# Size effect in ECC structural members in flexure

## Michael Lepech & Victor C. Li
Advanced Civil Engineering Materials Research Laboratory
Department of Civil and Environmental Engineering
The University of Michigan, Ann Arbor, Michigan, U.S.A.

ABSTRACT: The presence and severity of size effect in ECC structurally sized members in flexure is examined. High tensile strain capacity, fracture toughness, and characteristic length make ECC an ideal material for preventing brittle failures responsible for size effects present in concrete members. The primary focus of this study is the structural implications of size effect in ECC and steel reinforced ECC (R/ECC) members. Seven series of beam specimens fabricated of reinforced concrete, ECC, and R/ECC are tested in flexure to determine the severity of size effect. Test results of structurally sized members are presented. In this paper, special attention is given to the relationship between ECC ductility, quantified as tensile strain capacity, and the severity of size effect in these members is addressed.

## 1. INTRODUCTION

### 1.1 *Motivation*

As society becomes more dependent on infrastructure, the ability of civil engineers to meet the demands of an ever growing population becomes increasingly important. Often, these societal demands require larger structures than currently built today. Ranging from larger bridges to span wider channels to larger dams to store adequate drinking water for millions, larger structures will be necessary to meet the needs of future society.

While this may seem a straightforward process, current construction materials cannot meet the engineering challenges which very large structures pose. As building elements grow in scale, failure modes typically found in most applications are replaced by new modes of failure. One such case is exhibited in the current use of high strength concrete to replace normal strength concrete in many large structures. While high strength concrete material is stronger than normal strength concrete, it is prone to fracture failure as a result of higher brittleness. This change of failure modes, particularly evident in larger structures, leads ultimately to a lower structural strength despite the use of higher strength materials.

Among materials designers it is well accepted that the nominal strength of quasi-brittle and brittle members, such as concrete members, decreases with increasing scale. Therefore, the simple "sizing up" of concrete structures, such as bridges and dams, is not always viable for increasing their structural strength. This phenomenon has been incorporated into a number of concrete design codes, particularly in Europe (CEB 1990) and Japan (JSCE 1996), through a reduction of nominal strength capacity with increasing member dimension. However, there comes a point at which further enlarging is no longer practical, necessitating a material change in the design stage. The ability to overcome the accompanying reduction of strength with increasing size becomes extremely valuable to engineers, along with the ability to modify the severity of diminishing strength. Control of this phenomenon is essential for the design and construction of the largest structures of the future.

## 1.2 *Background*

As mentioned above, the trend of decreasing nominal strength accompanying increasing size of quasi-brittle and brittle members due to fracture failure is well recognized. First documented by Hillerborg et al (1976), and further defined by Bazant et al (1984, 1987, 1991) and others, this topic has been heavily studied and is still undergoing further work. One achievement of this work has been the generalized size effect law (Figure 1), combining strength theory, non-linear fracture mechanics, and linear elastic fracture mechanics (LEFM) to represent the behavior of increasingly larger quasi-brittle or brittle members.
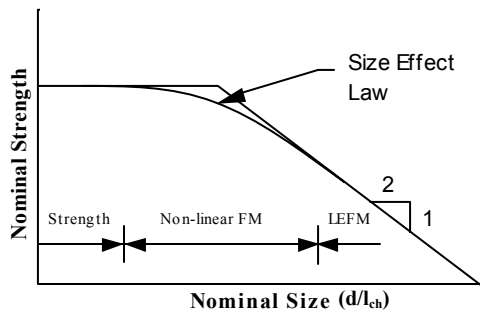


Figure 1. Generalized Size Effect Law

In normalizing member sizes, a material property with length dimension is typically chosen to standardize the generalized law for all materials. In this instance, Hillerborg's material characteristic length ($l_{ch}$) is used for normalization. One intriguing observation from this representation is the dependence of facture based size effect upon the brittleness of the material. For instance, highly brittle materials with small characteristic lengths are expected to exhibit a more severe size effect, even with reasonably small member dimensions (d). However, as the brittleness of the material decreases, or characteristic length grows, even relatively large members may exhibit a less severe size effect. This phenomenon may allow for the control of the severity of fracture based size effect.

Combining research results from a number of studies, the relation between material brittleness and the severity of size effect becomes clear. Figure 2 illustrates tests of plain concrete (Bazant and Kazemi, 1991), steel FRC (Akimaha et al, 1984), and carbon FRC (Ward and Li, 1990) members exhibiting various degrees of size effect based upon material brittleness. While these three testing series are based upon different geometries and loading scenarios, the trend among them is

apparent. Quantifying size effect as an exponential function, the severity of size effect is captured through the magnitude of the exponent (x) in Equation 1. This exponent is plotted in Figure 2.

$$\sigma_n = \alpha d^{-x} \tag{1}$$

where $\sigma_n$=nominal strength; $\alpha$=scaling coefficient; d=member size; and x=size effect exponent
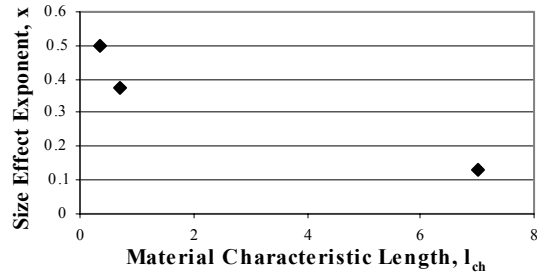


Figure 2. Size Effect in Quasi-Brittle Materials

While steel and carbon fiber reinforcement may be able to reduce the severity of size effect in concrete, they are not able to eliminate this fracture based phenomenon entirely. This is due to the tension-softening behavior of FRC composites. While the ability of these composite materials to resist brittle fracture failure is significant, FRC failures remain similar to plain concrete such that a single crack develops and all subsequent deformation is localized at that single crack face. To effectively eliminate fracture based size effect completely, it is necessary to suppress the fracture failure mode, leading structural members to fail in a ductile manner without consequence of member size. This requires a material with high tensile ductility which does not allow a single crack to localize prior to overall structural failure.

To meet these requirements, Engineered Cementitious Composites (ECC), a class of high performance fiber reinforced composites (HPFRCCs) have been developed (Li et al 2001). Using micromechanical design principles, this material exhibits a fracture toughness far higher than most cementitious materials, and similar to many ductile metals (Maalej et al 1995). Further, the composite pseudo-strain hardening capabilities of ECC material, achieved through formation of closely spaced microcracks, allow loaded members to "plastically deform" rather than fracture to accommodate deformation. This suppression of the fracture failure mode may eliminate the size effect phenomenon. The capability of ECC material to reduce the severity of size effect has

been demonstrated in a number of smaller scale studies, such as those discussed previously by Lepech and Li (2003).

While these previous studies are of great value, a need exists to evaluate ECC material performance within more realistic sized building members. The objective of this study is to validate the capability of ECC material to eliminate fracture based structural size effect in structural scale members, up to nearly 3 meters in span. Additionally, the ability to tailor the degree of ductility of ECC material is also investigated to allow for control of the severity of size effect within commonly sized structural members.

## 2. EXPERIMENTAL PROCEDURES

### 2.1 Materials

Three different ECC mix compositions were used in this study. Each mix was comprised of standard mortar components, cement (C), sand (0.1mm nominal grain size) (S), fly ash (FA), water (W), and admixtures for control of fresh properties such as superplastizcizer (SP) and methyl cellulose (MC). However, each of the three ECC compositions contains a different volume fraction of poly-vinyl-alcohol (PVA) fibers ($V_f$) for control of mechanical properties. ECC mix compositions, along with a reference concrete, are given in Table 1.

Table 1. Material Mix Proportions

| Mix | C | AGG* | S | FA | W | SP | MC | $V_f$ |
|---|---|---|---|---|---|---|---|---|
| ECC-1 | 1.0 | 0.0 | 1.0 | 0.1 | 0.45 | 0.01 | 0.0015 | 0.01 |
| ECC-3 | 1.0 | 0.0 | 1.0 | 0.1 | 0.45 | 0.01 | 0.0015 | 0.015 |
| ECC-5 | 1.0 | 0.0 | 1.0 | 0.1 | 0.45 | 0.01 | 0.0015 | 0.02 |
| Concrete | 1.0 | 2.0 | 2.0 | 0.0 | 0.45 | 0.01 | 0.0 | 0.0 |

*Aggregate used in Concrete (17mm nominal size)

To investigate the effect of material ductility upon the severity of size effect it was necessary to process a number ECC mixtures which show varying levels uniaxial tensile strain capacity. Three ECC compositions were designed using various fiber volume fractions to achieve different levels of strain capacity, as seen in the stress-strain curves for ECC-1 (Fig 3), ECC-3 (Fig. 4), and ECC-5 (Fig. 5).
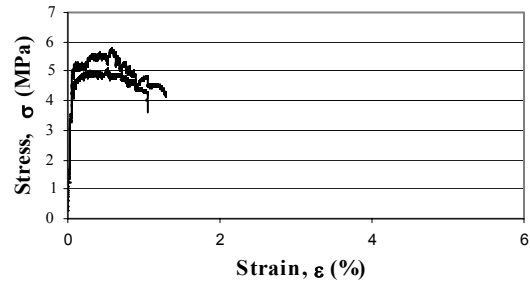


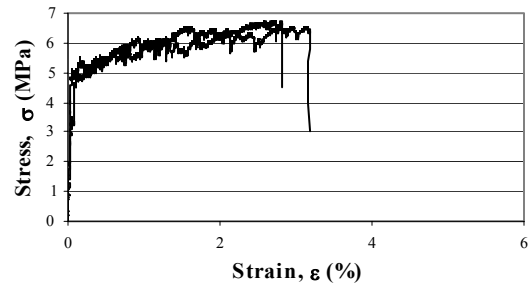Figure 3. ECC-1 Stress-Strain Curves
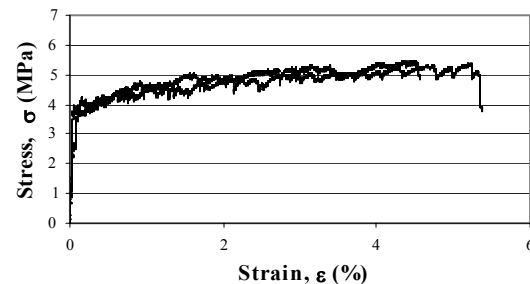


Figure 4. ECC-3 Stress-Strain Curves



Figure 5. ECC-5 Stress-Strain Curves

First cracking strength of these three composites ranged between 2.8MPa and 5.0MPa with a first cracking strain approximately 0.05%. Ultimate strengths for these composites varied between 5.0MPa and 6.7MPa. Compressive strength for each material ranged between 50MPa and 65MPa.

In addition to fiber volume fraction, curing time was also used in this study to control composite strain capacity. Due to the delicate balance of time dependent matrix and matrix-fiber interface properties, the ultimate strain capacity of ECC material changes throughout early aging of the material. The development of ECC-5 tensile strain capacity is shown in Figure 6.
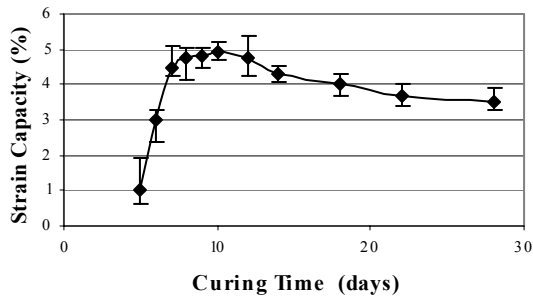
Figure 6. Development of ECC Strain Capacity

As seen, the maximum strain capacity of this ECC mix is attained at an age of 10 days, with approximately 5% strain capacity. However, past this age, capacity steadily drops to an equilibrium strain capacity of roughly 3%, which remains constant for the remaining material lifespan. While this technique for control of strain capacity is not practical for structural members, it will be used in this study purely as an additional control for material strain capacity. For structural applications, ultimate strain capacity will be controlled primarily through material ingredient tailoring.

Concrete mix proportions, also reported in Table 1, consisted of coarse aggregate (10mm nominal grain size), sand, cement, and water. Admixtures were used if necessary to optimize fresh concrete properties. The compressive strength of concrete used for R/C specimens was 45MPa.

Steel reinforcement used deformed bars with a yield strength of 410MPa, a yield strain of 0.02%, ultimate strength of 620MPa, and ultimate strain of 14%.

## 2.2 Specimen Configuration

This study is comprised mainly of three series of seven flexural specimens. For concrete and ECC-5 material test series, specimens ranging in span length from 0.175m to 2.8m were tested. Beams were tested in mid-point loading to determine the load-deflection curve for individual specimens and ultimately the nominal load capacity of each. Nine individual beams were cast for each specimen size, comprised of three reinforced concrete beams, three ECC beams, and three steel reinforced ECC beams. Beam geometry is shown in Figure 7a and 7b along with beam sizes detailed in Tables 2a and 2b.
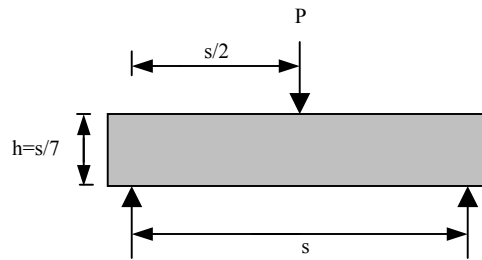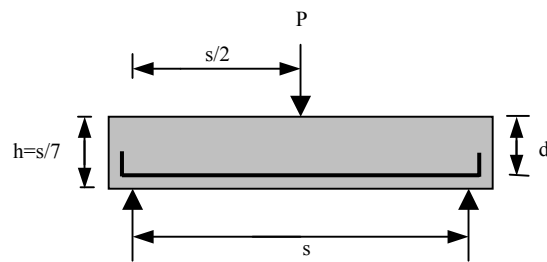


Figure 7a



Figure 7b

Figure 7. Test Specimen Geometry

Table 2a. Test Series Notation

| Test Series | Notation |
|---|---|
| Reinforced Concrete | RC-# |
| ECC-1 | ECC-1-# |
| ECC-3 | ECC-3-# |
| ECC-5 | ECC-5-# |
| Reinforced ECC | R/ECC-# |

Table 2b. Test Series Dimensions

| Test Number (#) | Span Length, s (meters) |
|---|---|
| 1 | 0.175 |
| 2 | 0.263 |
| 3 | 0.35 |
| 4 | 0.7 |
| 5 | 1.4 |
| 6 | 2.1 |
| 7 | 2.8 |

Reinforced concrete beams were tested as a validation test, quantifying the level of concrete size effect over the range of specimen sizes tested. These specimens contained only longitudinal reinforcement with no shear reinforcing. ECC specimens were tested to evaluate the ability of ECC material itself to prohibit fracture failure within the specimens, thereby eliminating size

effect. Finally, steel reinforced ECC beams, containing only longitudinal steel identical to the reinforced concrete specimens, were tested for comparison to R/C specimens along with evaluating the effect of reinforcement upon the ECC response. While testing of concrete beams without longitudinal reinforcement may have been desirable for comparison with ECC specimens without steel reinforcing bars, plain concrete specimens were deemed unnecessary due to their known brittle failure mode, which would only serve to duplicate R/C member testing. A constant longitudinal reinforcing ratio of 1.6% was chosen for each steel reinforced specimen. The reinforcement was kept a constant depth of 85% of overall member height.

In addition to the series of ECC-5 members, limited series of ECC-1 and ECC-3 members were tested which ranged in size from 0.175m to 0.7m. These members were tested to examine the effect of varying degrees of ductility, quantified through ultimate strain capacity, upon the severity of size effect. None of the ECC-1 and ECC-3 specimens contained steel reinforcing bars and all were tested identically to ECC-5 specimens, in midpoint flexure, as shown in Figure 7a.

### 3.0 EXPERIMENTAL RESULTS

Experimental test values are summarized in Table 3 and Table 4. These tables report nominal strength values for each specimen. Test results for reinforced concrete specimens exhibit a significant size effect phenomenon, as expected. Similar results have been reported by other researchers for similar sized concrete beams (Bazant and Kazemi, 1991).

Concrete beams typically failed in diagonal shear failure, except for the smallest specimens which failed through a combination of flexure and shear. ECC-5 and R/ECC beams however, failed primarily in flexure resulting in a very ductile performance for the full range of sizes tested.

Over the entire range of ECC-5 specimens, the mode of failure remained virtually identical. Beams first underwent a typical elastic response followed by initial microcracking. Following the formation of the initial crack, rather than localizing and failing in a brittle nature, ECC-5 beams developed a distinctive microcracking region throughout the center portion of the beam in maximum curvature. After extensive deformation, a single macro-crack begins to localize and the load capacity begins to drop. This same behavior was exhibited for all ECC-5 beams.

Table 3. Results for RC, ECC-5, R/ECC Tests

| Series | Strength (MPa) | Series | Strength (MPa) | Series | Strength (MPa) |
|--------|------|--------|------|--------|------|
| RC-1 | 19.76 | ECC-5-1 | 12.52 | R/ECC-1 | 14.92 |
|  | 22.92 |  | 12.84 |  | 16.83 |
|  | 22.07 |  | 11.28 |  | 13.49 |
|  |  |  | 15.13 |  | 16.55 |
| RC-2 | Not Tested | ECC-5-2 | 9.88 | R/ECC-2 | 12.88 |
|  |  |  | 12.71 |  | 16.79 |
|  |  |  | 16.31 |  | 18.47 |
| RC-3 | 13.88 | ECC-5-3 | 14.45 | R/ECC-3 | 18.38 |
|  | 12.72 |  | 11.54 |  | 13.00 |
|  | 15.03 |  | 12.17 |  | 14.76 |
|  |  |  | 18.77 |  | 19.40 |
| RC-4 | 12.77 | ECC-5-4 | 11.54 | R/ECC-4 | 12.79 |
|  | 11.88 |  | 13.17 |  | 14.99 |
|  | 10.51 |  | 14.77 |  | 16.41 |
| RC-5 | 8.62 | ECC-5-5 | 11.98 | R/ECC-5 | 13.75 |
|  | 9.91 |  | 12.80 |  | 15.21 |
|  | 9.71 |  | 14.89 |  | 18.23 |
| RC-6 | 8.20 | ECC-5-6 | 14.99 | R/ECC-6 | 14.90 |
|  | 9.23 |  | 13.83 |  | 15.99 |
|  | 9.40 |  | 11.98 |  | 13.30 |
| RC-7 | 7.24 | ECC-5-7 | 15.19 | R/ECC-7 | 16.90 |
|  | 8.50 |  | 10.18 |  | 14.19 |
|  | 8.29 |  | 12.14 |  | 16.40 |

Table 4. Results for ECC-3 and ECC-1 Tests

| Series | Strength (MPa) | Series | Strength (MPa) |
|--------|------|--------|------|
| ECC-3-1 | 11.53 | ECC-1-1 | 9.53 |
|  | 13.85 |  | 11.85 |
|  | 10.28 |  | 8.28 |
|  | 14.14 |  | 12.14 |
| ECC-3-2 | 8.88 | ECC-1-2 | 5.60 |
|  | 11.71 |  | 9.71 |
|  | 15.31 |  | 13.31 |
| ECC-3-3 | 15.60 | ECC-1-3 | 11.20 |
|  | 10.68 |  | 8.54 |
|  | 10.87 |  | 7.10 |
| ECC-3-4 | 13.90 | ECC-1-4 | 10.49 |
|  | 11.58 |  | 7.10 |
|  | 11.48 |  | 7.85 |

Failure of R/ECC beams was similar to that of ECC-5 beams. The response was characterized by an elastic pre-cracking regime, followed by initial microcracking at mid-span. This was followed by development of a microcraking region, also concentrated near midspan. These microcracks propagated through the level of reinforcement up to the neutral axis of the specimens. Finally, as in ECC-5 specimens, a single macrocrack formed, but

rather than failing after formation of this crack, the reinforcing steel began to dominate the response allowing the beam to undergo further displacement until yielding of the reinforcing steel. This results in a highly ductile response for the R/ECC composite.

The overall effect of specimen size on nominal strength is shown in Figure 8. In this representation, beam dimensions are normalized by a common length dimension, in this case beam width which was kept constant for two dimensional geometric similarity among all specimens.
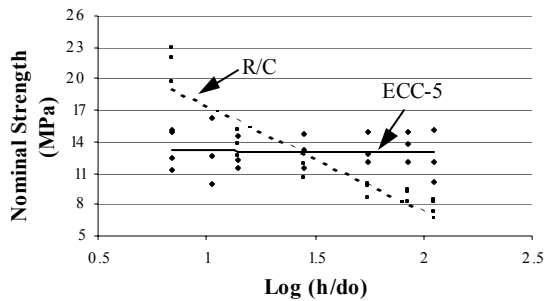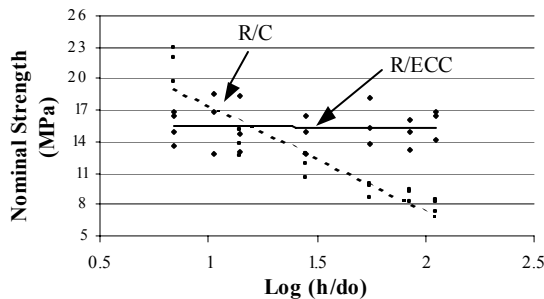


Figure 8a



Figure 8b

Figure 8. Size Effect in R/C, ECC-5, and R/ECC Specimens

In these figures, nominal flexural strength is plotted against normalized beam height. From Figure 8a it is evident that reinforced concrete, undergoing brittle shear failure, exhibits a significant fracture based size effect. However, ECC-5 specimens which develop a distributed microcracking region rather than failing in a brittle nature through fracture in shear or flexure, exhibit a negligible size effect up to a span of 2.8 meters, on the scale of reasonable structurally sized members.

In addition to negligible size effect among ECC-5 beams, R/ECC beams also exhibit little, if any, size effect phenomenon. This is due to the response of the ECC material prohibiting fracture

localization in a similar manner to the ECC-5 beams. By suppressing the tendency of fracture failure within the specimens, fracture based size effect can effectively be eliminated.

While the elimination of size effect is possible with adequate material ductility, as shown above, it is also possible to control the severity of fracture based size effect through the use of materials with various levels of ductility. This is highlighted in Figure 9.
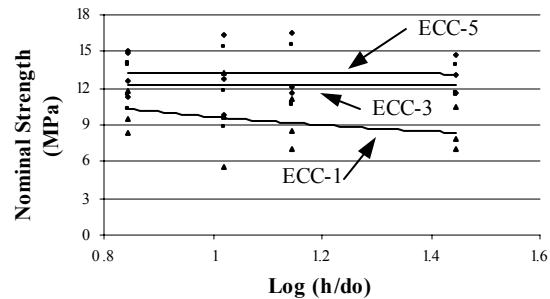


Figure 9. Size Effect in ECC-5, ECC-3, and ECC-1 flexural specimens

As seen earlier, flexural specimens made with ECC-5 material show negligible size effect over the range of specimens tested. However, ECC-3 specimens, which exhibited a significantly lower strain capacity, also showed minimal size effect over the range of sizes tested. ECC-3 and ECC-5 specimens tended to fail in identical ductile failure modes, developing multiple cracking regions throughout the sections of maximum curvature within the beams. Specimens cast from ECC-1 material however, exhibited a more severe size effect than ECC-5 and ECC-3 specimens, suggesting that the limited ductility was insufficient to fully suppress the fracture failure mode. This allowed for the introduction of fracture based size effect within these specimens. ECC-1 specimens developed limited multiple cracking within the beams prior to localization of a single macro-crack and failure of the specimen. All failures in ECC-5, ECC-3, and ECC-1 specimens remained primarily flexural in nature, suggesting that even limited amounts of material ductility were adequate to prevent shear failures.

## 4.0 DISCUSSION

ECC-5 members exhibit negligible size effect even in members ranging on a life-like structural scale. Conversely, reinforced concrete specimens showed a substantial loss in nominal strength over the same

range of specimen sizes. Further, ECC specimens reinforced with steel reinforcing bars showed a negligible size effect nearly identical to that in ECC specimens without additional steel reinforcing. The results of this study will likely prove critical to the civil engineering community in the future, as larger structures are built to support an expanding world population. The cyclical over sizing of reinforced concrete members or the use of additional steel reinforcement to accommodate lower nominal material strength can be overcome.

The existence of fracture based size effect is a consequence of the brittle or quasi-brittle nature of concrete materials. It is commonly thought that through the use of strain hardening materials, fracture based size effect can be eliminated. This is primarily due to the ability of true strain hardening materials to suppress fracture failure and shift the failure mode from brittle fracture to ductile "yielding", or multiple microcracking in the case of cementitious composites. Through this series of tests, these hypothesis have been proven correct.

Size effect in brittle or quasi-brittle materials is primarily due to the localization of a single macro-crack which accommodates nearly all deformation within the material. As specimen size increases, there exists greater stored elastic energy within the member to drive a single crack, leading to failure at lower loads. However, through the suppression of fracture localization, and fracture failure altogether, the fracture based size effect phenomenon does not exist in strain hardening materials, such as ECC, which develop multiple cracks rather than one single crack.

This theory is even further exemplified through the examination of ECC materials with varying levels of ultimate tensile strain capacity. As seen in ECC-5 and ECC-3 specimens, little size effect was seen in the range of member sizes tested. However, within ECC-1 specimens the presence of a fracture based size effect became apparent. As mentioned previously, this was accompanied by limited multiple cracking and early localization of a single crack. These results suggest that there is a minimum ductility limit which was not met by the ECC-1 members, but which was surpassed by both the ECC-5 and ECC-3 specimens. Beyond this ductility limit, fracture failure is adequately suppressed and fracture based size effect can be considered negligible.

With the establishment of negligible size effect in highly ductile ECC members, it becomes necessary to examine how this phenomenon fits into the overall size effect theory. As mentioned above, current fracture based size effect theory is captured by the generalized size effect law. This was shown previously in Figure 1.

As mentioned, three regions combine to from the overall generalized size effect law. These three regions are dominated by linear elastic fracture mechanics (LEFM), non-linear elastic fracture mechanics (NLEFM), and strength theory.

LEFM predicts the most severe fracture based size effect. This theory rests on the formation of a K-dominant zone within the structure or member, therefore leading to the assumption of ideally brittle failure. In this case, all fracture energy is consumed at the crack tip. From LEFM theory, nominal strength is proportional to the square root of flaw size and therefore member size. When plotted logarithmically, this is characterized by a size effect with a slope of –0.5.

As structure size becomes smaller, or the material becomes less brittle, the process zone surrounding the crack tip becomes large with respect to the overall member dimensions and the K-dominant zone fails to exist. However, through the application of non-linear fracture mechanics, such as fictitious crack models which account for the presence of a large fracture process zone, the severity of size effect due to fracture is seen to decrease.

The dependence of size effect on brittleness can be realized through the use of Hillerborg's characteristic length for normalizing specimen dimensions. As materials become less brittle (i.e. characteristic length increases) process zones within these materials inherently become larger, placing these structures within the NLEFM portion of the size effect law. Thereby, structures which use materials with very large characteristic lengths are subjected to a less severe size effect, and are placed closer to strength theory on the size effect law.

Ultimately, as characteristic length grows very large, fracture based size effect becomes negligible and structures comprised of these materials closely follow the strength theory portion of the size effect law. With a characteristic length of nearly 100m (Maalej, 1995), structures made of ECC material will likely comply to this portion of the size effect law. This may be one explanation for the negligible size effect exhibited by the specimens examined within this project. If this is the case, size effect will only be seen in extremely large structures, far larger than those built today.
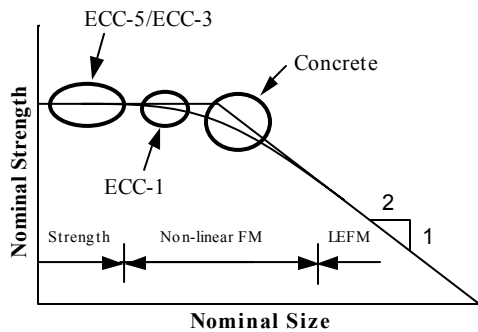
Figure 10. Placement of ECC and Concrete on Generalized Size Effect Law

However, the generalized size effect law is developed for brittle or quasi-brittle materials which fail in fracture. Truly ductile materials, such as ECC, are capable of completely suppressing the fracture phenomenon and shifting failure to non-brittle modes. This was seen through the ductile "yielding" of ECC and R/ECC beams undergoing flexural failure. Since ECC material can suppress the fracture phenomenon, then no size effect should be seen, regardless of structure size.

While both theories may serve to explain the negligible size effect seen in ECC members, this material cannot eliminate size effect derived from sources other than fracture. These may include size effect due to boundary conditions (i.e. fiber alignment at beam edges), or a Weibull type size effect due to flaw size variation or fiber distribution inhomogeneity. Reduction of these separate size effect phenomenon may be studied in future work.

## 5.0. CONCLUSIONS

Ranging from small scale to structurally sized components, ECC members tested in flexure exhibit negligible size effect when compared to reinforced concrete specimens of identical shape and size. Additionally, ECC beams with longitudinal steel reinforcing bars also exhibited negligible size effect, similar to those without reinforcing steel. This response is a result of the ductility of the ECC material suppressing the brittle fracture failure mode within ECC structural components and allowing ductile "yielding" of the members.

Further, this study demonstrates that through tailoring of the material, in this case through control of the ultimate strain capacity, a specific severity of size effect can be achieved. This may allow designers to negate varying amounts of size

effect if complete elimination of the phenomenon is impractical. Due to the higher cost of ECC material, the highest performing ECC may not be economically feasible in many projects. By using ECC with limited ductility, benefits can be achieved while minimizing material costs. This can be seen as one of the initial steps toward fully integrating material and structural design.

The exceptional performance of ECC material may be attributed to the high ductility of the composite suppressing fracture failure within ECC members, and allowing them to fail in a ductile manner. The consequences of this performance places ECC members primarily on the strength portion of the generalized size effect law, or may simply violate the fracture based size effect law completely. In either case, the use of ECC in structurally sized members effectively negates fracture based size effect.

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