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An isotropic damage model for non linear creep behavior of concrete in compression

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ABSTRACT: An isotropic model for creep damage of concrete in compression is proposed, where the combined effect of non linear viscous strain evolution and crack nucleation and propagation at high stress levels is considered. Strain splitting assumption is used for creep and damage contributions. Creep is modelled by a modified version of solidification theory. Concrete damage is governed by a damage index based on positive strains. As particular cases, the proposed model reduces to linear viscoelasticity for long time low stress levels whereas, for very high stresses, tertiary creep causing failure at a finite time can be described. The model is validated through comparison with experimental results. Some numerical example are also presented, where the roles of concrete ageing and strength variation with time are investigated.

1 INTRODUCTION

In common practice, creep of concrete is considered to produce deformations increasing with time with no significant material damage, and linear viscoelasticity is used. Nevertheless, when ultimate limit state analysis is performed to design concrete structural members, the state of stress can be moderately high, even over that conventionally considered as the limit of application of linear theory (30 to 40 percent of compression strength). Many examples can be found of structures whose long term serviceability has been compromised by creep deformations, resulting in a drastic reduction of its designed life span. In the case of structures liable to creep buckling, neglecting non linear creep behaviour may result in the overestimate of safety coefficients against buckling.

Few studies can be found where the combined effect of non linear creep deformations and damage of concrete is estimated. Microcracking nucleation and growth produced by creep at high stress levels cannot be captured merely by a nonlinear generalization of the viscoelastic stress-strain relation (Bazant 1993). For medium to high compression stresses, non linear creep gives strains several times greater than viscoelastic counterpart, but for very high stresses, creep strains are associated also with microcracking growth with time and, consequently, to damage (Neville 1970), which can result in concrete failure after a finite time interval (the tertiary creep).

Starting from strain splitting assumption, Cervera et al (1999) recently proposed a creep-damage model where material degradation influences directly all strain contributions (through the damage parameter). Hence, non linear creep is obtained as the result of material damage. Nevertheless, experimental results clearly show that, for medium stress levels, creep strains can be much greater than those predicted by linear viscoelasticity using a damaged elastic modulus (see for instance Mazzotti & Savoia 2001a).

In the present paper, a creep-damage model for concrete is presented, which considers both nonlinear creep and damage growth with time for compression uniaxial stresses. The model is based on the strain splitting assumption. Creep strain is modelled extending solidification theory in the non linear range. A relaxation form is adopted in order to ensure a direct implementation in a displacement-based finite element code. Non linear creep strains are evaluated by defining, starting from experimental results, a non linear function of damage index. It is also assumed that most of creep strain does not produce damage (as in the case of low stresses) and only a fraction of creep strain contributes to damage evolution with time.

For concrete damage, a strain-based formulation is adopted, where damage is related to positive strains obtained from longitudinal strain through non linear Poisson' ratio. Moreover, to define damage-creep interaction, some experimental results of short-term creep test at high stress levels are used (Mazzotti & Savoia 2001a).

The proposed model is validated by comparison with experimental results. Some numerical examples are also presented, concerning creep tests at increasing stress levels and different ages at loading. It is shown that the present model reduces to linear viscoelasticity for low stress levels and predicts failure due to tertiary creep for high stresses.

2 A CREEP-DAMAGE NON LINEAR MODEL

In the proposed non linear creep-damage model, it is assumed that high stress level produces with time not only non linear creep deformations but also the onset and growing of matrix cracking with consequent deterioration of concrete mechanical properties (Meyers et al. 1969). These phenomena can be effectively described making use of damage mechanics, which is particularly suitable as a basis of simple computational algorithms.

Concrete damage is described by means of a damage index defined in terms of strains, so that damage evolution with time at constant stress (creep problem) can be followed. The proposed model is written in incremental form as:

$$\dot{\sigma} = E_{eff}\,\dot{\varepsilon}^{el}, \tag{1}$$

where $\dot{\varepsilon}^{el}$ represents the elastic strain rate and E_{eff} is the effective longitudinal modulus, which is lower with respect to initial tangent modulus E_0 due to damage but it can also increase with time due to concrete ageing. Total strain rate is defined as the sum of different contributions:

$$\dot{\varepsilon} = \dot{\varepsilon}^{el} + \dot{\varepsilon}^{d} + \dot{\varepsilon}^{v} + \dot{\varepsilon}^{irr}, \tag{2}$$

where $\dot{\varepsilon}^{el}$ is the elastic strain rate (appearing in the incremental relation (1)), $\dot{\varepsilon}^{d}$ is the strain rate due to damage, $\dot{\varepsilon}^{v}$ is the non linear creep strain rate and $\dot{\varepsilon}^{irr}$ is the irreversible instantaneous strain rate. The strain splitting assumption (2) is the basis of several formulations in the framework of infinitesimal deformations (see for instance Rabier 1989). Making use of (2), Equation (1) takes the form:

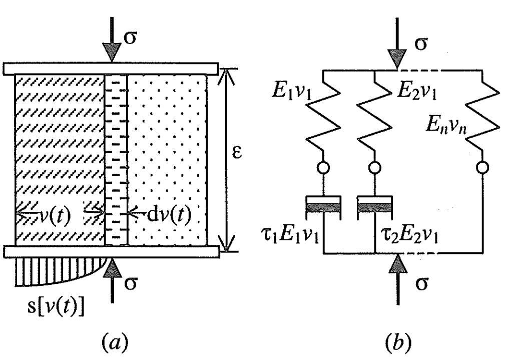

Figure 1. (*a*) Mechanical model of Bazant solidification theory; (*b*) Maxwell chain equivalent to the proposed model.

$$\dot{\sigma} = E_{eff}\left(\dot{\varepsilon} - \dot{\varepsilon}^{d} - \dot{\varepsilon}^{v} - \dot{\varepsilon}^{irr}\right). \tag{3}$$

The main advantage of assuming strain splitting is the possibility of defining single strain contributions in Equation (3) with reference to uncoupled models, whose mechanical parameters are easily identifiable with standard experimental tests (Cervera et al. 1999). For instance, in the present formulation creep strain rate is defined making use of solidification theory (Bazant 1977), while damage strain rate and irreversible instantaneous strain rate are defined according to a scalar one-parameter elastoplastic damage model.

3 THE CREEP MODEL

For creep strain of concrete in compression a non linear version of solidification theory is proposed.

According to solidification theory, originally proposed by Bazant (1977) for linear viscoelasticity, the age-dependent behaviour of concrete is described by treating the solidifying material (hydrating cement paste) as a varying composite whose components are characterised by age-independent mechanical properties. Ageing is modelled as a change of concentrations of solidified matter, whose volume variation is denoted by $v(t)$, with $0<v(t)\leq 1$ (Bazant & Prasannan 1989). This simple mechanical model is aimed to describe many different physical-chemical phenomena occurring during concrete ageing. Growth of mass of cement hydration products per unit volume of concrete is the main physical mechanism. In addition, during concrete hardening, relaxation of localised high stress peaks in the microstructure of cement gel also occurs, as recently described in microprestress-solidification theory (Bazant et al. 1997). This phenomenon is not considered in the present form.

According to solidification theory, the subsequently deposited layers of solidified constituent are assumed to be coupled in parallel and subject to the same macroscopic strain increment $d\varepsilon$ when contributing to carry compressive stress (Fig. 1*a*). The layers of cement gel already solidified carry a finite stress s, while those before solidification are considered stress free ($s=0$).

The solidifying constituent is considered as a non ageing linear viscoelastic material characterised by micro-relaxation function $\Psi(\xi)$, where $\xi=t-t'$. With reference to time τ and denoting by $v(\tau)$ the volume of solidified matter at that time, the stress s at age $t > \tau$ in the solidifying layer $dv(\tau)$ is:

$$s(\tau,t) = \int_{\tau}^{t} \Psi(t-t')\,d\varepsilon(t'), \tag{4}$$

where the limits of the integral reflect the fact that only strain occurred after the solidification time of the layer must be considered. Making use of Equation (4), equilibrium equation between the macroscopic (applied) stress σ and (micro) stresses $s[v(t)]$ referred to general layers, can be written as

$$\sigma(t)=\int_{t'=0}^{t}\Psi(t-t')v(t')\mathrm{d}\varepsilon(t') \; . \tag{5}$$

Then, the relaxation function is the product of two functions: one of the concrete age at loading t' and the other of load duration t-t':

$$R(t,t')=v(t')\Psi(t-t') \, . \tag{6}$$

Adopting a Dirichlet series expression for micro-relaxation function $\Psi(t\text{-}t')$ (Carol & Bazant 1993):

$$\Psi(t-t')=E\,e^{-(t-t')/\tau}, \tag{7}$$

Equation (5) can be written in differential form as:

$$\dot{\sigma}(t)+\frac{\sigma(t)}{\tau}=Ev(t)\dot{\varepsilon}(t), \tag{8}$$

corresponding to a Maxwell unit with ageing modulus $E(t)$=$v(t)E$ and constant dashpot viscosity η=τE, where τ is the characteristic relaxation time.

Bazant & Prasannan (1989) recently proposed a general solidification theory based on n different solidification processes characterised by volume increments $\mathrm{d}v_\mu(t')$ (μ=1,...,n). The following system of first-order differential equations is obtained

$$\begin{cases} \sigma(t)=\sum\limits_{\mu=1}^{n}\sigma_\mu(t) \\ \dot{\sigma}_\mu(t)+\dfrac{\sigma_\mu(t)}{\tau_\mu}=E_\mu v_\mu(t)\dot{\varepsilon}(t) \qquad \mu=1,\ldots,n, \end{cases} \tag{9}$$

corresponding to an ageing Maxwell chain of n units with n different constant relaxation time τ_μ. For a general Maxwell chain, the identification problem of a set of elastic ageing moduli $E_\mu(t)\equiv E_\mu\, v_\mu(t)$ is ill posed (Bazant & Prasannan 1989), due to the high deviation and the very short observation time of experimental data available. It is then preferable to adopt two solidification functions at most $v_1(t)$, $v_n(t)$: the first function $v_1(t)$ governs ageing of first n-1 Maxwell units, whereas $v_n(t)$ modulates the last unit.

In the present model, the last unit is a degenerated unit constituted by an elastic spring of modulus $v_n(t)E_n$ (see Fig. 1*b*), in order to ensure an asymptotic value of creep strain for t=∞, as prescribed by CEB MC90 (Papa et al. 1998, Ceccoli et al. 1999). Moreover, CEB MC90 relaxation curves are used to evaluate the parameters defining elastic ageing moduli for the set of Maxwell units adopted.

Hence, starting from Equation (9), stress rate can then be given the following form:

$$\dot{\sigma}(t)=\sum_{\mu=1}^{n}\dot{\sigma}_\mu(t)=\sum_{\mu=1}^{n}E_\mu v_\mu(t)\left[\dot{\varepsilon}(t)-\frac{\sigma_\mu(t)}{E_\mu v_\mu(t)\tau_\mu}\right]. \tag{10}$$

It is noteworthy that, admitting an incremental form, solidification theory does not require storage of the whole stress history.

Solidification theory can be extended to cover the case of non linear creep, since it does not require the definition of convolution operators but can be written in term of incremental equations. Non linear creep strains can be very important for medium to high stress levels, as confirmed by experimental tests reported in Section 6.

In the present model, non linear creep is introduced by defining a stress rate reduction factor $\lambda(d)\leq 1$, as a function of damage index d, so that Equation (10) is replaced by:

$$\dot{\sigma}(t)=\sum_{\mu=1}^{n}E_\mu v_\mu(t)\left[\dot{\varepsilon}(t)-\frac{\sigma_\mu(t)}{E_\mu v_\mu(t)\tau_\mu}\right]\lambda(d). \tag{11}$$

For step-wise loading histories, the reduction function $\lambda(d)$, for the general time step Δt, is defined as

$$\begin{cases} \lambda=f(d) & \text{if} \quad \Delta\sigma(\Delta t)=0 \\ \lambda=1 & \text{if} \quad \Delta\sigma(\Delta t)\neq 0. \end{cases} \tag{12}$$

The second condition is required to ensure that, when a loading step is applied, function $\lambda(d)$ does not affect instantaneous stress-strain relation.

Equation (11) can be viewed as the dual formulation of non linear creep model proposed by Bazant & Prasannan (1989), originally formulated in terms of Kelvin units (creep problem).

4 THE DAMAGE MODEL

Continuum damage describes mechanical effects of progressive microcracking, void nucleation and growth at high stress levels making use of a set of state variables modifying the material behaviour at the macroscopic level (Lemaitre & Chaboche 1985). In the present study, damage is governed by one scalar parameter d (isotropic damage) which degrades the elastic stiffness of concrete through a variation of the secant Young's modulus. For a damaged continuum isotropic medium, the constitutive law is then written as:

$$\sigma_{ij}=\frac{E_0(1-d)}{1+\nu}\varepsilon_{ij}^{el,d}+\frac{E_0(1-d)\nu}{(1+\nu)\cdot(1-2\nu)}\varepsilon_{kk}^{el,d}\delta_{ij}, \tag{13}$$

where σ_{ij} and $\varepsilon_{ij}^{el,d}$ are stress and damaged elastic strain tensor, E_0 is the undamaged Young's modulus, d is the damage parameter, δ_{ij} is the Kronecker symbol and ν is the Poisson' coefficient. Some authors (see for instance Pijaudier-Cabot 1995, di Prisco & Mazars 1996) consider a damaged value of Poisson' coefficient in the form ν=$\nu_0(1+\delta)$, where δ is a second damage parameter which is not independent but related to d through a relation based on experimental data.

Starting from Equation (13), strain tensor can be written as a function of stress tensor in the form:

$$\varepsilon_{ij}^{el,d} = \frac{1+\nu}{E_0(1-d)}\sigma_{ij} - \frac{\nu}{E_0(1-d)}\sigma_{kk}\delta_{ij}. \tag{14}$$

With reference to a uniaxial state of stress characterised by $\sigma_{11}\neq 0$, $\sigma_{22}=\sigma_{33}=0$, by expressing transverse strains as a function of longitudinal strain as

$$\varepsilon_{22}^{el,d} = \varepsilon_{33}^{el,d} = -\nu\,\varepsilon_{11}^{el,d}, \tag{15}$$

the constitutive relation is rewritten in terms of longitudinal stress and strain components as:

$$\sigma_{11} = E_0(1-d)\varepsilon_{11}^{el,d}. \tag{16}$$

The degradation of Poisson' ratio is taken into account in the evolution law of the damage index d, as it will be described in Section 6. In Equation (16), the damaged elastic strain is defined as the total strain minus the irreversible (plastic) strain:

$$\varepsilon_{11}^{el,d} = \varepsilon_{11}^{tot} - \varepsilon_{11}^{irr}, \tag{17}$$

and takes the role of an effective instantaneous strain, being related to stress component through a damaged Young's modulus. In this way the effective strain component is the sum of the elastic strain and of the damage strain component (Fig. 2).

Taking into account Equation (17), Equation (16) can be rewritten in incremental form as:

$$\dot{\sigma}_{11} = E_0(1-d)\left[\dot{\varepsilon}_{11} - \frac{\dot{d}}{1-d}\varepsilon_{11}^{el,d} - \dot{\varepsilon}_{11}^{irr}\right], \tag{18}$$

where the three terms in brackets represent the total strain rate, the damaged strain rate and the irreversible strain rate, respectively.

According to Mazars & Pijaudier-Cabot (1989), the isotropic damage parameter d is a function of the equivalent strain defined as:

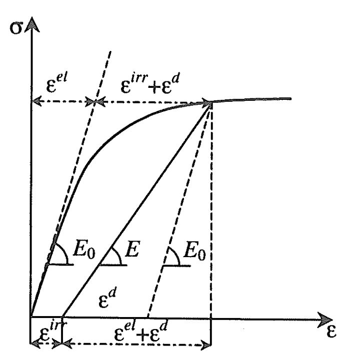

Figure 2. Notation adopted for strains for concrete under uniaxial compression.

$$\tilde{\varepsilon} = \sqrt{\sum_{i=1}^{3}\left\langle \varepsilon_i^{el,d}\right\rangle_+^2}, \tag{19}$$

where $\langle\varepsilon_i^{el,d}\rangle_+$ are the positive principal strain components. Then, a damage criterion is introduced:

$$g(\tilde{\varepsilon}, K(d)) = \tilde{\varepsilon} - K(d) \leq 0, \tag{20}$$

where $K(d)$ is the damage threshold at the current time t. It must be defined so that:

$$K(d) > K_0 \quad \forall t \in \mathbb{R}^+ \tag{21}$$

where K_0 denotes the initial damage threshold, in order to assure damage to be an irreversible processes. Damage evolution is governed by the normality rule:

$$\begin{cases} \dot{d} = \dot{\mu}\dfrac{\partial g(\tilde{\varepsilon},K)}{\partial \tilde{\varepsilon}} \\ \dot{\mu} = H(\tilde{\varepsilon},d)\dot{K} \end{cases} \tag{22}$$

where $\dot{\mu} \geq 0$ is a damage consistency parameter defining damage loading/unloading condition according to Kuhn-Tucker relations

$$\dot{\mu} \geq 0, \qquad g(\tilde{\varepsilon},K) \leq 0 \qquad \dot{\mu} g(\tilde{\varepsilon},K) = 0 \tag{23}$$

From Equations (22, 23), it can be easily shown that

$$\dot{d} = \frac{\partial g(\tilde{\varepsilon},K)}{\partial \tilde{\varepsilon}}\dot{\tilde{\varepsilon}} = \frac{\partial G(\tilde{\varepsilon})}{\partial \tilde{\varepsilon}}\dot{\tilde{\varepsilon}}, \tag{24}$$

where G is the damage accumulation function (Simo & Ju 1987). For concrete like materials and instantaneous loadings, the following damage accumulation function has been considered (originally proposed by Mazars & Pijaudier-Cabot (1989) without the irreversible strain contribution):

$$G(\tilde{\varepsilon}) = 1 - \frac{(1-A)k_0}{\tilde{\varepsilon}} - Ae^{-B(\tilde{\varepsilon}-k_0)}, \tag{25}$$

where A,, B and k_0 are parameters characteristic of the material. This expression has been found to give results in agreement with experimental data for standard tests. In an uniaxial compression test, positive strains are represented by transverse strain components which are determined as a function of longitudinal strain according to Equation (15) and considering Poisson' ratio as a function of longitudinal strain itself, $\nu=\nu(\varepsilon_{11})$.

5 CREEP-DAMAGE MODEL: INCREMENTAL FORMULATION

Substituting the Maxwell unit elastic moduli with the correspondent secant damaged moduli in Equation (11), the effective modulus $E_{eff}(t)$ in Equation (3) takes the form

$$E_{eff}(t) = (1-d)\sum_{\mu=1}^{n} E_{0\mu}\,\nu_\mu(t). \tag{26}$$

Correspondingly, Equation (3) can be rewritten as:

$$\dot{\sigma}(t)=\sum_{\mu=1}^{n}\dot{\sigma}_{\mu}(t)=\sum_{\mu=1}^{n}E_{0\mu}v_{\mu}^{\cdot}(t)(1-d)\lambda(d)\cdot$$
$$\cdot\left[\dot{\varepsilon}(t)-\frac{\dot{d}}{1-d}\varepsilon^{el,d}-\frac{\sigma_{\mu}(t)}{E_{0\mu}v_{\mu}(t)(1-d)\tau_{\mu}}-\dot{\varepsilon}^{irr}\right]. \quad (27)$$

5.1 *The Effective Strain for Damage Evolution*

The present model is based on the assumption that only a fraction of total creep strain contributes to damage evolution with time. The motivation is that, for medium stress levels, creep strain may be several times that predicted by linear viscoelasticity, but no significant variation of elastic modulus of reloading branch is observed.

An effective strain is then defined, replacing the equivalent strain for damage evaluation in the case of instantaneous loadings, see Equation (19). The effective strain is the sum of instantaneous damaged elastic strain $\varepsilon^{el,d}=\varepsilon^{el}+\varepsilon^{d}$ and of a percentage of the creep strain, i.e. $\varepsilon_{eff}=\varepsilon^{el,d}+\beta\cdot\varepsilon^{v}$, where β is considered for simplicity independent of loading level. Good correlation between numerical and experimental results has been obtained by adopting β=0.1÷0.2 (see the following Section).

5.2 *Strength variation*

Concrete strength variation with time is also taken into account. Ageing of concrete results in increase of both stiffness and strength. Stiffness variation is already considered into creep mechanism for ageing materials. Strength increase must be also included to correctly evaluate damage grow with time in the framework of a creep-damage model. This phenomenon can be particularly significant for concretes loaded at early age.

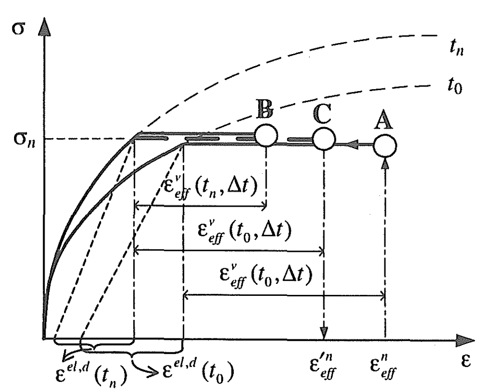

Figure 3. Reduction of effective strain due to strength growth with time: limit cases A and B and reduced effective strain (C) used in the present model.

Some recent models can be found in the literature where strength variation is taken into account (see for instance Cervera et al. 1999).

Due to the limited experimental data available about the damage-strength increase interaction, a simple model is introduced in the present study. First of all, CEB MC90 stress-strain law, which depends on concrete strength, is used to define the concrete behaviour as a function of its strength which increases with time.

The effective strain at time t_n (where, as define before, effective means strain to be used to evaluate damage grow) for a concrete initially loaded at time t_0 must then be defined (see Fig. 3). Strength increase is taken into account by defining a reduced effective strain, which is an intermediate value between the effective strains correspondent to two limit conditions (Fig. 3):

A. Concrete with stiffness and strength evaluated at time t_0: the effective strain is the sum of instantaneous strain $\varepsilon^{el,d}(t_0)$ and viscous strain $\varepsilon^{v}_{eff}(t_0, \Delta t)=\beta\cdot\varepsilon^{v}(t_0, \Delta t)$.

B. Concrete with stiffness and strength evaluated at time t_n: the effective strain is the sum of instantaneous strain $\varepsilon^{el,d}(t_n)$ and viscous strain $\varepsilon^{v}_{eff}(t_n, \Delta t)=\beta\cdot\varepsilon^{v}(t_n, \Delta t)$.

In the present model, an effective strain satisfying the two limit cases (which will be called reduced effective strain since it must be smaller than that calculated with mechanical properties evaluated at time t_0) is defined as the sum of instantaneous strain $\varepsilon^{el,d}(t_n)$ and of effective viscous strain $\varepsilon^{v}_{eff}(t_0, \Delta t)$ (point C in Fig. 3). Comparing the reduced effective strains obtained in subsequent time steps t_n, t_{n+1}, a damage activation criterion is then introduced:

$$\dot{d}>0 \quad \text{iff} \quad \varepsilon'^{n+1}_{eff}>\varepsilon'^{n}_{eff}. \quad (28)$$

Hence, especially for young concrete, strength increase with time may stop damage grow with time.

The proposed model has been numerically implemented using a modified version of the exponential algorithm (Bazant & Wu 1974). Allowing for the adoption of variable time steps, computational effort can be reduced preserving accuracy and stability, see Mazzotti & Savoia (2001b) for details.

6 NUMERICAL EXAMPLES

The features of the proposed creep-damage model to simulate damage grow with time in uniaxial creep tests are shown in this section.

6.1 *Definition of Creep and Damage Laws*

Starting from strain splitting assumption, parameters defining the creep response and damage evolution law can be defined separately. It is required that the two limit cases of linear creep and instantaneous non linéar behaviour are reobtained for low and very high stress levels.

As far as damage law is concerned, the CEB

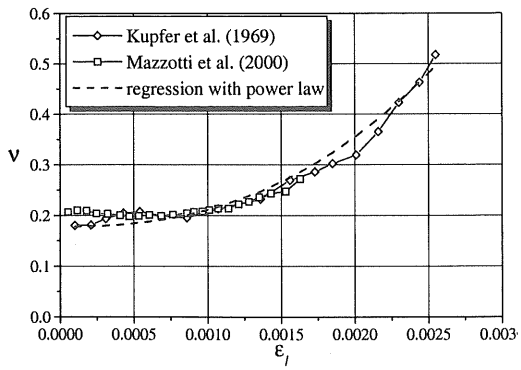

Figure 4. Poisson' ratio evolution in uniaxial compression test: experimental data and regression curve using a power law.

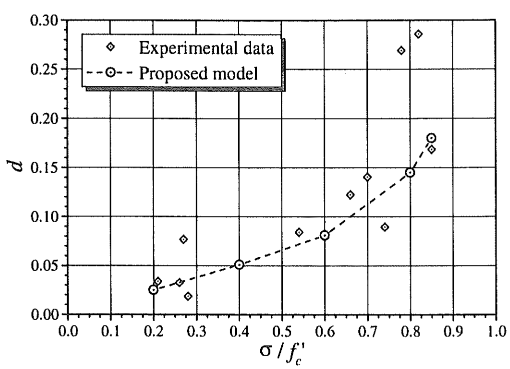

Figure 5. Damage index vs. stress level: experimental results (Mazzotti & Savoia 2001a) and numerical predictions by the present model.

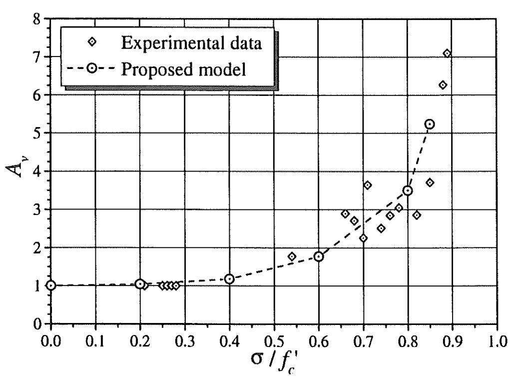

Figure 6. Non linear amplification of creep strains as a function of stress level: comparison between experimental and numerical prediction.

Model Code 1990 stress-strain curve is adopted. To describe Poisson' ratio evolution as a function of longitudinal strain, various proposals can be found (see, for instance, Krajcinovic & Fonseka 1981); in the present model a power law is used, determined through a best fit procedure starting from some classical experimental results (Kupfer et al. 1969) and results recently obtained by the present authors using ESPI technique (Mazzotti et al. 2000) (Fig. 4).

As for creep deformation in the linear range (low stress levels), solidification theory with nine Maxwell units is used, adopting relaxation times varying from 0.003 up to 30000 days. Moreover, the last unit has infinite relaxation time, in order to govern the asymptotic behaviour for $t \rightarrow \infty$; the parameters involved (ageing laws and elastic moduli) are calibrated using relaxation curves given by CEB MC90.

Finally, parameters adopted for the amplification function $\lambda(d)$ defining non linear creep and those required to identify the more suitable form of the effective strain are obtained by best fit procedure starting from a set of experimental creep short-term results at high stress levels (Mazzotti & Savoia 2001a). In Figures 5 and 6, damage evolution and non linear creep amplification obtained from numerical simulation are compared with experimental results presented in Mazzotti & Savoia (2001a).

The creep amplification factor A_v reported in Figure 6 is the ratio between the non linear specific creep and its counterpart obtained using linear viscoelasticity. The substantial agreement between experimental and numerical results (taking into account the unavoidable scattering of experimental results) confirms the validity of laws used in the proposed model.

6.2 *Comparison with Experimental Results*

The results of a numerical simulation at 75 percent of compression strength are compared with those obtained in a short term experimental test (1 hour of loading), see Mazzotti & Savoia (2001a). The results are depicted in Figure 7, where creep strain obtained applying linear viscoelasticity is also

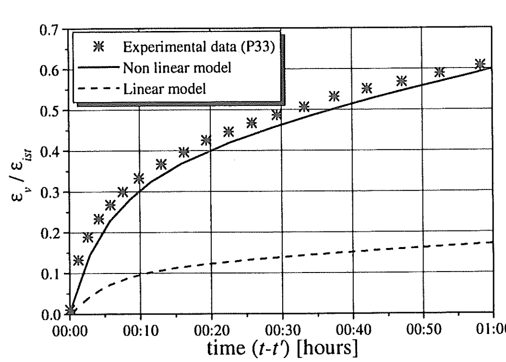

Figure 7. Short term uniaxial creep test: comparison between experimental results and those given by present model.

reported. The Figure shows substantial agreement between experimental and numerical results obtained from the proposed model, and a significant strain amplification with respect to linear solution.

6.3 *Some numerical example*

A series of creep tests of concrete with 28-day age at loading and different stress levels is simulated and compared with results obtained from linear viscoelasticity. Figure 8 shows that, for low stress levels (20 to 40 percent of compression strength f_c'), the non linear model gives substantially the same creep strain obtained from linear viscoelasticity, since non linear amplification of creep strains and concrete damage are negligible. Increasing the stress level (up to 80 percent of f_c'), a significant non linear amplification of creep strain can be observed (strains more than three times greater than those predicted by linear viscoelasticity), in accordance with experimental data reported in Figure 5. Specimen failure is not reached, at least before 30000 days of loading. Finally, for stress levels higher than 80 per cent of compression strength, tertiary creep can be observed, i.e., concrete failure due to growth of damage with time. This result is in agreement with prescriptions given by most important international standards (MC90, BS-PC110, DIN 1045) and with experimental findings about the sustained strength of concrete (Meyers et al 1969, Shah & Chandra 1970).

The effect of concrete ageing before loading application on creep behaviour and damage process has been also investigated. To this purpose, the same creep test has been simulated considering 7- and 360-day age at loading (see Figs 9, 10). The concrete loaded at early age exhibits higher creep strains, especially for medium-to-high loading levels. Moreover, failure due to tertiary creep is reached for 75 per cent of compression strength. The opposite behaviour is observed for the concrete loaded at 360-day age, with tertiary creep for compressive load between 90 and 95 per cent of strength.

The sensitivity of the proposed model to strength variation mechanism, included as described in Section 5, is also verified. As an example, Figure 11 shows the numerical predictions of creep tests with age of 28 days at loading, including and excluding the mechanism of damage reduction due to strength growth with time. As is to be expected, at low stress levels (20 to 40% of f_c') this phenomenon has no significant effect, since no damage occurs. On the

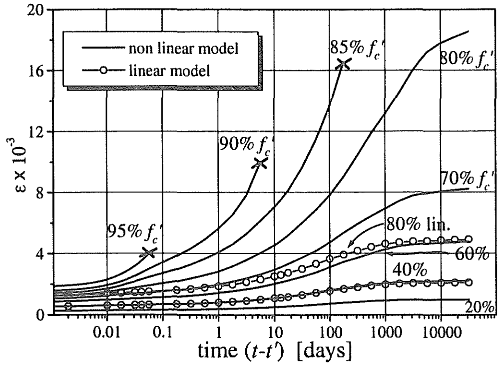

Figure 8. Simulation of creep tests at different stress levels: 28-day age of concrete at loading: comparison between results given by the present model and linear viscoelasticity.

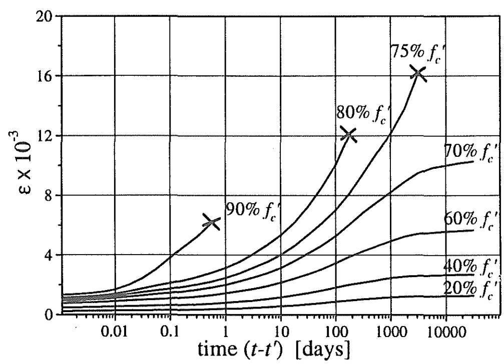

Figure 9. Simulation of creep tests at different stress levels: 7-day age of concrete at loading.

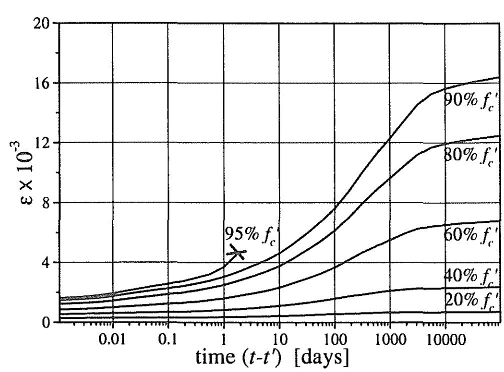

Figure 10. Simulation of creep tests at different stress levels: 360-day age of concrete at loading.

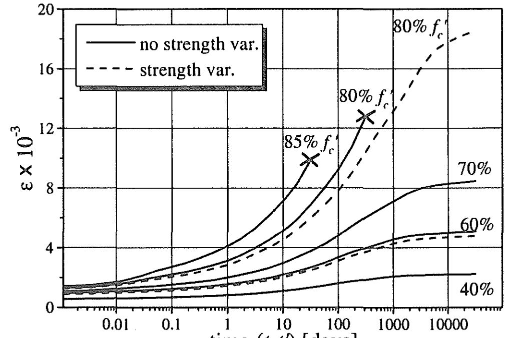

Figure 11. Simulation of creep tests at different stress levels with concrete loaded at an age of 28 days: results given by the proposed model with or without strength variation with time.

contrary, for high stress levels, strength variation with time may reduce damage growth due to creep strain process; for instance, for 80 percent of f_c', failure due tertiary creep does not occur.

7 CONCLUSIONS

In the present paper, a creep-damage model has been proposed for non linear creep behaviour of concrete in uniaxial compression. Starting from strain splitting assumption, a strain based isotropic model and the solidification theory are used for damage and (non linear) creep, respectively.

The model is able to cover the whole range from viscoelastic behaviour for low stress levels, to non linear creep for medium-level stresses (not accompanied by significant concrete damage) to tertiary creep for high stresses (i.e., failure due to mechanical degradation of concrete with time). Strength variation with time is also taken into account; it may reduce damage growth with time especially for concretes loaded at early age.

The present model has been validated by comparison with experimental results obtained from short term creep tests. These tests confirmed that non linear creep of concrete at medium stress levels cannot be justified only by concrete damage processes, since it is not accompanied by a correspondent variation of modulus of reloading branch.

Some numerical examples have been presented. The model is able to predict the level of compression stress over which tertiary creep may occurs. As is to be expected, this level is lower for concretes loaded at early age. Finally, a more comprehensive experimental investigation has to be carried out to better clarify some numerical aspects of the model and to extend the validity of the results to more general loading conditions (in particular longer loading duration).

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