Residual fatigue life of RCC: Deterministic & probabilistic approach

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ABSTRACT: Fatigue in reinforced cement concrete (RCC) is considered as a progressive permanent internal damage of the material under fluctuating operational stress and depends on material composition, loading forms and environmental conditions the structure is subjected to. In this study, the residual life assessment for reinforced concrete members is done based on fatigue and fracture mechanics concepts. The residual strength (load carrying capacity) of the structure at a particular level of damage is computed by knowing the fatigue crack growth as a function of time and the effects of crack propagation on strength. Crack propagation rate is estimated by considering different parameters such as load history, percentage of steel reinforcement and size effect. Many of the parameters on which fatigue damage depends are random in nature. Therefore, a probabilistic study is undertaken to determine the residual fatigue strength of the structure by considering the external loads and fracture properties as the primary random variables.

Keywords: fatigue law, residual life, probabilistic, reinforced concrete, reliability index

1. INTRODUCTION

Fatigue loading in concrete causes failure in many structures, such as concrete pavements, airport runways, bridges and offshore structures. Majority of these structures that were built several decades ago have undergone continuous damage and degradation resulting in reduced load carrying capacity. Hence, it is necessary to develop tools in order to predict the residual strength or the residual life of these existing structures.

Residual life prediction broadly defines the lifetime - the structure can withstand an appreciable load under the presence of cracks. The assessment also requires that either the crack be detected before it has developed to a dangerous size or load be restricted in such a way, so that the crack will never reach a critical size corresponding to failure. Therefore, fracture control is the concerted effort by engineers, to ensure safe operations during the service life of the structures. In this study, the residual life of concrete members is determined using fracture mechanics. Residual life prediction includes two analyses [Broek (1978)] namely,

1. The effect of cracks on strength (margin against fracture)
2. The crack growth as a function of time.

The first analysis aims at determining the strength reduction as a function of continuously increasing crack length, while the second one talks about the rate of fatigue crack propagation.

2. FRACTURE MECHANICS BASED FATIGUE MODEL FOR CONCRETE

A number of empirical laws have been found in the literature, which describes fatigue behavior in metals. The most popularly used is the Paris law (1963) given by,

$$\frac{da}{dN} = C(\Delta K_I)^n \quad (1)$$

where ΔK_I is the stress intensity factor range, C and n are material constants. Fatigue behavior in concrete member depends on a number of parameters namely, grade of concrete, fracture properties, relative size of the structure, loading history and the frequency of the external loading. Slowik, Plizzari & Saouma (1996) proposed a law which is given by,

$$\frac{da}{dN} = C\frac{K_{Imax}^m \Delta K_I^n}{(K_{IC} - K_{I\sup})^p} + F(a, \Delta\sigma) \quad (2)$$

where *C, m, n* and *p* are material constants, K_{Imax} is the maximum stress intensity factor in a stress cycle, K_{IC} is the fracture toughness and K_{Isup} is the maximum stress intensity factor the structure has reached in its past history. These co-efficients (*m, n* and *p*) are determined by the authors through an optimization process using the experimental data and are 2.0, 1.1 and 0.7 respectively.

2.1 *Discussions on parameter C*

The parameter *C* in empirical Equation 2 basically gives a measure of crack growth per load cycle. In concrete members this parameter indicates the crack growth rate for a particular grade of concrete and is also size dependent. Slowik, Plizzari & Saouma (1996) have determined the value of *C* to be equal to 9.5×10^{-3} and 3.2×10^{-2} mm/cycle for small and large size specimen respectively. It should be noted here that the stress intensity factor is expressed in $MNm^{-3/2}$.
These values were determined for a particular loading frequency. Since the parameter *C* gives an estimation of crack propagation rate in fatigue analysis, it should also depend upon the frequency of loading. Further, the fatigue crack propagation takes place primarily within the fracture process zone and hence *C* should be related to the relative size of the fracture process zone, which itself is related to characteristic length. Therefore, *C* should depend on the characteristic length l_{ch} and ligament length *L*, where $l_{ch}=EG_f / f_t^2$, where, *E* is the elastic modulus of concrete, f_t is the tensile strength of the concrete and G_f is the specific fracture energy. A linear relationship was proposed by Slowik, Plizzari & Saouma (1996), between parameter *C* and the ratio of ligament length to characteristic length given by $C=(-2+25\ L/l_{ch})\ X\ 10^{-3}$ *mm/cycle*. In this study, this equation for *C* has been modified to account for the loading frequency, through a regression analysis, using experimental results of different investigators. Figure 1 shows a plot of *C* times *f (Cf)* versus the ratio of ligament to characteristic length (L/l_{ch}).

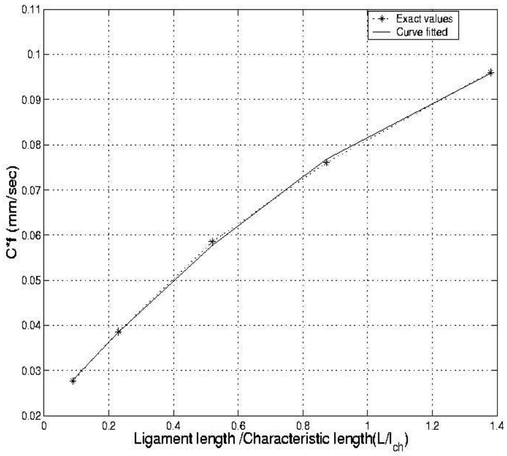

Figure 1: Relation between Cf and the ratio of ligament length to characteristic length

In this plot, experimental results of Slowik, Plizzari & Saouma (1996) performed on compact-tension (CT) specimens and Bazant & Kangming (1991) for three-point bend specimen have been used for normal strength concrete with load frequencies 3 and 0.033 Hz. respectively, for small, medium, and large sized specimens. The specimen details are tabulated in Table 1.

Table 1: Specimen Details

Specimen	Depth mm	width mm	Initial notch	E Mpa	K_{IC} $MN/m^{3/2}$
CT	900	400	230	17000	1.48
CT	300	100	50	16000	0.95
3-Pt bend	152.4	38.1	25.4	27120	1.41
3-Pt bend	76.2	38.1	12.7	27120	1.51
3-Pt bend	38.1	38.1	6.35	27120	1.66

From Figure 1, the resulting best-fit curve represents a quadratic polynomial given by,

$$Cf = 0.0193\left(\frac{L}{l_{ch}}\right)^2 + 0.0809\left(\frac{L}{l_{ch}}\right) + 0.0209 \ mm\ /\sec \qquad (3)$$

where f is external loading frequency.

From this equation one can obtain the value of parameter C for any load frequency, grade of concrete and size.

2.2 *Effect of overloads*

In Equation 2 the function $F\ (a,\ \Delta\sigma)$ describes the sudden increase in equivalent crack length due to an overload. In this work, an analytical expression is developed for $F\ (a,\ \Delta\sigma)$ by considering the rate of crack propagation to depend on the inherent property of concrete and stress amplitude, given by

$$F = \left(\Delta K_I / K_{IC}\right)\Delta a \qquad (4)$$

where, ΔK_I indicates the instantaneous change in stress intensity factor from the normal load cycle, to certain overload cycle, i.e.

$$\Delta K_I = K_{I\,overload} - K_{I\,normal\,load}$$

Here, $K_{I\ overload}$ represents the maximum stress intensity factor due to overload and $K_{I\ normal\ load}$ is the maximum stress intensity factor due to normal load, just before the overload. Δa is the crack length reached just before the overload.

3. STRENGTH OF CRACKED REINFORCED CONCRETE

In this work, the strength of a cracked under-reinforced concrete member is obtained by considering a model originally proposed by Carpinteri (1984) and later modified by Baluch, Azad & Ashmawi (1992). The fracture mechanics model for bending of reinforced concrete beams used in the analysis is shown in Figure 2. The section is subjected to a moment M, and the effect of steel bar is simulated by a force F_S acting at the centroid of the steel bar. Based on linear elastic fracture mechanics, the stress intensity factors of the beam subjected to M and F_s are,

$$K_{IM} = \frac{MY_M\left(a/d\right)}{d^{3/2}t} \qquad (5)$$

$$K_{IF} = -\frac{F_S Y_F\left(a/d\right)}{d^{1/2}t} - F_S\left(\frac{d}{2} - d_S\right)\frac{Y_M\left(a/d\right)}{d^{3/2}t} \qquad (6)$$

In the above equations, K_{IM} and K_{IF} are the stress intensity factors due to M and F_S, respectively, d is the depth of the beam, d_s is the distance between the bottom surface of concrete and the centroid of the steel bar, and Y_M and Y_F are geometric factors, given for three point bend specimens [Carpinteri (1984)]. The two geometric factors are applicable for relative crack depth $\alpha<0.7$.

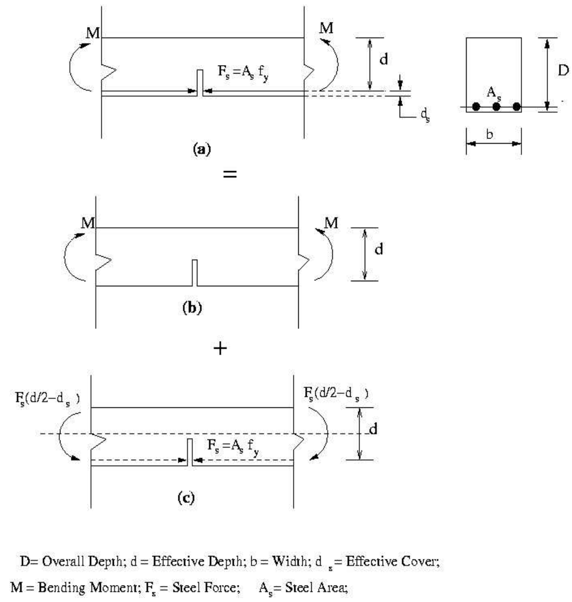

Figure 2: Cracked beam element model for reinforced concrete

The total stress intensity factor K_I can be obtained by superposing both K_{IM} and K_{IF}. Thus,

$$K_I = \frac{1}{d^{3/2}t}Y_M\left(a/d\right)\left[M - F_S\left(\frac{d}{2} - d_S\right)\right] - \frac{F_S}{d^{1/2}t}Y_F\left(a/d\right) \qquad (7)$$

The criterion for crack propagation or in otherwise for unstable fracture is $M=M_f$, when $K_I=K_{Ic}$, which leads,

$$M_f = \frac{K_{IC} d^{3/2} t}{Y_M(a/d)} + \frac{F_S d}{Y_M(a/d)} \left[Y_F\left(\frac{a}{d}\right) + Y_M\left(\frac{a}{d}\right)\left(\frac{1}{2} - \frac{d_S}{d}\right)\right] \quad (8)$$

The above equation describes a relationship between M_f, F_S, and a. If the values of F_S and the corresponding crack length a are known the value of M_f can be calculated. If the applied moment corresponds to steel yielding, then F_S will be the plastic force F_P. Equation 8 can be rewritten as,

$$M_f = \frac{K_{IC} d^{3/2} t}{Y_M(a/d)} + \frac{F_P d}{Y_M(a/d)} \left[Y_F\left(\frac{a}{d}\right) + Y_M\left(\frac{a}{d}\right)\left(\frac{1}{2} - \frac{d_S}{d}\right)\right] \quad (9)$$

In non-dimensional form the above equation can be rewritten as,

$$\frac{M_f}{K_{IC} d^{3/2} t} = \frac{1}{Y_M(a/d)} + N_P \left[\frac{Y_F(a/d)}{Y_M(a/d)} + \frac{1}{2} - \frac{d_S}{d} \right] \quad (10)$$

In which,

$$N_P = \frac{f_Y d^{1/2} A_S}{K_{IC} A}$$

N_P is a non-dimensional number, which is basically a function of material, geometric and sectional properties. For a particular grade of concrete and steel it is inversely proportional to the size of structure, indicating an increase in brittleness property, with increase in size.

4. PROCEDURE FOR COMPUTING THE RESIDUAL LIFE OF REINFORCED CONCRETE

The fatigue crack propagation model, described by Equation 2 together with Equation 3 is used for determining crack propagation nature in a reinforced concrete section. The stress intensity factor for cyclic load, in Equation 2 is to be evaluated using Equation 7, which is the algebraic sum of K_{If} and K_{IM}.

To determine the residual strength of a reinforced concrete member at a particular crack length, Equations 8 and 9 are used. The fracture moment is calculated as a function of two variables namely steel force F_S and crack length a. For a given beam geometry, if the steel area is known, then by varying the stress in the steel bars, we can obtain the variation of fracture moment with different crack lengths. In the residual life prediction analysis, considering the applied moment as unknown, the moment carrying capacity is computed for increasing crack length, and different steel force, as long as steel does not yield. If the applied moment is such that steel force reaches its yield value, then the moment carrying capacity is to be determined using Equation 9, more specifically as a function of two parameters namely crack length a and the non-dimensional number N_P. According to linear elastic fracture mechanics the fracture strength takes an infinite value as the crack length approaches zero. Since no material can sustain infinite stress, the strength corresponding to zero crack length is limited to the ultimate strength of the material.

5. PARAMETRIC STUDY FOR RESIDUAL LIFE ASSESSMENT OF REINFORCED CONCRETE

The proposed method is applied to obtain the residual service life of a reinforced concrete beam subjected to three-point bending, in terms of load carrying capacity and fatigue crack propagation rate. A parametric study is performed by varying the percentage reinforcement and ultimate compressive design strength. The material and geometric properties of the specimen analyzed is given below,

Overall depth (D)	380 mm
Effective depth (d)	355 mm
Clear cover (d_s)	25 mm
Width (b)	102 mm
Initial notch length (a_0)	38 mm
Area of tension steel	100 mm^2
f_{ck}	23 N/mm^2
K_{IC}	31 $N/mm^{3/2}$
Modulus of elasticity	

(steel) 205 Gpa
Steel yield stress 400 Mpa
Fatigue load cycle P_{max}=44kN, P_{min} =0 kN

Initially, constant amplitude loading cycle of frequency 3 Hz has been considered, which produces a minimum zero and a maximum 11×10^{-3} MN-m bending moment at the mid-span region of the beam. Using Equation 3, the parameter *C* is computed as 34.98×10^{-3} mm/cycle. Applying Equations 2 and 8, fatigue crack propagation curve is obtained by taking the values of coefficients *m*, *n*, and *p* as given earlier. Figure 3 shows the fatigue crack propagation curve obtained. It is seen that the curve becomes asymptotic, indicating failure at a crack length of about 110 mm when the load has undergone about 10,000 cycles.

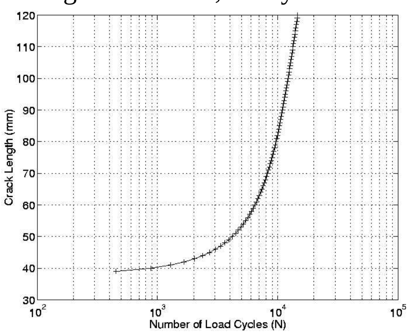

Figure 3: Crack propagation curve for reinforced concrete specimen

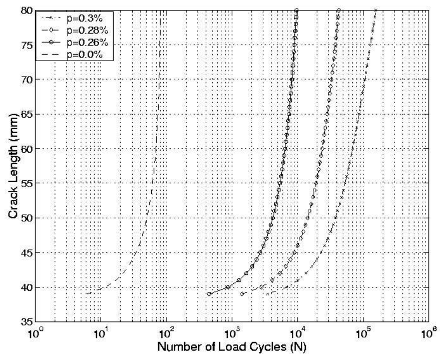

Figure 4: Crack propagation curve for reinforced concrete specimen for different reinforcement ratios

Figure 4 indicates the variation in fatigue behavior with different percentage of reinforcement. It is seen from this figure that the slope of curves decreases slightly with increasing reinforcement ratio indicating increase in ductility.

In the next part of the analysis the change in moment carrying capacity as a function of crack length is determined. For the given percentage of steel varying the stress induced in the steel bars up to the yield limit, variation of fracture moment along with different crack length is obtained using Equation 8, as shown in Figure 5.

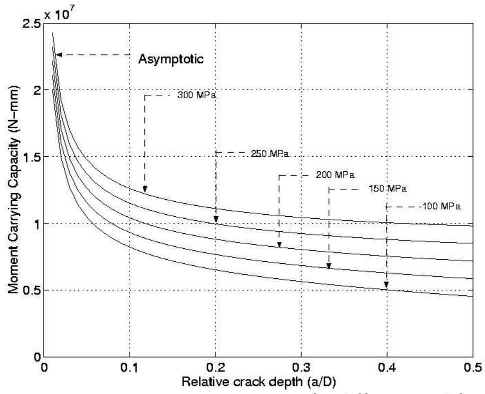

Figure 5: Moment carrying capacity for different steel force

As seen in this figure the curves become asymptotic as the crack length approaches zero. In reality, since no material can have infinite capacity the curves have to be terminated at the ultimate design moment value. This value depends on the stress-strain model used for concrete as described later.

In the case of an under-reinforced beam section, the force in the steel reinforcing bars may exceed the yield value when using Equation 8. Under this circumstance Equation 9 would be the governing equation. Using this equation, the normalized moment carrying capacity is plotted against the relative crack depth for varying steel reinforcement N_P as shown in Figure 6.

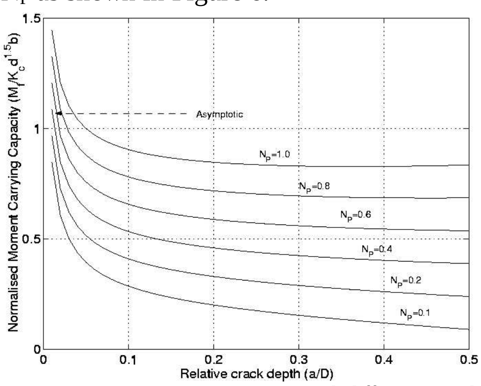

Figure 6: Moment carrying capacity with different N_P values

It is seen from this plot that the moment carrying capacity increases with increasing reinforcement for any given crack size. Further, as the crack size increases, initially there is a huge drop in the moment carrying capacity. For N_P value close to zero (Figure 6), i.e. for low reinforcement areas or very small cross sectional area of the beam, the fracture moment decreases continuously as the crack length increases leading to a unstable crack propagation. But for higher N_P values, the curves rise up slightly indicating a stable branch following an unstable one. As N_P values increase further, the stable branch becomes steeper. Hence, as stated by Carpinteri (1984), we can assert that the fracture process in reinforced concrete members become stable only when the beam is sufficiently reinforced or the cross section of the beam is sufficiently large when the crack has propagated quite deep.

However, it is important to observe that the curve relating to N_P=1, which represents the fracture process for common reinforced concrete beams is slightly deflected downwards, indicating an unstable crack propagation.

Concrete is characterized by a softening type post-peak load deflection behavior. Hence, it is assumed that concrete will resist some part of the tensile stress coming in the tension zone according to the softening law. Therefore, the stress-strain diagram in tension zone (Figure 7) will be combination of steel concrete tension. The total tension (T) can be obtained by, $T=T_1+T_2+F_S$, where T_1 is the total force coming from the pre-peak zone, T_2 is the tension contributed by the softening branch of the curve, and F_S is the steel force. Maintaining force equilibrium, $C=T$, ultimate moment has been evaluated.

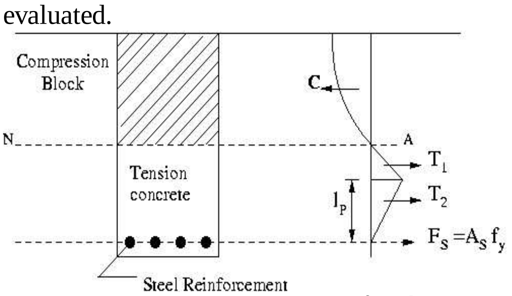

Figure 7: Stress-strain variation in a reinforced concrete section (compression and tension)

As explained earlier, at zero crack length the moment carrying capacity should correspond to the ultimate moment capacity of an uncracked section and this depends upon the constitutive law used for concrete in compression. Different constitutive laws for compression in concrete such as bilinear law, parabolic linear law etc, are proposed and used by different investigators. Figures 8 and 9 represent the moment carrying capacity against relative crack depth when bilinear and parabolic-linear models for concrete are used, respectively.

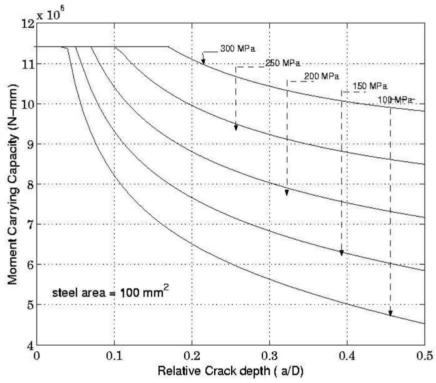

Figure 8: Design moment curves for varying stress in steel rebars (constitutive law for concrete in compression is bi-linear)

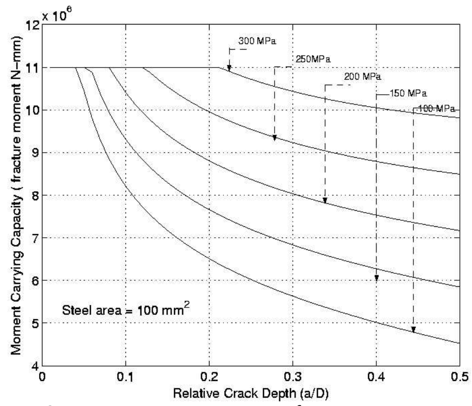

Figure 9: Design moment curves for varying stress in steel rebars (constitutive law for concrete in compression is parabolic-linear)

It is seen that the moment carrying capacity decreases with increasing crack length and decreasing steel force that is induced by the applied loading. For small crack lengths, the moment carrying capacity is limited to an ultimate value depending on the constitutive law for concrete in compression and the yield stress in steel.

6. PROBABILISTIC ASSESSMENT OF FATIGUE CRACK PROPAGATION

The analysis of fatigue crack growth in concrete is complicated by its heterogeneous nature. Thus, a statistical/probabilistic framework is needed for modeling of crack growth. Furthermore, a wide range of parameters may influence fatigue crack growth rates in concrete. These include mechanical as well as material parameters, such as fracture toughness, stress amplitude and stress range. In this work a reliability assessment has been performed for fatigue crack propagation considering fracture toughness and the applied stress as primary random variables.

Considering LEFM approach for reliability analysis of fatigue crack propagation, two separate types of failure criteria can be used.

1. Failure occurs when the crack of size *a* developed exceeds the predetermined or specified critical size a_c and the failure function is represented by, $g = a_c - a$,
2. Failure occurs when the stress intensity factor K at the leading edge of the crack exceeds the fracture toughness K_C and the failure function can be represented by, $g = K_C - K$.

As described earlier the failure due to fatigue crack propagation takes place when the crack propagation curves become asymptotic at a particular load cycle. Here, we determine the probability of failure when the crack reaches a specified final size. The probability of failure can be obtained using

$$P_f = \Phi\left(-\beta\right) \qquad (11)$$

where β is the Reliability Index and defined as the minimum distance of the limit state function with respect to origin in a space of standard random variables [Melchers (1999)]. Hence, our objective is to determine the reliability index β, specifying different final crack length corresponding to failure.

For the sake of simplicity, neglecting the history and overload effect, Equation 2 can be simplified as,

$$\frac{da}{dN} = C\frac{K_{\mathrm{Im}ax}^{m}\Delta K_I^{n}}{K_{IC}^{p}} \qquad (12)$$

If we consider the minimum stress value in a load cycle to be zero, then $\Delta K_I = K_{\mathrm{Im}ax} - K_{\mathrm{Im}in} = K_{\mathrm{Im}ax}$.

Therefore Equation 12 can be rewritten as

$$\frac{da}{dN} = C\frac{\left(K_{\mathrm{Im}ax}\right)^{m+n}}{K_{IC}^{p}} \qquad (13)$$

For a three point bend specimen,

$$K_I = \frac{P}{b\sqrt{d}}\, f\left(\alpha\right) \qquad (14)$$

Hence,

$$\Delta K_I = K_{\mathrm{Im}ax} = \frac{\Delta P}{b\sqrt{d}}\, f\left(\alpha\right) = \frac{P_{\max}}{b\sqrt{d}}\, f\left(\alpha\right) \qquad (15)$$

For a given beam size one can consider ($P_{max}/b\sqrt{d}$) as a new parameter with b and d being constants. The failure criteria can now be defined as,

$$\frac{da}{dN} = \infty, \quad \text{or,} \quad \frac{dN}{da} = 0.$$

Hence, the failure function will now be,

$$g\left(X\right) = \frac{K_{IC}^{p}}{C\left(K_{\mathrm{Im}ax}\right)^{m+n}} = 0 \qquad (16)$$

The random variables together with the type of distributions, their mean and standard deviation are shown in Table 2.

Table 2: Description of the random variables

Random Variable	Distribution	Mean (σ)	Std. Dev. (μ)
K_{IC}	Normal	0.982	0.169
$\Delta P/b\sqrt{d}$	Normal	0.356	0.0356

The response surface method is used to compute the reliability index since the failure function is nonlinear. The basic aim of response surface method is to replace the original failure function g(X) by an equivalent function R(X) in a polynomial form, given by,

$$R(X) = a + \sum_{i=1}^{n} b_i X_i + \sum_{i=1}^{n} c_i X_i^2 \qquad (17)$$

where X_i= 1,….n, are basic variables. Parameters a_i, b_i, c_i are constants to be determined. In this present study, since we have chosen two random variables namely, fracture toughness and normalized load, n=2.

Now evaluating the functional values of both R (X) and g (X) at points $X_i=\mu_i+t\sigma_i$, we get a set of (2n+1) linear equations to solve for parameters a, b, and c's.

In Equation 16 (or more specifically 15) stress intensity factor K_{IC} depends on the relative crack length α (a/D). At failure α will be α_F (a_F/D). A parametric study is done to obtain the reliability index as a function of the final crack length a_F as shown in Figure 10.

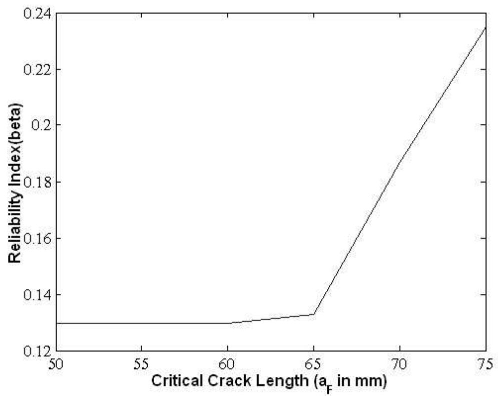

Figure 10: Reliability Index as a function of critical crack length

From this plot it is seen that there is no appreciable change in β value up to a final crack length of 65 mm. After 65 mm, a sharp increase in reliability index is observed. From this plot, one can determine the allowable final crack length at which failure takes place for a given probability of failure.

7. CONCLUSIONS

In this study, a fracture mechanics based fatigue crack propagation law for concrete is proposed which takes into account the specimen size, loading history, strength of concrete and frequency of applied loading. Using this law, a procedure for computing the residual life of damaged reinforced concrete members is proposed. It is seen that the rate of fatigue crack propagation decreases as the reinforcement ratio increases and that the fracture process becomes stable only when the member is sufficiently reinforced or the cross-section is made sufficiently large when the crack has propagated to a considerable depth. Further, using probabilistic theory it is shown that the critical crack length can be computed for any given probability of failure.

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