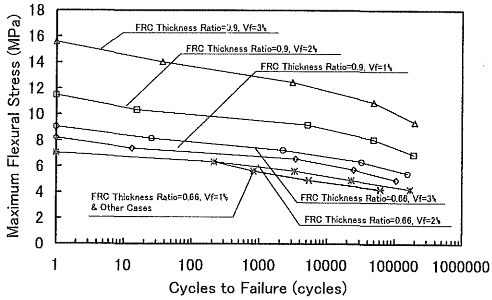

Figure 8. Fatigue life of overlaid beams with varied FRC thickness ratio for beam height = 0.1 m.

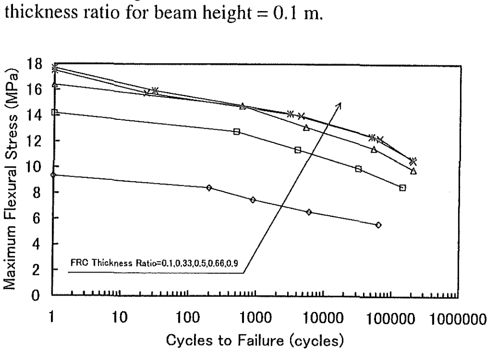

Figure 9. Fatigue life of underlaid beams with varied FRC thickness ratio for beam height = 0.1 m. $V_f = 3$ % only.

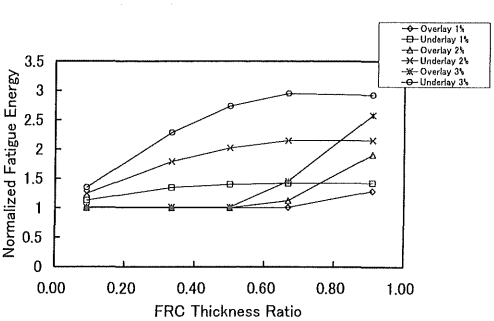

Figure 10. Normalized fatigue energy for beam height = 0.1 m.

4 CONCLUSIONS

In the case of overlay, resulting fatigue crack growth plots and theoretical S-N diagrams show that neither small dosage of fibers such as 1 and 2 % nor thin FRC repair layer is effective to suppress the fatigue crack growth. This is because the FRC layer on the compression side requires strong crack tip shielding of bridging stress in order to retard the fatigue crack growth and therefore prolong the fatigue life.

In the case of underlay, fatigue crack growth plots show the suppression of fatigue crack growth in the early stage, and theoretical S-N diagrams show improvements in fatigue strength because the FRC layer on the tension side contributes to the ultimate strength improvement.

Overall, overlay and underlay exhibit different characteristics in the role of fatigue crack tip shielding. This is clearly shown in the relation between normalized fatigue energy and FRC thickness ratio.

Further study should consider the presence of rebars. The presence changes the discussions above, since rebars control flexural cracks and FRC is rather beneficial for shear crack suppressions. Further possible studies should include punching shear fatigue computation of RC slabs. Also, interlayer delamination and reflective cracking must be accounted for when a repair layer is added to existing slabs.

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