Size effect experiments on granular materials in compression

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ABSTRACT: Heterogeneous granular materials exhibit a size effect under tensile loading. thus strength determined in experiment is not a material property. Only few experiments in literature are available on the size effect under compression and the maximum size range of these experiments was 1:4. In this paper a size effect series on sandstone specimens and a size effect series on concrete specimens is presented. These series are the largest with size ranges of 1:32 and 1:16 published so far. Obviously large size ranges are advantageous for experimental studies on the size effect. Experimental studies on that topic are very important, because heterogeneous materials are mostly analyzed and designed with a compressive strength determined on small specimens, that is assumed constant for all sizes. In the experiments a considerable size effect was detected.

Keywords: Size effect, concrete, sandstone, compression

1 INTRUDUCTION

Material failure criteria of concrete in codes or numerical models are usually expressed by strength or yield criteria. These descriptions imply that constitutive parameters do not change for different geometries and sizes, thus the strength is a material parameter that is independent of size. However, for concrete under tensile loading it is well known from experiments that the nominal strength is decreasing with increasing size. In reinforced concrete structures compressive failure is the more important and dangerous failure mode. Less size effect experiments on concrete under compressive loading are available. Nowadays also high strengths concretes are applied in real structures and size effect investigations become more important because the higher the strength the more pronounced is the brittleness and the size effect.

In the last three decades numerous researchers have studied size effect intensively. Most of the size effect investigations were performed under tensile loading on compact test specimens (e.g. Slowik 1995) and direct tension on dog bone specimens (e.g. Van Vliet 2000). Very few experimental investigations were performed on the size effect of concrete under compressive loading. Such series were performed by Bažant and Kwon (1994) on microconcrete, by Hollingworth (1998) and Sener et.al. (1999) on plain concrete columns and by Němeček (2000) on reinforced concrete. The largest size range of the mentioned test series performed on plain and reinforced concrete under compression was 1:4, see Figure 1. The size effect series with the largest size ranges (1:16 and 1:32) tested on granular materials under compressive loading are indicated in Figure 1 as "Burtscher Concrete" and "Burtscher Sandstone". The specimens of the mentioned test series are compared with the others in Figure 1. The dimensions and the reinforcement in all the series were scaled by a factor for every size, thus geometrical similarity was fulfilled. In this paper the "Burtscher Sandstone" and "Burtscher Concrete" series are presented. See Figure 2 and 3 for an overview of the specimen sizes. For a detailed description of the experimental results see Burtscher 2002 and Burtscher et.al. 2003a, b.

The experimental investigation on the size effect is a difficult task, because one has to deal with different sizes of geometrically equal specimens, where the same boundary conditions have to be

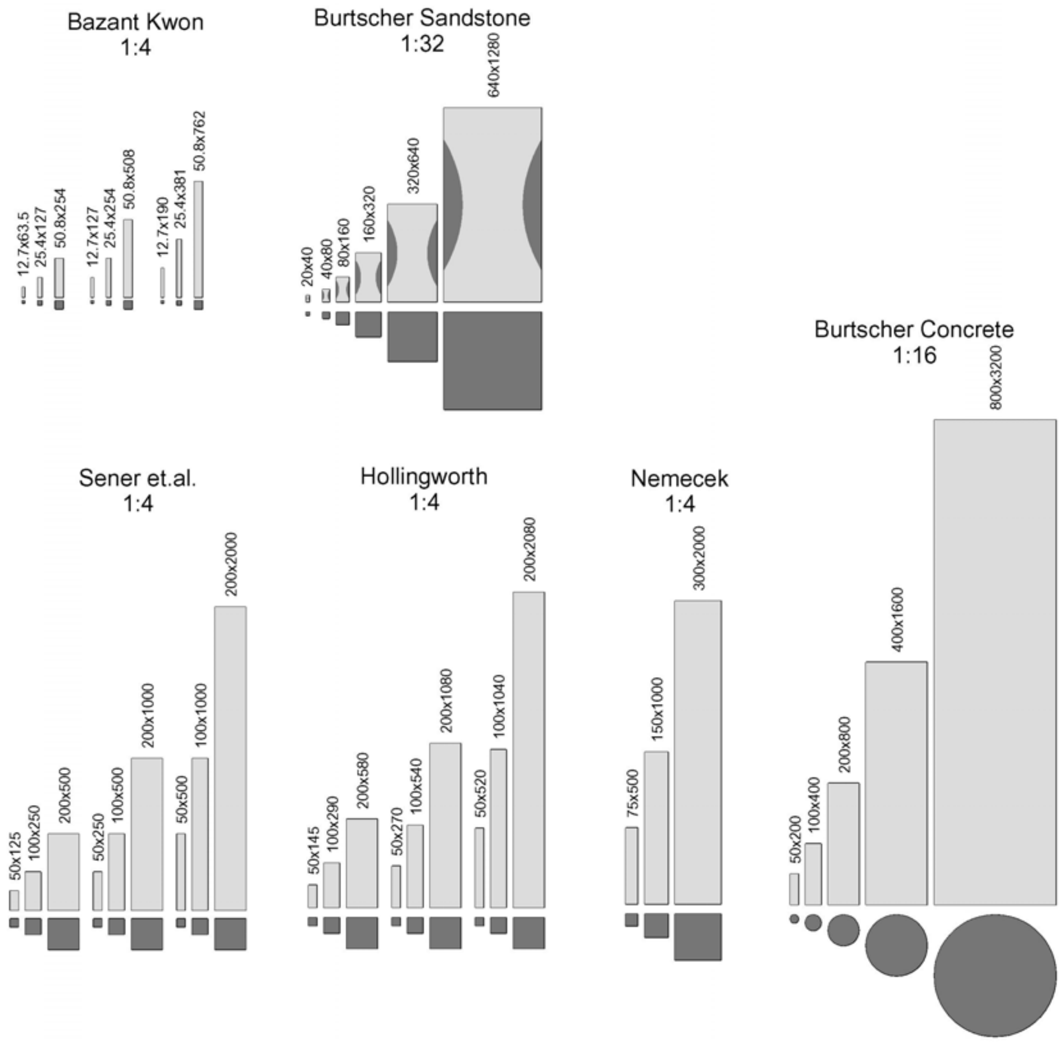

Figure 1: Overview of specimens in size effect series.

met for all specimens and specimen sizes. All geometric dimensions of the specimens have to be scaled by the same factor. The larger the factor from the smallest to the largest specimen, the more pronounced is the effect of size on the observed quantity. When multiplying all specimen dimensions by a scale factor the length changes linearly with the size factor, the cross section changes by the order of two with the size factor and the volume by the order of three. This means practically that the testing frame has to be adapted to the length of each specimen type and that the maximum load changes by the order of two with the size factor from one size to the other (disregarding the size effect). Due to strongly varying maximum loads different testing frames have to be used.

2 TESTING FACILITIES

The experiments were performed using the Column Tester, the D2 machine in the joint Laboratory of the Institute for Structural Concrete and the Institute for Steel Structures at the Vienna University of Technology and the Inova machine at the Department of Structural Mechanics at CTU Prague. The maximum loads for the D2 frame and the Inova machine are 500kN and 2000kN, respectively. The largest machine used for the investigations was the Column Tester, with this

Figure 2: Specimens of the concrete series.

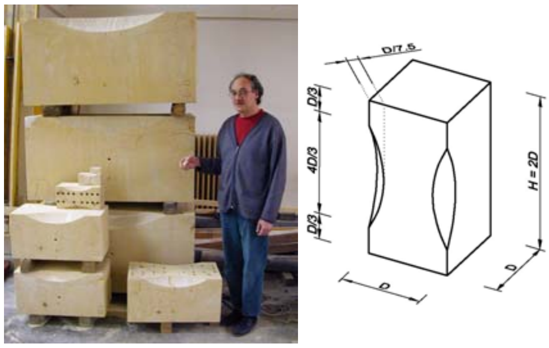

Figure 3: Specimens of sandstone series.

machine it is possible to perform experiments with closed loop control on specimens with a length from 40cm to 5m, a cross section up to 120x100cm and with a maximum load of 17000kN. This testing frame allows to test real size structures. Hence it is possible to compare the structural behavior of real size structures with laboratory size structures.

3 PREPARATION OF THE CONCRETE AND SANDSTONE SPECIMENS

The large dimensions and the high maximum load of the Column Tester allows testing of very large specimens and therefore increases the total size range of the experimental series. The size range of a size effect series is determined by the magnification factor of the characteristic dimension of the smallest and the largest specimen size. The smallest size is limited by the representative volume. The largest size is limited by the maximum load and dimension of the testing machine. The Column Tester at the Vienna University of Technology allows to increase the size range because of its high testing load and large dimensions, thus larger size ranges are possible.
In order to reduce the influence of the high scatter that is common in granular materials a very high number of specimens was tested. All dimensions were scaled for every size by the same factor, thus geometrical similarity was provided. The aim of these series was to find the influence of size on strength, namely the size effect.

3.1 *Preparation of the concrete specimens*

In order to minimize sources of size effect that affect the material strength for the different sizes, namely diffusion phenomena, hydration heat and top layer strength, special demands on casting and formwork were necessary. The formwork was made of cardboard tube with 400 μm PVC foil inside, see specimen XXL in Figure 2. The foil was advantageous for several reasons:

- The formwork did not absorb water at the beginning, which would have led to drying of the concrete surface at very early ages.
- No moisture gradient could develop and therefore no diffusion phenomena could take place.
- The water that was available for the concrete to hydrate was for every specimen the water that was added during mixing.
- The cardboard tube was a good isolator. Therefore no temperature gradient could develop, that could influence the hydration and produce regions with different microcracks. However the temperature due to hydration was certainly higher in the XXL specimens than in the S size specimens.

All concrete specimens were produced from one batch of concrete at the concrete plant, where the formwork had been erected and was mixed. 7 m3 were necessary to produce the specimens. The specimens were cast in a standing position to minimize influence of the lower strength of the concrete below the surface, which is especially pronounced when specimens are produced horizontally. All concrete specimens were produced with spiral reinforcement at the specimen ends, see Figure 4 for XL size. In the XXL and XL specimens the region with lower strength was inside the region with spiral reinforcement. For the sizes L, M and S the weak part would have been in the not reinforced region and the strength of the

Table 1. Quantity, dimensions, eccentricity of loading and weight of specimens of concrete size effect series.

Specimen	Quantity	Diam.	Length	Weight	e
		cm	cm	kg	cm
XXL	3	80	320	4020	2.0
XL	3	40	160	500	1.0
L	6	20	80	65	0.5
M	6	10	40	8	0.25
S	6	5	20	1	0.125

Table 2. Quantity, dimensions, eccentricity of loading and weight of specimens of sandstone size effect series.

Specimen	Quantity	Diam.	Length	Weight	e
		cm	cm	kg	cm
XXL	2	64	128	1150	3.2
XL	3	32	64	145	1.6
L	12	16	32	18	0.8
M	29	8	16	2.3	0.4
S	65	4	8	0.3	0.2
XS	82	2	4	0.04	0.1

specimen would have been reduced due to the preparation process. This was prevented by adding an additional cardboard tube with a length of 20 cm at the top. This cardboard tube and the additional top concrete were removed approximately 30 minutes after casting of the specimens. The formwork was removed between one hour and 8 hours before testing.

3.2 *Dimensions and geometry of the concrete specimens*

In the concrete series 5 different sizes from S to XXL with a total size range of 1:16 were tested. The number of S sizes was 6, where the number of XXL sizes was 3, see Table 1. The columns tested had a circular cross section. The diameter of the cross sections varied from 5cm to 80cm. The specimen length was always four times the diameter. All dimensions were scaled by a scaling factor. Therefore the specimens of the different sizes were all geometrically equal. The ratio of smallest specimen dimension (50mm) versus aggregate size (8mm) was 6.25. Thus the smallest specimen was not influenced by aggregate size. In order to prevent failure and to ensure a proper load transfer from the loading platen to the specimen, the specimens were equipped with steel plates on both ends and spiral reinforcement. The spiral reinforcement was placed over the height of the diameter on both ends of the specimen and was scaled for the different sizes. The specimens are shown in Figure 2.

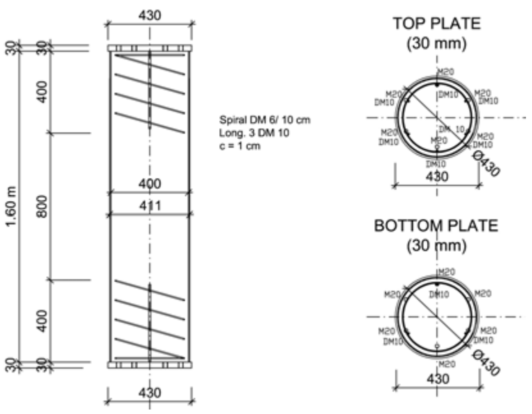

Figure 4: Geometry and reinforcement of specimen XL.

3.3 *Advantages of sandstone*

In order to determine the fracture mechanics size effect of a granular material it is important that the material is the same in the whole volume of each specimen. In the case of concrete this is not so easy to achieve, because of the boundary layer effect, the microcracks due to hydration effects and due to diffusion phenomena. These phenomena have different influence for different sizes and therefore may change the material response, which is unfavorable for such kind of tests. These effects are introduced into the concrete during the casting process. To avoid such effects, tests on sandstone were performed. This material has several advantages.

- The material shows no or a vanishing boundary layer effect. In concrete specimens the layer on the surface has different properties than the interior concrete. The thickness of this layer is dependent on the maximum grain size, is equal for every specimen size and may therefore influence the test results.
- No hydration or diffusion effects of took place in the material.
- Preliminary tests showed that the scatter of strength values was very low for different specimen.
- The average grain size is 0.5 mm (average pore size is 0.6 mm), which allows to test very small specimen sizes, without going beyond the conditions for a representative volume.

- The stress-strain response is nearly linear up to peak load.

The big disadvantage of sandstone is that the material parameters can scatter strongly, when inclusions or other inhomogeneities are present. There were inhomogeneities in the sandstone. But due to a proper sawing scheme all visible inhomogeneities were outside the notched region and did therefore not affect the strength, because the notched region has a smaller cross section, see Figure 3. In the notched region the sandstone showed a very homogeneous structure.

3.4 *Dimensions and geometry of the sandstone specimens*

The total number of specimens was 193, where the number of the smallest specimens was 82, see Table 2. The sandstone specimens had a square cross section and the length was two times the width.

When a prismatic specimen is tested under compression, the failure mode is often the spalling of edges near the loading surface. The specimen is influenced in that region by the boundary conditions. If failure occurs in that region the damage that precedes the failure is influenced by the interface and thus may affect the maximum load or the post peak behavior. Additionally and even more important is that it cannot be distinguished if the failure is due to improper boundary conditions (e.g. uneven loading surface) or by exceeding the material strength. Thus the specimens should be notched, that the location where fracture starts is predefined and is not located on the specimen ends. This allows an assessment of the test and makes the control of the test easier. For tests on specimens loaded in tension or for three-point-bending tests notches are often used. In compression this is more problematic because the fracture band propagates inclined to the loading direction (e.g. Bazant 1999). The inclination of the notch has to be equal to the propagating crack band. In this investigation smooth notches were used to circumvent this problem. In a smooth notch the conditions are also closer to a real structure, because there is no sharp notch tip where the stresses become theoretically infinite. The notch was investigated in a preliminary test series and is shown in Figure 3.

4 TESTING OF THE CONCRETE AND SANDSTONE SPECIMENS

All specimens were loaded with a small eccentricity. The eccentricity was also scaled with the specimen size and was for all sandstone specimens equal to e=D/20 (here D is the specimen width) and for all concrete specimens equal to e=D/40 (here D is the diameter) from the centroid of the (notched) cross section, see table 1 and 2. This was done to circumvent failure by axial splitting, which is a failure mode that does not show a size effect. Additionally, the exact centric loading of a column is difficult to establish experimentally and due to the heterogeneity of the material the stress states in the cross section are never uniform. Another advantage is that due to the higher stress concentration, the surface where the failure starts, was predefined and could be observed during experiment.

In all experimental set-ups the loading platens were placed on longitudinal hinges on both ends of the specimen. The position of the force was defined by the axis of the hinge. Due to the rotating capability in one direction no moment could be transferred or generated during the experiment, i.e. rotation of the loading platen was possible without moment build up. To introduce the eccentricity the specimen was shifted in direction normal to the hinge axis by the amount of the predefined eccentricity. The load did not change the position and did not rotate during experiment.

The experiments were performed under displacement control. The displacement rate was varied, so that all specimens were loaded with the same strain rate. In the concrete series the strains along the whole specimen and along the not reinforced region were considered. Responsible for control of the experiment was the larger of the two values. In the sandstone series one strain was the strain along the compressed specimen face and the other was the lateral strain multiplied by a constant factor. The factor was determined such, that the longitudinal strain correlates with the lateral strain in the elastic region. The larger of the two values was used for test control.

5 RESULTS

5.1 *Concrete series*

One of the main issues of the size effect theory is the influence of size on strength of the specimens. The strength was determined on the compressed side as maximum stress according to linear elastic continuum mechanics. All tests were performed between the 48th and the 75th day after casting. The increase of concrete strength in this time range was determined on accompanying tests on concrete cylinders. It was assumed that the strength of the cylinders increased by the same ratio as the strength of the specimens. All strength values were calculated for the 52nd day after casting. The strengths and the standard deviations for each size are shown in double logarithmic scaling in Figure 5. It can be easily observed that a size effect on the nominal strength was present. The mean value for the XXL(80cm) size was 32.6 N/mm² and for the S(5cm) size was 41.7 N/mm². Thus, the reduction of the nominal strength was 22% for a specimen that is 16 times larger. The specimens were tested up to different strains in the post peak branch, which makes comparison of failure modes not possible. Additionally, the failure was not predefined by a notch and was initiated randomly in the not reinforced region.

5.2 *Sandstone series*

In Figure 6 the mean strengths with standard deviations are plotted versus specimen size. In the plots a decrease of strength with increasing specimen size could be determined and a size effect was present in the test series again. After testing the fracture patterns of the specimens were determined to check for invisible inclusions. All specimens presented in this paper showed a fracture mode of a transversally propagating band in the notched region and were therefore used for comparison of the results.
The mean strength decreases from the XS (2cm) to the XXL (64cm) sizes from 36.6 N/mm² to 19.4 N/mm², which corresponds to a strength decrease of 47% for a size range of 1:32. Thus the mean strength of the XXL size is only 53% of the XS size.
All specimens that initiated fracture outside the notched region were excluded. The fracture mode was a transversally propagating band and was similar for all sizes.

6 SIZE EFFECT LAWS AND ASYMPTOTES

6.1 *The Size Effect Law by Bažant*

The Size Effect Law (SEL) was presented by Bažant (1984) for mode I failure. This theory is based on stress redistributions, where small sizes are bound to plasticity limit, and large sizes follow Linear Elastic Fracture Mechanics. The SEL was derived for geometrically similar structures with notches as

$$\sigma_N = \frac{B\,f_t}{\sqrt{1+\dfrac{D}{D_0}}}\ , \tag{1}$$

where f_t is the tensile strength, D is a characteristic dimension of the structure, B is constant and D_0 is the transition size from plasticity limit to Linear Elastic Fracture Mechanics, which is also constant. The constants B and D_0 have to be determined by fitting of experimental data.
Compression failure is often fitted to the SEL (equation 1), by arguing that the failure mode of compressed concrete specimens can be considered as resulting from local tensile mechanisms.
In the paper by Bažant & Xiang (1997) compressive failure was regarded as a combination

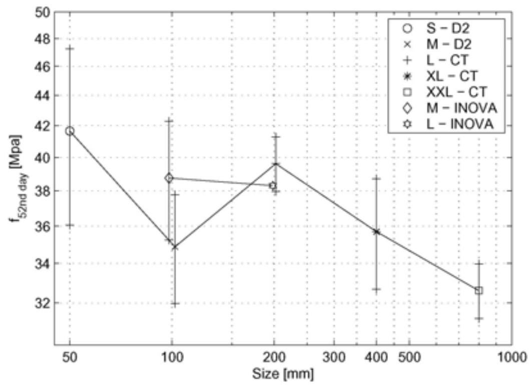

Figure 5: Mean strength with standard deviation of concrete series calculated for the 52nd day.

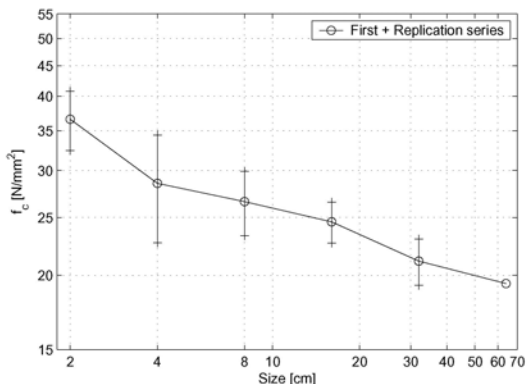

Figure 6: Mean strength with standard deviation of sandstone series.

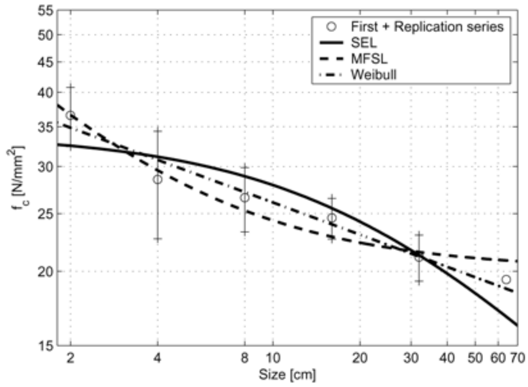

Figure 7: Mean strengths of sandstone series with standard deviation and size effect laws

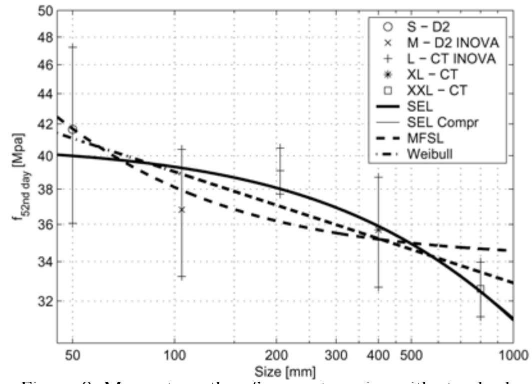

Figure 8: Mean strengths of concrete series with standard deviation and size effect laws.

of tensile and shear mechanisms of buckling plates and a dependence of the strength on size was derived as

$$\sigma_N = k_c\, D^{-2/5} \,. \tag{2}$$

For data fitting the size effect law for compression (SEL compr) was used in similar format as the SEL

$$\sigma_N = \frac{B\, f_t}{\left(1+\dfrac{D}{D_0}\right)^{\frac{2}{5}}} \,. \tag{3}$$

6.2 *The Multifractal Scaling Law by Carpinteri*

The Multifractal Scaling Law (MFSL) proposed by Carpinteri (1994), Carpinteri & Ferro (1994) is defined as

$$\sigma = f_c \sqrt{1+\frac{l_{ch}}{D}} \,, \tag{4}$$

where σ_N is the nominal compressive stress, f_c is the compressive strength of an infinitely large specimen and l_{ch} represents a internal material length and f_c is the strength for infinitely large specimens. These constants need to be determined by fitting of the experimental data.

6.3 *Application of the laws to experimental results*

The SEL and the MFSL were fitted with the nonlinear Levenberg-Marquardt algorithm and plotted in Figure 7 for the concrete series and in Figure 8 for the sandstone series. The parameters for the data fit are given in Table 3. The trend of all laws shows a clearly decreasing strength with increasing size. For larger sizes the MFSL approaches a constant value for increasing size (f_c in Table 3). The difference between SEL and SEL-compression is small for the whole investigated size range.

The trend from the SEL and SEL-compression follows better the data than the trend from the MFSL.

6.4 *Asymptotes to the data*

In the double logarithmic plot in Figure 8 (sandstone series) the strength decrease follows nearly a straight line, where the slope from the XS to the XXL size is -1/5.5. In the Weihbull statistical theory the decrease of strength is followed by a straight line in the double logarithmic plot. The classical Weibull law says

$$\sigma_N \ \mathit{proportional}\ D^{-n_d/m} \ , \tag{5}$$

where m is the Weibull modulus and n_d =3 for three dimensional similarity. Thus the Weihbull modulus can be calculated as m=16.5 (16.7 with data fit, see Table 3). In literature it is stated that the Weibull modulus should be between m=12 and 24.

The results of the concrete series in Figure 7 are

Table 3. Parameters for the different size effect laws determined from the sandstone and the concrete series. D_0, l_{ch}, in cm, $B\,f_t$, f_c in N/mm^2

Material	SEL	SELcompr	MFSL	Weibull
Sandstone	$B\,f_t$=34.1 D_0=20.5	$B\,f_t$=35.1 D_0=12.3	f_c=20.14 l_{ch}=4.62	m=16.7
Concrete	$B\,f_t$=40.7 D_0=140.1	$B\,f_t$=40.8 D_0=104.8	f_c=34.15 l_{ch}=2.45	m=40.5

not so close to a straight line than the sandstone series. The slope between the S and XXL size, L and XXL, XL and XXL are -1/11.3, -1/7.6 and -1/7.7, respectively. The Weibull modulus determined from the slope between the S and XXL size can be determined as 33.9 (40.5 with data fit, see Table 3). The Weibull modulus is for concrete outside the range that is stated in literature, where for sandstone it is not. The material heterogeneity of the sandstone is smaller than for concrete. Another argument could be that the sandstone is very porous. However, the description of the strength decrease by Weibull theory seems to be good for sandstone and concrete.

7 CONCLUSIONS

The results presented here show the largest size range of tests on concrete and sandstone specimens under compressive loading published so far. The results clearly show a size effect on the compressive strength of concrete and sandstone. On a size range of 1:16 for concrete specimens the mean strength of the largest specimens was only 78% of the mean strength of the smallest specimens. On a size range of 1:32 for the sandstone specimens the mean strength of the largest specimens was only 53% of the smallest specimens. Thus, the compressive strength for granular materials changes with specimen size. Thus size effect exists also in compression and the compressive strength is not a material parameter.

The design of concrete structures is usually based on continuum mechanics, which assumes constant strength for different sizes. The application of continuum mechanics for design is widely used and relatively simple. The experiments showed that the size effect may reduce the safety of large structures. Ways to consider varying strength for different sizes are proper size effect laws. In the paper the most common size effect laws were fitted to the data. The comparison showed that different types of granular materials decrease by a slightly different tendency. From this point it cannot be judged, which of the investigated laws is the most suitable for the description of size effect.

8 REFERENCES

Bažant, Z.P. 1984. Size effect in blunt fracture: Concrete, rock, metal. *Journal of Engineering Mechanics* 110(4), 518-535.

Bažant, Z.P., Kim, J.-J. H., Daniel, I.M., Becq-Giraudon, E. & Zi, G. 1999. Size effect on compression strength of fiber composites failing by kink band propagation. *International Journal of Fracture* 95, 103-141.

Bažant, Z.P. & Kwon, Y. 1994. Failure of slender and stocky reinforced concrete columns: tests of size effect. *Materials and Structures*, RILEM, 27, 79-90.

Bažant, Z.P. & Xiang, Y. 1997. Size effect in compression fracture: Splitting crack band propagation, *Journal of Engineering Mechanics - ASCE*, 123(2), 162-172.

Burtscher, S.L. 2002. Size effect of Concrete and Sandstone in compression, PhD thesis*, Institute for Structural Concrete, Vienna University of Technology*, Report No. 3, ISBN 3-902089-02-4.

Burtscher, S.L., aus der Schmitten, H., Watzl, B. & Kollegger, J. 2003a. Experiments on the size effect of sandstone under compression, *Institute for Structural Concrete, Vienna University of Technology,* Report No. 4, ISBN 3-902089-03-2.

Burtscher, S.L., Suda, G., Mihaylov, V. & Kollegger, J. 2003b. Size effect tests on concrete columns, *Institute for Structural Concrete, Vienna University of Technology,* Report No. 3, ISBN 3-902089-04-0.

Carpinteri, A. 1994. Scaling laws and renormalization groups for strength and toughness of disordered materials. *International Journal of Solids and Structures* 31(3), 291-302.

Carpinteri, A. and Ferro, G. 1994. Size effects on tensile fracture properties: A unified explanation based on disorder and fractality of concrete microstructure, *Materials and Structures* 27, 563-571.

Hollingworth, S. 1998. Structural and mechanical properties of high strength concrete, PhD thesis, *Division of Civil Engineering, Cardiff*, University of Wales.

Němeček, J. 2000. Modelling of compressive softening of concrete, PhD thesis, *Department of Structural Mechanics, CTU Prag*, ISBN 80-01-02298-6.

Sener, S., Barr, B. & Abusiaf, H. 1999. Size effect tests in unreinforced concrete columns, *Magazine of Concrete Research*, 51(1), 3-11.

Slowik, V. 1995. Beiträge zur experimentellen Bestimmung bruchmechanischer Parameter für Beton, Technical Report 3*, ETH Laboratory for Building Materials*.

van Vliet, M. 2000. Size effects in tensile fracture of concrete and rock, PhD thesis, *Technical University Delft*.