# Toughness in testing and design, the FRC experience

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ABSTRACT: The paper discusses implementation of material concepts such as toughness in testing and subsequently in design. These considerations are general enough to cover other material aspects than toughness and at the same time provide an explanation as to why it has proven so difficult to move from a relatively simple material, FRC, to structural design and application.

Further, the paper discusses various concepts for materials testing and tries to characterize the requirements for a testing method, which can be used in practice to characterize FRC in a simple yet flexible and meaningful way. The paper points to an already suggested testing method, which with a very simple and straight forward means of interpretation can yield fracture mechanical properties for both manual and design computerized FEM analysis. Finally, the paper summarizes some of the simple design models already available utilizing the fracture mechanical material characterization suggested in the paper.

Keywords: Fiber reinforced concrete, testing, design, cohesive crack, stress-crack opening relationship.

## 1 INTRODUCTION

### 1.1 *Scope*

The use of Fiber Reinforced Concrete (FRC) today is typically limited to non-structural applications or secondary elements, where the fibers are employed to serve functions such as minimizing shrinkage cracking and limit crack widths in the serviceability state due to mechanical loading. Slabs on grade and industrial floors are the two dominant applications in this class of applications.

Structural use of FRC is scarce, and attempts to use fiber reinforcement as structural reinforcement has so far been concentrated on replacement of shear reinforcing stirrups in structural members such as beams as well as replacement of complicated reinforcement arrangements in areas where concentrated loading is applied to the structure. Noteworthy is also the application of steel fiber FRC in pre-cast tunnel segments in the Netherlands (Kooiman 2000). The attempts to use FRC in structural applications have so far been of an experimental nature and often conducted in collaboration with universities.

Most FRC used today is based on normal strength concrete with a relatively small amount of steel fiber (0.4-1.0 vol.%) known as Steel Fiber Reinforced Concrete (SFRC). This type of FRC can be ordered from larger ready-mix concrete companies today as a standard item.

In contrast to FRC materials, which have concrete-like performance, high performance Fiber Reinforced Cementitious Composite, HPFRCC, materials have been developed. These materials are typically characterized by their ability to strain harden under multiple cracking in uni-axial testing (the effect known as pseudo strain hardening). Also this type of material is commercially available today. A Danish developed, highly specialized, high strength material, Densit®, is applied using high amounts of steel fiber – up to 6 vol. %. (Bache 1987, 1992). The Densit® material itself is an extremely densely packed material containing micro-silica and characterized by a very low water –binder ratio. The material itself is very strong and wear resistant, however also very brittle – thus the need for high amounts of steel fiber. Another material, Compact Reinforced Composites (CRC), is based on Densit®, (Bache 1987). CRC is built up of strong, densely arranged main reinforcement placed in fiber concrete. The fiber concrete is Densit®. The composition ensures extreme strength, ductility and impact resistance. The water/binder ratio in both Densit® and CRC is as low as 0.18, which requires special production techniques including very intense vibration. Densit® and CRC are used in specialized structural elements, in the safety product industry and for wear lining. Both Densit® and CRC were developed at Aalborg Portland cement factory in Aalborg, Denmark in the nineteen eighties. Later similar commercial products have been developed elsewhere e.g. Ductal® in France applying steel as well as polymeric fiber (Chanvillard & Rigaud 2003).

While extensive amounts of research have been carried out regarding FRC and HPFRCC from a materials point of view, the expected extensive use of FRC in the construction industry has not materialized even though both the low fiber volume FRC and materials like Densit®, CRC and Ductal® are commercially available and have obvious potential for structural applications. There are a number of possible reasons for this phenomenon: Conceptually, while FRC is generally recognized as tougher than concrete, a logical and systematic translation of this property into structural performance is all but absent. On a practical level, current design codes for structures in Denmark and many countries do not cover FRC materials. Without design guidelines, engineers find it difficult to incorporate this material into their structural design. Secondly, test methods that are robust and at the same time properly characterize FRC are still under debate. Thirdly, without a rational methodology for selection of fiber, matrix and control of interface, the resulting composite usually does not achieve optimal behavior, thus negatively affecting the performance to cost ratio.

In the case of HPFRCC the situation is more or less the same, standard design codes do not take the special properties of this class of material into account, if standard design code are used the material proves un-economical because the performance cannot be not taken into account and finally there is a lack of standardized test methods yielding material properties applicable in structural analysis.

## 1.2 *History*

The fact that it is primarily the toughness that distinguishes FRC from regular concrete has long been realized. Naturally, the need to characterize the toughness of FRC has been realized for a long time as well. In fact, a large amount of different testing methods have been proposed over the last 20-30 years in order to characterize mechanical properties of FRC materials and in particular the toughness – some becoming national standards in various countries such as Germany (DBV 1991), Belgium (NBN B 15-238 1992) and Japan (JCI-SF 1984). An overview and state-of-the art was provided in (Brite-EuRam Project BRPR-CT98-0813 2000). Many of these testing methods, however, did not provide materials parameters with a direct interpretation in terms of constitutive material parameters, such as stress, strain and crack width, thus making it difficult the apply the testing results in structural design.

At present local guidelines for testing and design do exist based on various concept. A Swedish guideline (Svenska Betonföreningen 1995) is used in design of most ground slabs and industrial floors in Scandinavia. These guidelines are based on the concept of a linear elastic interpretation of a four point bending test in combination with the well know toughness index (ASTM C 1018-97 standard 1998).

In the case of HPFRCC materials testing standards and code regulations are almost non-existent, however recently, French design guidelines were published aiming particularly on HPFRCC or *Ultra High Performance Fiber Reinforced Concretes*, taking the tensile response of these particular materials such as Ductal® into account (Setra, AFGC, 2002).

In the recognition of the obstacles identified above hindering the use of FRC and in particular the lack of consensus regarding identification of material parameters and corresponding test methods a considerable effort was made recently in the framework of RILEM in order to provide general guidelines for testing and design of FRC. This work resulted in 4 recommendations (RILEM Committee TDF 162 2000a,b, 2001, 2002), which was supplemented by the work in a European Community funded research project, Brite-EuRam Project BRPR-CT98-0813. Since FRC mechanical behavior is not fundamentally different from that of plain concrete and since FRC is characterized by a variable (fracture) toughness, higher than that of plain concrete, it seems straight forward to use fracture mechanical concepts for the characterization of FRC (Stang 1991). Further, the Fictitious Crack Model by Hillerborg (1976) by now recognized as highly suitable for the description of fracture of concrete and related materials, lends itself in a very natural way to the description of FRC, with fiber bridging and pullout contributing to the cohesive stress-crack opening relationship, which forms the basis of the FCM, see

(Hillerborg 1982). Two of the RILEM recommendations (RILEM Committee TDF 162 2001, 2002) take the fracture mechanical concept of the Fictitious Crack Model as a stating point, the first giving recommendations for the determination of the stress-crack opening relationship using the uni-axial tensile test, while the other gives recommendations for the application of the stress-crack opening relationship in structural design.

### 1.3 *This paper*

In the following considerations regarding implementation of material concepts such as toughness in testing and subsequently design are described. These considerations are general enough to cover other material aspects than toughness and at the same time provide an explanation as to why it has proven so difficult to move from a relatively simple material improvement (fiber reinforcement) to structural design and application.

Further the paper discusses various concepts for materials testing and tries to characterize the requirements for a testing method, which can be used in practice to characterize FRC in a simple yet flexible and meaningful way. The paper points to an already suggested testing method (RILEM Committee TDF 162, 2000a), which with a very simple and straight forward means of interpretation can yield fracture mechanical properties for both manual and design computerized FEM analysis. Finally, the paper summarizes some of the simple design models already available utilizing the fracture mechanical material characterization suggested in the paper.

## 2 TOUGHNESS FOR FRC

### 2.1 *The connection between toughness characterization and design*

Traditional design of concrete and reinforced concrete structures typically involves a very few material parameters describing the mechanical behavior of concrete, typically only the compressive strength and the Young's modulus. Tensile strength is – if applied – typically deduced from compressive strength. Alternatively, design formulae take tensile strength into account in an implicit way. To the author's knowledge, no design formulae contain explicit information about fracture toughness, even though it is recognized that toughness plays a significant role for the structural behavior, both in the serviceability and in the ultimate limit state. To take the influence of toughness in design into account implicitly is permissible only if the ratio between strength parameters, typically the compressive strength, and toughness is more or less constant in the materials under consideration. For special types of concrete such as high strength concrete and fiber reinforced concrete this turns out not to be the case.

In order to make it possible for structural designers to design structures made from FRC is was clearly necessary to introduce the concept of toughness in the structural calculations. This has been recognized for decades, however it has turned out that the task has been more complicated than expected. Part of the reason can be found by looking at Figure 1, which is a general representation of the relationships governing structural performance, which is one of the primary goals of structural design.

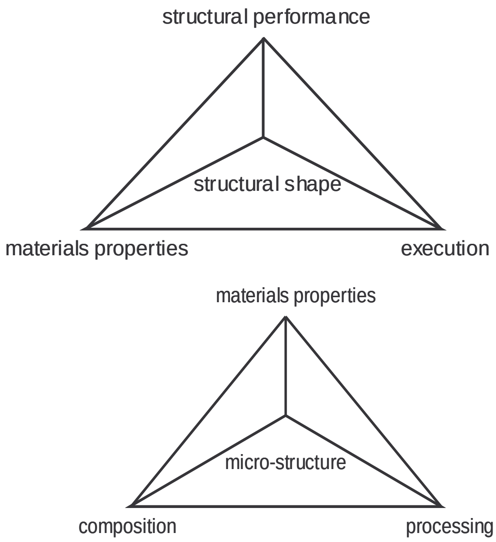



Figure 1. Illustration of the relationship between structural performance, material properties, structural shape and execution (top) and the relationship between material properties, composition, processing and micro-structure (bottom).

The top part of the figure represents the world of the structural engineer, where structural performance is considered primarily as a function of material properties and structural shape, even though it is recognized that the construction process and execution plays a role as well. Material properties are taken more or less for granted and used as input for the structural design process using various design tools, code formulas, FEM analysis etc.

The bottom part of the figure, on the other hand, illustrates the world of the material scientist or material developer. Here material performance is the goal and material performance - characterized in terms of material properties - is considered a function of material composition, the micro-structural arrangement of material phases and the processing techniques applied. It is clear form the figure that the material characterization (expressed in terms of material properties) forms the link between the material developer and the structural engineer. In practice testing form this link and it is clear that the testing used should make sense in both worlds. This is not always a logical requirement in materials development and where will always be materials testing carried out yielding results, which are not directly applicable by the structural engineer. However, if test methods creating the link between materials development and structural design are lacking, new materials will never be implemented on a structural level. For many years the test methods used for characterizing the toughness of FRC did not yield results, which could be implemented in structural analsys.

### 2.2 *Toughness in terms of fracture parameters*

Recently, much focus has been put on the use of fracture mechanical concepts in testing and design of FRC, (Rossi 1995, Stang & Olesen 2000, RILEM Committee TDF 162 2002). This has been done in recognition of the fact that fibers in conventional concrete primarily have a toughening effect while fibers play little or no role with respect to stiffness and strength. Thus, the effect of the fibers is a pronounced change in ratio between the (compressive) strength and the toughness. As a consequence the use of standard design tools for concrete structures in the design of SFRC becomes questionable.

The basic fracture mechanical concept suggested to be applied in SFRC design is the stress-crack opening relationship, which is associated with the so-called fictitious crack model. The model was originally suggested by Hillerborg to be used in concrete and FRC, (Hillerborg et al. 1976, Hillerborg 1980) and in recent years its applicability in SFRC design has been demonstrated in a number of different contexts.

The fictitious crack model can be thought of as a cohesive crack model relating the cohesive stress on the surface of the fictitious crack, $\sigma_w$, with the crack opening, $w$. This relation is called the stress-crack opening relationship. For $w$=0, $\sigma_w(0)=f_t$.

Typical stress-crack opening relationships for steel fiber reinforced concrete, determined by the uni-axial tensile test recommended by RILEM (RILEM Committee TDF 162 2001) are shown in Figure 2.

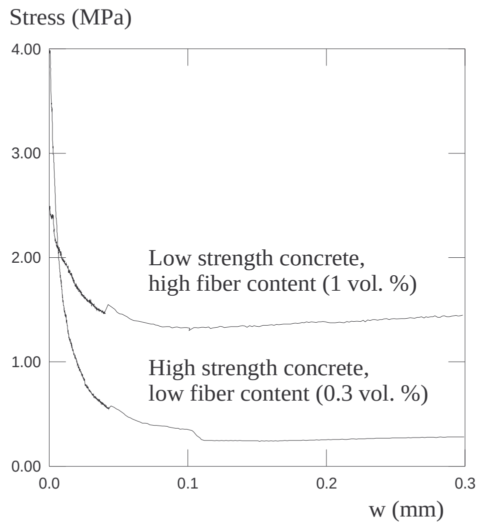



Figure 2. Stress-crack opening relationships obtained from uni-axial tensile testing according to RILEM TC TDF 162 (2001).

### 2.3 *Fracture parameters in design*

There are a number of advantages associated with the use of a fracture mechanical approach in the design of FRC. Several micro-mechanical models are available in order to provide understanding of the connection between material composition and stress-crack opening relationship. When failure is not pure compression failure it is well known that there are significant size effects associated with the failure stress. These size effects originate for a large part in the fracture process, thus design formulae based on fracture mechanics automatically takes the structural size into account. Finally, design formulae based on fracture mechanical concepts typically contain information on crack widths inherently, which is a significant advantage when designing in the serviceability limit state.

The disadvantages of applying fracture mechanics in design are that fracture mechanics concepts are not very well known in civil engineering community, and that the stress-crack opening relationship is not a property very well suited for design, especially since the relationship is highly non-linear and since the stress level as well as the shape depend on concrete, fiber type, and amount of fiber. To overcome this problem a number of different simplified stress-crack opening relationships have been suggested, (RILEM Committee TDF 162 2002), among which the bi-linear and the drop-constant relationships seem to be the most operational. The simplest, the drop-constant relationship is shown in Figure 3. This relationship prescribes a tensile strength $f_t$ and constant stress, $\sigma_y$, called the residual stress up to a maximum crack opening $w_{max}$.

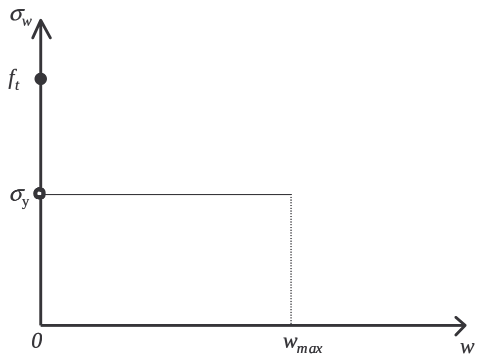



Figure 3. Simple representation of the stress-crack opening relationship suitable for design purposes.

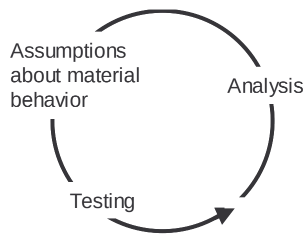
The bi-linear relationship is well suited for FEM analysis, while the drop-constant relationship is well-suited for implementation in simple design formulae.

## 3 TESTING FOR TOUGHNESS

### 3.1 *Concepts for testing*

When carrying out material testing the objectives can of course be many, typically information about material performance is sought. For the sake of simplicity we will concentrate here on two types of testing, primarily to illustrate the need for simple robust and economical testing methods and the difference between testing carried out for research and practical purposes.

One motivation for carrying out material testing can to obtain as detailed information as possible about the mechanical behavior in order to investigate e.g. the connection between fiber content and fracture mechanical behavior in FRC materials. The principles of this type of testing are outlined in Figure 4, top. This kind of testing requires basic assumptions about the material behavior, i.e. a constitutive model. In the case of a fracture mechanical approach to FRC this would be the assumption of the validity of the fictitious crack model. After the testing has been carried out the constitutive model is used to interpret the results of the testing. The interpretation of the test often requires structural analysis of the test specimen and loading configuration. The interpretation will help to shed further light on the assumptions initially made about the material behavior i.e. of the validity of the constitutive model. Furthermore, numerical values for the parameters in the constitutive model are provided, in this case detailed information about the stress-crack opening relationship. The testing in principle is recursive in its nature, as indicated in Figure 4, since it is not know in advance how the material is behaving and thus how the test should be interpreted.



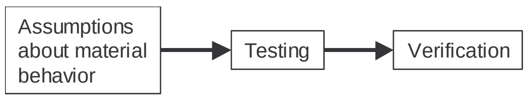




Figure 4. Concepts of material design. The top figure illustrates the research approach used in order to *investigate* hypothesis about material behavior while the bottom figure illustrates the approach used in quality control where expectations (e.g. in the design process) concerning the value of certain material parameters need to be *verified*.

Another motivation for the testing could be quality control or determination or verification of the material parameters introduced in a design situation. In Figure 4, bottom the typical relationship between design and testing is outlined from a practical viewpoint. Structural design is only rarely carried out with detailed information about a certain material behavior as input, rather the design is carried out under the *assumption* of certain material performance expressed in terms of a few material parameters. In the case of applying a fracture mechanical design approach to FRC these parameters would be the material parameters associated with the drop-constant stress-crack opening relationship outlined in Figure 3. Subsequent testing of the materials intended for use or actually used in a structure should be able to determine or verify if the materials actually meets the requirements defined in the design. The test should do only that in a simple and direct way while it is less important if the test verifies fundamental assumptions about material behavior. The testing is linear in nature as indicated in Figure 4.

### 3.2 *Testing for design*

Application of Fiber Reinforced Concrete (FRC) in structural applications requires testing methods, which are comparable to testing methods applied for conventional concrete with regards to simplicity and reliability. Furthermore, as explained above, it is a requirement that the parameters determined in the testing have a direct link to material properties used by the structural designer. Finally the test method should be of the linear type, this is sould be able to – in a simple and straight forward way – to verify if an assumption about material behavior in the structural design is met or not.

In order to arrive at practical test methods that meet the above the requirements it is important to leave the recursive test methods behind and concentrate on the linear ones which basically only tells if the structure build or to be build is safe or not.

In this context it is useful to consider the compressive test of concrete, which in fact introduces a very complex stress and strain state in the test specimen and an even more complex failure state. Non the less the test can be interpreted in a very straight forward way, and this simple test has been almost the only basis for structural design of concrete structures for many years whether designed using linear elasticity, plasticity or more complicated methods.

### 3.3 *A simple interpretation of the 3 point bending test*

The uni-axial tensile test seems the most direct and logical way of determining the stress-crack opening relationship. Recently, RILEM technical committee TC 162-TDF, “Test and design methods for steel fibre reinforced concrete” published a recommendation for uni-axial testing of SFRC with the aim of determining the stress-crack opening relationship directly (RILEM Committee TDF 162 2001). The test rely on the assumption that it is possible to restrain rotation of the crack surfaces in order to make it possible obtain more or less uniform crack opening over the whole specimen. This concept has been discusses at great length in the literature (Van Mier et al. 1996), but recent studies at DTU seem to confirm that the completely or sufficiently restrained uni-axial test specimen indeed does determine the stress-crack opening relationship correctly at least in the case of plain concrete (Østergaard 2003).

The method determines the stress-crack opening curve directly, in the sense that no structural model is necessary for the interpretation of the results. The stress is determined on the basis of the load by direct calculation and the crack opening is determined directly from the average reading of the clip gauges measuring the crack opening apart from a small correction due to the elastic deformation of the material next to the crack surfaces. Consequently the test method can be used both to obtain detained information about material behavior and to determine or verify simplified stress-crack opening relationships used for design. However, the test method is demanding both with respect to time and laboratory equipment. Furthermore, practical experience with the method for FRC has shown problems with achieving the expected number of fibers on the fracture surfaces, particularly in the case of relatively high fiber contents, the source of these problems still being investigated.

The beam test is well known as a tool for the determination of fracture energy $G_F$ of concrete (RILEM Committee FMC 50. 1985). RILEM technical committee TC162 proposes a 3 point bending test on a test specimen with a notch. The standard specimens proposed has a span $l$ of 500 mm, a height $h$ of 150 mm, a width, $b$, of 150 mm and a notch depth $a_0$ of 25 mm. The load $P$ as well as the deflection $\delta$ is measured. Optionally, the Crack Mouth Opening Displacement, CMOD, can be measured at a distance $d$ from the bottom of the beam. (RILEM Committee TDF 162 2000a).

Overall the beam test is less demanding with respect to time and laboratory work than the uni-axial test, however it should be emphasized that the test is still significantly more demanding than e.g. a standard compression test. The beam test is intended for use in a design method based on a non-linear stress-strain relationship. The result of the bending test is interpreted in a way that yields so-called equivalent flexural tensile strengths, which can subsequently be applied in design according to recommendations by the same committee (RILEM Committee TDF 162 2000b). The design is based on a non-linear stress-strain relationship in which key elements are determined by the equivalent flexural tensile strengths determined in the test.

Recently, it has been shown that it is possible to model the behavior of a FRC beam with or without a notch with good results using a fracture mechanical approach. This can be done both using non-linear finite elements and an analytical approach introducing a non-linear hinge, where the crack is propagating, in an otherwise elastic beam. The approach is discussed at some length in a paper on structural analysis of FRC structures based on fracture mechanics from RILEM technical committee TC162 (RILEM Committee TDF 162 2002). The analytical analysis can be based on analytical solutions for the non-linear hinge in terms of moment versus angular deformation relations. Closed form solutions are available for both the bi-linear as well as the drop-constant stress-crack opening relationship (Olesen 2001a).

The existence of such relative simple solutions for the beam test based on fracture mechanics obviously opens up for using the beam test for determination of the fracture mechanical properties, i.e. the stress-crack opening relationship. When detailed information about the stress-crack opening relationship is required a so-called back analysis is needed, because it is not possible based on knowledge of the beam response (load-deflection or load-CMOD) to solve directly for the underlying stress-crack opening relationship. Back analysis is based on a comparison between the observed response and the response calculated with a certain choice of stress-crack opening relationship. This comparison is quantified in terms of an error. The best choice of stress-crack opening relationship can now be determined by minimizing the error. Back analysis for the beam test has been studied extensively for concrete and SFRC (Nanakorn & Horii 1996, Kitsutaka 1997). However, is was shown by Stang & Olesen (1998), that it is very difficult with this method to distinguish between tensile strength and the initial part of the stress-crack opening relationship which seems to indicate that back analysis should not be attempted unless independent information about the tensile strength exist, especially in the case of SFRC, where the ratio between strength and toughness can vary significantly.

Even though there are problems with the inverse analysis, it turns out that the beam test is suitable for verification of the fracture parameters in the simplified stress-crack opening relations applied in design, such as the bi-linear or the drop-constant relationship. In the case where the simple drop-constant stress-crack opening relationship has been applied, the expected beam response in terms of either a load-deformation or a load-CMOD relation can be calculated using the analytical model based on the non-linear hinge as outlined above. For a given test specimen geometry, this calculation can be based on the assumption of vanishing tensile strength and a certain value $\sigma_y$ of the residual stress. Choosing different values for the residual stress, $\sigma_y$, a series of curves is produced which can be interpreted as a *verification chart*. Since the influence of the Young's modulus is very weak for practical purposes only a single verification chart is needed for each type of test specimen. In Figure 5 a verification chart for the beam suggested by RILEM technical committee TDF 162 is shown together with the relationship between the deflection and the crack opening displacement, COD, at the bottom of the ligament. A given test result obtained using a certain test specimen geometry and instrumentation (e.g. according to the RILEM TDF 162) can be compared with the corresponding verification chart. A certain assumed design value $\sigma_y$, valid up to a certain maximum crack opening $w_{max}$ is verified if the measured load-deflection or load-CMOD curve lies above the curve in the verification chart corresponding to the same value of $\sigma_y$ for all deflections or CMODs less than certain values, $\delta_{max}$ or $\mathrm{CMOD}_{max}$. It can be shown that deflection, $\delta$, and CMOD are approximately linearly related to COD (see also Figure 6), which again can be related to $w_{max}$ in design guidelines. This makes use of the verification charts particularly simple. The use of the verification charts does not involve tensile strength and again the tensile strength should be determined/verified independently from the beam test.

The method can easily be expanded to cover other test geometries (the Wedge Splitting Test specimen e.g. seems to have significant potential also for testing of FRC and the non-linear hinge analysis of this specimen has been established, Østergaard et al. 2002) and other types of stress-crack opening relationships.

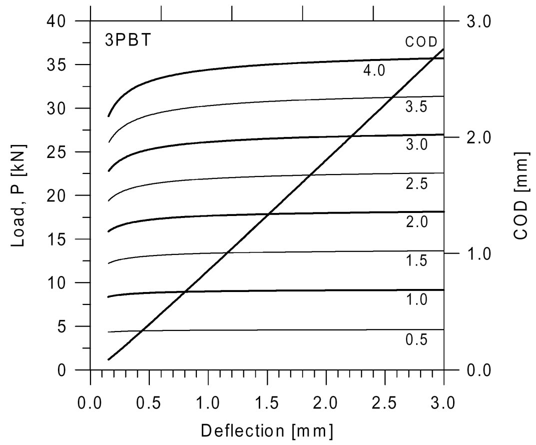



Figure 5. Verification chart for the RILEM 3 point bending test (3PBT), proposed for steel fibre reinforced concrete by the technical committee TC 162-TDF. The verification charts can be used to verify or determine the residual stress $\sigma_y$ used in the drop-constant stress-crack opening relationship. The numbers next to the curves refer to the residual stress $\sigma_y$. The left axis is load, the right axis is the COD and the almost straight line represents the relationship between deflection and COD.

## 4 DESIGNING WITH SIMPLE FRACTURE PARAMETERS

When applying the simple drop-constant stress-crack opening relationship a number of simple structural models are already available. These include the simple cross-sectional analysis of FRC cross sections subjected to a combination of bending and compression (Olesen 2001a), which opens up for structural analysis of e.g. beams and pipes. Further an analysis of cross sections with a combination of FRC and reinforcement has been carried out giving crack-openings as well as ultimate load carrying capacity (Olesen 2001b). Finally, a method for prediction of crack width in slabs on grade subjected to shrinkage has been suggested (Olesen and Stang 2000).

More importantly, the fracture mechanical approach to design of FRC opens up for a consistent implementation in FEM using the bi-linear stress-crack opening relationship or even more complicated if necessary. This can be done either through the discrete, the smeared of the relatively new XFEM approach. No doubt in the future more flexible formulations of fracture mechanics in FEM taking the cohesive crack models as a starting point will improve possibility of carrying out design of FRC structures.

## 5 CONCLUSIONS

A fracture mechanical approach to design of SFRC structures is gradually becoming more and more realistic, also from a practical point of view. A sound basis for such an approach seems to be the so-called fictitious crack model, which uses the stress-crack opening relationship as a basic input. Advantages connected to a fracture mechanical approach include: micro-mechanical models are available in order to provide understanding of the connection between material composition and stress-crack opening relationship, design formulae based on fracture mechanics automatically takes the structural size into account and design formulae based on fracture mechanical concepts typically contain information on crack widths inherently. A number of standards for test specimens have now been recommended by RILEM. These specimens include a uni-axial and a bending test specimen. The results of both tests can be interpreted in terms of fracture mechanical properties. This can be done either in order to get detailed information about the fracture mechanical parameters or in order to determine or verify simpler stress-crack opening relationships applied in design. A simple method for determining the residual stress $\sigma_y$ in the drop-constant stress-crack opening relationship is suggested. A number of structural design models are available utilizing the simple drop-constant relationship while general analysis using the bi-linear stress-crack opening relationship is becoming more and more flexible.

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