Rate Effect of Concrete with a Simplification of Crack Interaction

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ABSTRACT: This paper deals with the rate effect on concrete strength with consideration of free water viscosity. A crack in an infinite solid subjected to linear loading normal to the crack surface is considered accounting for effects of free water viscosity and inertia. Crack interaction is also considered by a simple method. Results are obtained for the relationships between the dynamic fracture initiation toughness and the fracture initiation time, and between the dynamic strength enhancement and the strain rate, both with and without considering free water. A comparison between the theoretical result and the experimental data in the literature indicates that a good agreement is achieved, which implies that the model can be used to explain the rate-dependent properties of concrete.

Keywords: Concrete, Rate effect, Fracture, Microcrack, Free water.

1 INTRODUCTION

Many concrete structures may be subjected to high rate dynamic loadings (earthquake, impacts, explosions, etc.). It is therefore necessary to know the behavior of this material in order to predict the response of the structure. The mechanical properties of cement-based material are sensitive to strain rate. Under dynamic loading, the increases of strength and fracture toughness are observed; this phenomenon is called as “rate effect”.

Recently, many experimental results show that the free water in concrete plays an important role in concrete property under dynamic loading (Rossi et.al 1992, Cardoni 2001). The strength of concrete after drying is not sensitive to lower loading rate (< 1s^{-1}) (Reinhart 1990, Rossi 1991, Rossi et al. 1992). Furthermore, the effect of loading rate on concrete increase with the water/cement ratio of the material (Rossi et al. 1994). The rate effect is less for high-strength concrete than that for normal concrete (Rossi, 1997).

Rossi (1991, 1997) attributed the rate effect in concrete to water viscosity produced by Stefan effect, and proposed a visco-plasticity model to simulate the properties of concrete under dynamic loading.

On the other hand, under higher loading rate (>1 s^{-1}), both dry and wet samples exhibit significant strain rate sensitivity. The inertial effect should also be included to analyze the rate effect.

Kipp et al. (1980) have employed the theory of linear elastic dynamic fracture mechanics to formulate the response of dynamic loading of a single crack. Liu et al. (1984) have adapted stress criteria near the crack tip, and analyzed the relationship between dynamic initiation toughness in brittle solids and loading rates. Ross et al. (1996) derived an analytical expression included both crack velocity and fracture toughness based of energy equilibrium equation.

Many experiments have been done, yet no physical mechanism can clearly explain the rate effect for concrete. Accordingly, the objective of the present paper is to propose a dynamic fracture model, which can quantitatively explain the rate effect for concrete both under higher and lower loading rates.

In fact, many interfacial bond micro-cracks and micro-cracks within the cement matrix are inherent in concrete material due to manufacturing procedures and shrinkage during hardening of cement-based composites. A simple method considering multiple microcracks interaction is also presented.

2 DYNAMIC FRACTURE MODEL OF SINGLE CRACK

2.1 Crack configuration

A crack with length 2a (Fig.1) located in an infinite medium is employed to analyze the loading rate effect of a single crack. The crack surfaces are subjected to an opening mode linearly increasing tensile load $\sigma(t) = \dot{\sigma}t$ (Fig.2). While the crack surfaces begin to separate from each other, the cohesive force σ_c known as Stefan effect delays the movement of the crack surface, then delays the initiation and propagation of the crack. Using the Irwin crack growth criteria, the crack will begin to propagate when the dynamic stress intensity factor $K_I(t)$ reaches the critical fracture stress intensity factor (called fracture initiation toughness) K_{IC}^{D}.

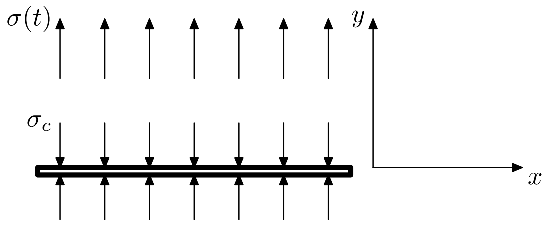

Figure 1: Crack configuration

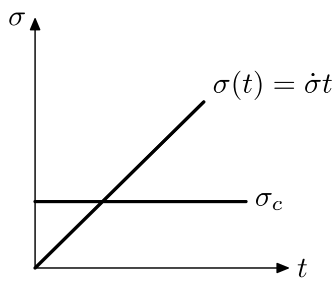

Figure 2: Loading history

2.2 *Stefan effect*

The viscous cohesive force σ_c of free water can be considered as followed. Cotterll (1964) has analyzed the viscous force between two parallel, circular plates. If the two plates are apart in a direction perpendicular to the plates, the force F needed to pull the plates will be (Fig.3)

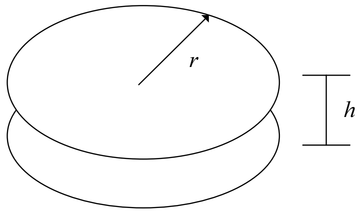

Figure 3: Stefan effect

$$F = \frac{3\pi k r^4}{2h^3} v \tag{1}$$

where, k is the liquid viscosity, h is the liquid thickness, r is the radius of plate, v is the velocity of plate.

Then, the viscous tensile stress between the plates is

$$\sigma_c = \frac{3kr^2}{2h^3} v \tag{2}$$

This equation shows that the viscous force is proportional to the v and k. According to the linear fracture mechanics, the crack displacement u is proportion to the stress σ, so the velocity $v = du/dt$ is proportional to the loading rate. Then we can approximately assume that the cohesive force between the crack surfaces is proportional to the loading rate.

2.3 *Solution for the model*

The theory of linear dynamic fracture mechanics has provided a clear understanding of the response of cracks to transient tensile loads. Parton et al. (1989) and Freund (1990) have discussed the response of an elastic solid containing a crack and subjected to impact loading normal to the crack surface.

The solution for stress intensity factor of the model, shown as Figure 1, can be achieved by some simple transformations (Zheng & Li, in press). With the presence of external loading $\sigma(t)$ and the viscous force of free water σ_c, the applied loading is

$$\sigma(t) = \dot{\sigma}t - \sigma_c = \dot{\sigma}(t - T_0) \tag{3}$$

This can be regarded as linear increasing loading added at time T_0. Where T_0 is a constant related to the geometry of crack and the viscosity of free water.

The variation of dynamic stress intensity factor with the normalized time is plotted in Figure 4.

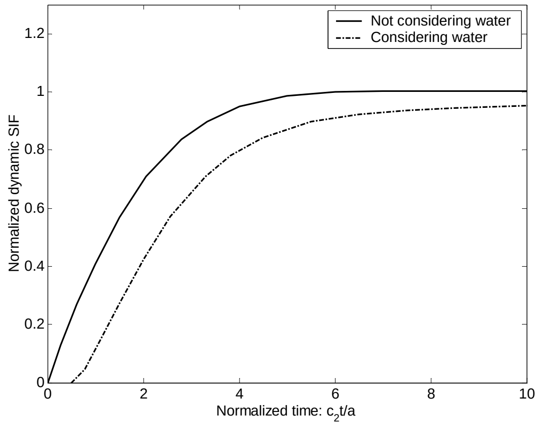

Figure 4: Dynamic SIF variation with normalized time of linear increasing loading

$$K_I^{\mathrm{d}} = \sigma(t)\sqrt{\pi a}\, f(c_2 a / t) \qquad (4)$$

where c_2 is the shear wave velocity.

It can be seen that when c_2a/t >>1, the stress intensity factor approaches to the corresponding static values, and when c_2a/t is small, the stress intensity factor is much less than the corresponding static values.

3 MUTIPLE MICROCRACK INTERACTION

For concrete, micro-cracking has long been known to be the dominant source of nonlinear or inelastic behavior. And the mechanical response of the concrete is mainly controlled by micro-cracking in the transition zone between the aggregate and the HCP (Zaitsev, 1983). The propagation and coalescence of cracks is the dominant mechanism of concrete material failure.

Under increased loading, microcracks in the transition zone grow and coalesce. Eventually, a continuous crack system forms, resulting in the loss of load capacity. Under tensile loading, increasing load acts directly to increase the SIF at the crack tip and drive crack propagation. As a result, for tensile loading, when a crack begins to propagate into the matrix and coalesce, the sequence of cracking leads up to the development of a continuous crack system and failure of concrete occurred very rapidly. So we can regarded that the crack with the largest radius $a_{\max}$ dominants the concrete failure process.

In fact, concrete contains many micro-cracks interacting with each other. In this section, a simple method is presented to consider the microcrack interaction.

The macro-crack and micro-crack interaction has been extensively discussed in the literature (Kachanov 1987, Kachanov et.al 1990). This method is called as pseudo-traction method. To solve the SIF, one replaces the original problem by an equivalent configuration: the plate is stress free at infinity, but with uniform traction σ^0 applied along the faces of every micro-crack. The latter problem is further reduced to superposition of N problems; each involves an infinite plate with a single crack at the designated location. The crack faces are loaded by normal tractions to be solved. With the consideration of crack interaction, the stress intensity factor of single crack tip can be written as:

$$K_I = \sigma_0\sqrt{\pi a_0}\, F\left(\frac{c}{a_0}, \frac{a'}{a_0}\right) \qquad (5)$$

where a_0 is the crack radius, a' is the crack radius adjacent to the crack, c is the space between cracks, F is dimensionless shape factor. It can be seen that the SIF of crack is strongly related to the crack distribution. If the crack are collinear, and $a' = a_0$ the typical value of F is plotted as Figure 5.

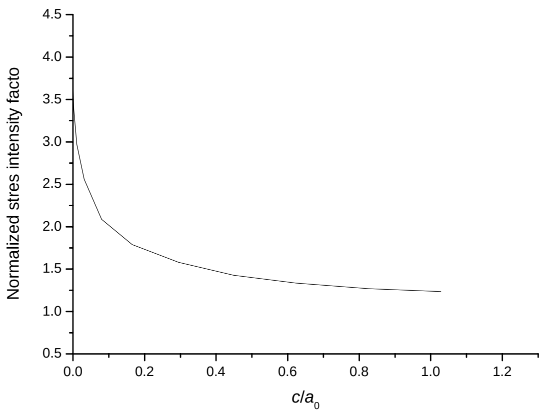

Figure 5: Typical curves of normalized stress intensity factor

Because of the crack interaction, the actual SIF of the crack is always bigger than the origin value. Therefore, we can idealize the concrete material as a single dominant crack of radius $a = a_{\max}\sqrt{F}$.

The actual distribution of micro-cracks in the concrete is very complex and almost impossible to determine. While the space of cracks is not smaller than the crack radius, the influence is not significant.

In this paper, from Figure 5, we take F=1.2.

4 RESULTS AND DISCUSSION

Dynamic stress intensity factor of the crack has been achieved in Section 2. Assume that the real (or micro) fracture toughness of the medium does not vary with loading rate, i.e. $K_{IC}^{D}=K_{IC}$ =constant, then a higher corresponding stress under dynamic loading is needed than that under static loading to reach the same critical fracture intensity factor. Then the dynamic enhancement factor can be deduced as

$$K_I = \sigma_c \sqrt{\pi a} = K_{IC} \tag{7}$$

and

$$K_I^{\mathrm{d}}\Big|_{\sigma = D\sigma(t)} = \sigma(t)\sqrt{\pi a} = K_{IC}^{d} \tag{8}$$

in which

$$D = 1/f(c_2 a/t) \tag{9}$$

Then if the crack begins to propagate at time t, the dynamic strength σ_c (or macro equivalent fracture toughness $\sigma_c\sqrt{\pi a}$) enhancement will be D, which is plotted in Fig.6.

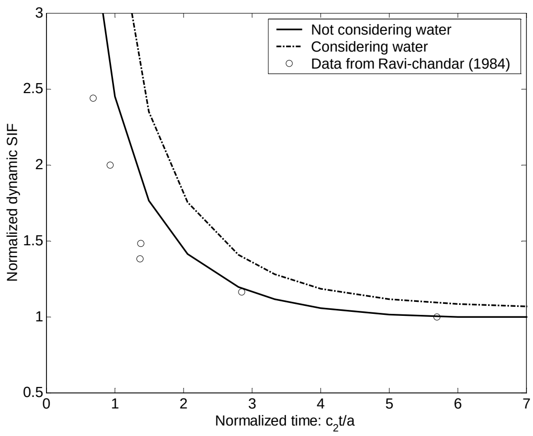

Figure 6: Dynamic enhancement factor variation with fracture initiation time

Ravi-Chandar and Knauss (1984) carried out experiments on thin sheets of Homalite-100 to get fracture initiation time under dynamic loading. According to Figure 6, the dynamic strength (or the macro equivalent fracture toughness) varies with the fracture initiation time, its trend is similar to the results. Because of the material difference, the result in this paper is only a qualitative analysis of the experiment.

From Figure 6, we can see that if the fracture initiation time is less than about $3a/c_2$, the dynamic strength (or the macro equivalent fracture toughness $\sigma_c\sqrt{\pi a}$) depends strongly upon the fracture initiation time. However, if the fracture initiation time is higher than about $3a/c_2$, the dynamic enhancement factor keeps almost constant 1. So we can take this phenomenon as the inertia effect under dynamic loading, especially under higher loading rate. And if free water viscosity is considered, the dynamic enhancement factor will be a little bit higher than 1.0 under lower loading rate.

Because under linear increasing loading $\dot{\sigma}$ = Constant, the fracture initiation time is

$$t = \frac{D\sigma_0}{\dot{\sigma}} \tag{10}$$

where σ_0 is the static strength of concrete, then from Eqs. (9) and (10), we can get the variation of dynamic enhancement factor with loading rate $\dot{\sigma}$.

$$\frac{D}{f^{-1}(1/D)} = \frac{a\dot{\sigma}}{c_2\sigma_0} \tag{11}$$

The experimental results of Ross et al. (1996) are employed to prove the model, in which the parameters are: a = 3cm, c_2 = 300m/s, σ_0 = 3MPa, E = 3 × 10^4MPa. D is evaluated by substituting these parameters into Eq. (24), which is plotted versus $\log_{10}$ ($\dot{\varepsilon}$) in Figure 7 along with experimental data. It can be seen that a good correlation is obtained between the tensile strengths of concrete got by this model and the experiments of Ross et al. (1996).

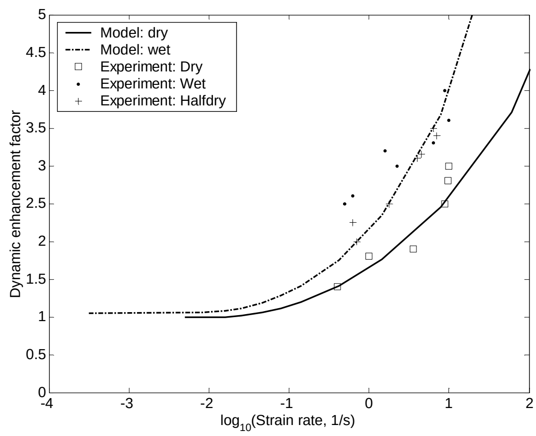

Figure 7: Dynamic enhancement factor variation with strain rate $\dot{\varepsilon}$

It should be pointed out that the Stefan effect in concrete is very hard to quantify because it is difficult to measure the diameter of the micro-pores in the hydrated cement paste. But it can be calibrated from the experiments on the macro properties of concrete.

5 CONCLUSIONS

In this paper a method based on dynamic fracture mechanics with both the free water viscosity and inertia effect included is presented. Multiple crack interaction is also considered.

It should be pointed out that the simplified model is only used to explain the rate effect of concrete. Because concrete failure is due to micro-crack nucleation, propagation and coalescence, the increasing of dynamic macro equivalent toughness for single crack and the simplification of cracks interaction can qualitatively explain the rate effect in concrete. It is not enough to calculate quantitatively precisely, especially for the compressive failure of concrete.

It has a long way to go to develop a precise mathematical model for an actual concrete specimen with arbitrarily distributed multi-cracks.

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