Analytical evaluation of the softening behaviour of fine grained concrete

W. Brameshuber; T. Brockmann; B. Banholzer
Institute of Building Materials Research, RWTH Aachen, Germany

ABSTRACT: The collaborative research centre „Textile Reinforced Concrete (TRC) – Development of a New Technology" (SFB 532) established at Aachen University (RWTH Aachen) is investigating the basic mechanisms of the new composite material. As the fine grained binder systems used show a more homogeneous structure compared to ordinary concrete the material properties of this component have to be known. Hence, it is the objective of the presented investigations to determine the softening behaviour of the concrete. 3-point bending tests are carried out, and a model is proposed which allows the analytical derivation of a multi-linear σ-w-relation without using FE-analysis. Measurements with a video extensometer give the displacements over the ligament height in the cracking zone during loading and allow the derivation of the corresponding input parameters. First results show a good agreement with the tension softening behaviour derived by FE-analysis.

Keywords: concrete, tension softening, analytical model, bending test, video extensometer

1 INTRODUCTION

The objective of the collaborative research centre „Textile Reinforced Concrete (TRC) – Development of a New Technology" (SFB 532) established at Aachen University (RWTH Aachen) is to provide guidelines and standards for designing thin-walled elements. For this reason the material properties of the single components have to be known and will be integrated in different models supporting analytical and numerical simulations of textile reinforced concrete structures. Thus, mechanical and fracture mechanical properties of the fine grained concrete matrix for TRC have to be determined. The fine grained binder systems typically show highly flowable consistencies which offer full penetration of the technical textiles. These special properties are achieved by using a small maximum grain size ($d_{max} = 0.6$ mm), high binder contents, and adding different pozzolanic additives, all of which lead to a more homogeneous and finer structure compared to ordinary concrete. Hence, it is necessary to investigate the new material's response and to evaluate the tension softening characteristics.

Fracture of concrete is closely related to the formation of a fracture process zone (FPZ) in front of a crack tip. In a FPZ, where softening is taking place, the fracture process of concrete shows a non-linearity. The fictitious crack model (FCM) was proposed by Hillerborg (1983 & 1985) to describe such non-linear fracture behaviour. This mechanical model replaces a FPZ by a fictitious crack able to transfer stress from one fracture face to the other. The cohesive stress changes in relation to the fictitious crack width w, hence forming a tension softening diagram. The determination of such a σ-w-relation usually requires a load-displacement curve obtained from a Mode I stable fracture test like e.g. a 3-point bending test on notched beams, and a subsequent numerical analysis by means of FEM. According to the FCM the development of a single fictitious crack in the mid-section of the beam is assumed, where points in the crack extension path are assumed to be first in (i) a linear-elastic state, then in (ii) a fracture state where the material is softened caused by cohesive forces in the FPZ and finally in (iii) a state of no stress transfer. Usually the FPZ within the FEM is modelled by means of spring or interface elements, while the remaining body is assumed to be linear elastic. For a given

displacement u known from the experiment in the mid-section of the beam the corresponding displacements can be calculated when the boundary conditions are met, i.e. no displacement of the rigid supports. The calculated displacement field also implies the displacements in the mid-section of the beam in x-direction, and hence in the region of the fracture zone (Figure 1). The corresponding stress distribution is then derived by applying the material relations of the concrete. Usually a linear σ-ε–relation defined by the tensile strength f_t and the Young´s modulus E is assumed to describe the pre-cracking behaviour, while a σ-w-relation is chosen to describe the softening behaviour of the concrete as described by Hillerborg´s FCM (1983). Hence, the resulting force P_{sim} can be calculated and is compared with the force P_{exp} known from the experiment for each load step (load-displacement curve from 3-point bending test). The assumed softening curve is adapted in an iterative fitting process such that the numerical simulation of the load-displacement curve and the experiment show a good agreement.
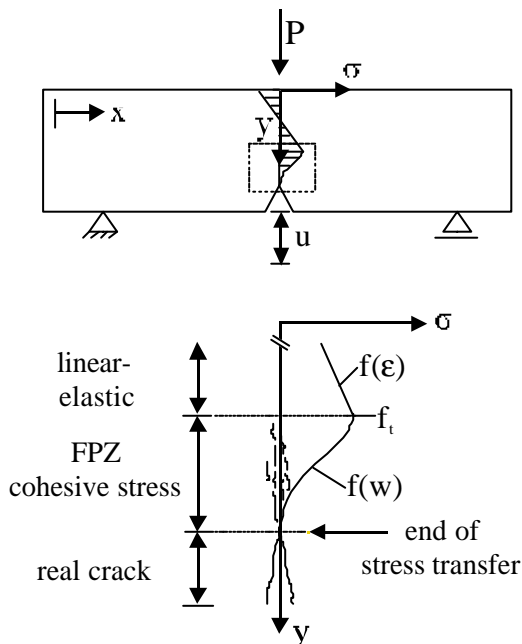

Figure 1. A loaded concrete beam with a crack and a fracture zone

As the identification of a multi-linear σ-w-curve which is defined by many parameters (or pair of values) and allows an unlimited characterisation of the softening is a complex and time consuming task, different simplifications have been proposed

in the recent years, like e.g. linear, bi-linear (e.g. Petersson 1981), multi-linear (e.g. Gustafson 1985) and exponential (e.g. Duda 1991) σ-w-relations. However, a different approach was proposed by Uchida (2001). He chose a method - based on the FCM – to estimate a multi-linear tension softening diagram by inverse analysis of an experimentally determined load-displacement curve of a 3-point bending test. In this method the σ-w-curve is determined step by step for each increment of crack propagation by assuming multiple slopes at the tip of the diagram, calculating the resulting load-displacement curves, and choosing one slope that suits the experimental results. This method allows a free fitting of the curve, but even within this model the knowledge of numerical simulation is required as still FEM are employed for calculations. As the fitting process and the application of FEM are considered to be time consuming, different analytical models have been proposed by e.g. Chuang and Mai (1989), Planas and Elices (1987), and Ulfkjaer et al. (1990 & 1995).

Ulfkjaer et al. (1990 & 1995) have presented an analytical one-dimensional model for the bending failure of concrete beams by means of a fictitious crack in an elastic layer with a thickness proportional to the beam depth. Outside the elastic layer the displacements are modelled by beam theory. Tensile tests give the tensile strength f_t, the Young´s modulus E, and also the critical crack opening displacement w_c as input parameters. Assuming a linear σ-w-relation as softening behaviour it is possible to determine the crack width for different load levels on the base of beam theory and thus to calculate the load-displacement curve of a 3-point bending test. However, in this approach only a linear softening relation has been used and also the model has to be calibrated by FE calculations in order to determine a calibration factor.

Sundara (1998) proposed an extension of the analysis given by Ulfkjaer et al. (1990 & 1995) for a 3-point bending problem with a bilinear softening relationship, which offers a more realistic description of the fracture behaviour of concrete.

All these aforementioned models have in common, that either by means of FEM or beam theory the distribution of displacements within the ligament of a concrete beam are calculated. Knowing these and using the (assumed) σ-w-curve allows the derivation of the stress distribution within the ligament, which finally allows to calculate the force P_{sim} which is compared with the experimentally determined force P_{exp}.

In contrast to these models, in the present paper a new approach is shown, where the distribution of displacements within the ligament of a beam in a 3-point bending test is already known from the experiment, as measured continuously during testing by a video extensometer. Combining the advantages of a step-wise determination of a multi-linear σ-w-relation as proposed by Uchida (2001) and the classical idea of beam theory the model allows a straightforward determination of the σ-w-relation. Knowing the displacements Δl over the ligament height and applying equilibrium of forces the determination of the σ-w-relation without the need of using FEM and further optimisation processes is possible.

First results of σ-w curves derived with the new model based on the results of 3-point bending tests using fine grained concrete beams are shown in the presented paper. The derived softening behaviour is then compared with σ-w-curves which have been determined in the usual way by means of FE-analysis.

2 DESCRIPTION OF THE MODEL

Similar to the previously introduced models the Young´s modulus E and the tensile strength f_t are used as input parameters, and the fracture process of the 3-point loaded concrete beam is considered according to the FCM. During the experiment the load P_{exp} and the displacements Δl (strain, and compression strain) in x-direction are measured continuously at the different measuring levels y_i with the measuring length $l_m(y_i)$ assembled over the ligament height y_n along the mid-section of the beam (Figure 3). Thus, the stress distribution $\sigma(y_i)$, which is assumed to be linear between the measuring levels (Figure 3), can be derived directly as will be described in the following section.

In the following i refers as well to the location y_i for reasons of simplification. In addition, the following considerations and equations refer to one load step only.

Assuming linear-elastic material behaviour until the tensile strength of the concrete is reached, the stress distribution within the ligament may be determined for the regions of compression or tension according to Hooke´s law by

$$\boldsymbol{s}_i = \boldsymbol{e}_i \, E = \frac{\Delta l_i}{l_m(y_i)} E \qquad (1)$$

where ε_i is defined by the measured displacement Δl_i related to corresponding measuring length $l_m(y_i)$ and E is the Young´s modulus of the concrete.

As the proposed model is based on equilibrium of forces the inner moment M_I at the mid-section of the beam must always equal the outer moment M_O ($M_I = M_O$), see Figure 3.

The outer moment M_O may be determined by

$$M_O = \frac{P_{exp} l_s}{4} \qquad (2)$$

where P_{exp} = measured load for each load step; l_s = span.

The inner moment M_I is determined by the following equation, see Figure 2,

$$M_I = \sum_{j=1}^{n-1} F_j h_j \qquad (3)$$

$$= \sum_{j=1}^{n-2} F_j h_j + \frac{\boldsymbol{s}_{n-1} + \boldsymbol{s}_n}{2} (y_{n-1} - y_n) h_{n-1} b$$

where j = i is defined as section between two measuring levels i and i+1 and as a result 1 < j < n-1; F_j = force of section j derived by integration of corresponding stresses; h_j = moment arm of considered stress distribution area related to neutral axis y_s; σ_i = stress at ligament height y_i; y_i = distance from the surface of the concrete beam in y-direction; b = width of concrete beam.
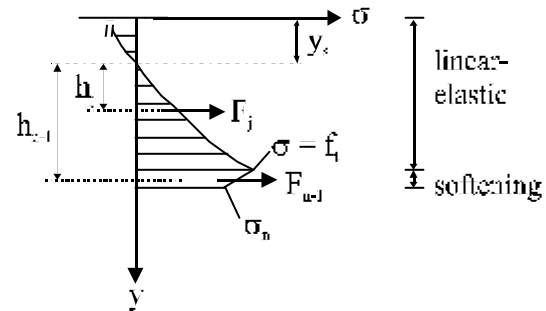

Figure 2. Principal stress distribution $\sigma(y_i)$ and corresponding section forces F_j
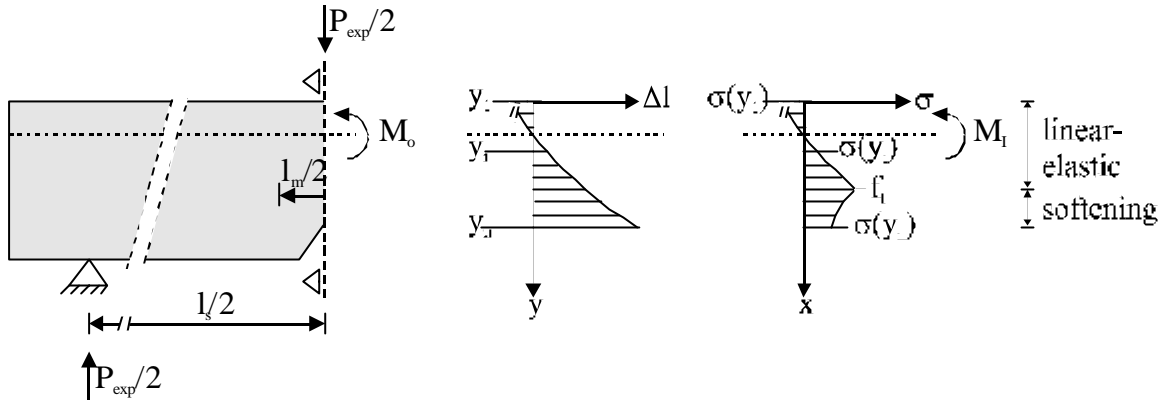
Figure 3. Distribution of displacements $\Delta l(y_i)$ and stresses $\sigma(y_i)$ in the mid-section of the concrete beam

To determine h_j the neutral axis y_s has to be known and hence is determined for each load step as follows. The section k where the neutral axis is crossing, i.e. $\Delta l = 0$, is found if $\Delta l_{k-1} < 0$ and $\Delta l_k > 0$. Then y_s yields

$$y_s = y_{k-1} + \Delta l_{k-1} \left(\frac{y_k - y_{k-1}}{\Delta l_k + \Delta l_{k-1}} \right) \qquad (4)$$

For a given load step T it is assumed that a small FPZ has developed in section n-1 because the tensile strength f_t was exceeded at y_n. In this case the equilibrium of forces, and hence moments, does not hold anymore if only linear elastic material behaviour is assumed in section n-1 because the material has already started to soften. Thus for this load step (Figure 4) the stress σ_n^T cannot be derived by linear-elastic assumptions but has to be described by the unknown softening function. As the moment equilibrium is still valid, $M_I = M_O$ may be solved for σ_n^T which then is the only unknown parameter of Equation 3. The corresponding displacement Δl_n^T is known from the experiment. Hence the first point of the softening function is found (Figure 4, load step T). Note, that a linear relation of the stress distribution between the known σ_{n-1}^T and the fitted σ_n^T is assumed.

For the following load step T+1 a further progression of the FPZ is assumed. This new load step is chosen such, that the recorded Δl_{n-1}^{T+1} is smaller than Δl_n^T determined in the foregoing load step. Hence, the corresponding stresses at the locations y_i for $1 \leq i \leq n-1$ can be found either by using the basic linear elastic equation stated in Equation 1 or by linear interpolation up to the last known σ_n^T as shown in Figure 4. As already shown for the previous load step now the actual displacement Δl_n^{T+1} at the low end (y_n) of the ligament height is measured but the corresponding σ_n^{T+1} is not known and hence has to be derived by equilibrium of moments as described above. This iterative process which is a combination of linear interpolation of stresses corresponding to measured displacements Δl_i and assuming equilibrium of forces carried out for the following load-steps $> T+1$ allows the step-wise determination of a multi-linear σ-Δl-curve.

As the elastic deformations in the regions of the cohesive crack (i.e. within the measuring length $l_m(i)$) are neglected in the proposed model so far the determined σ-Δl-relation corresponds to the demanded σ-w-curve.
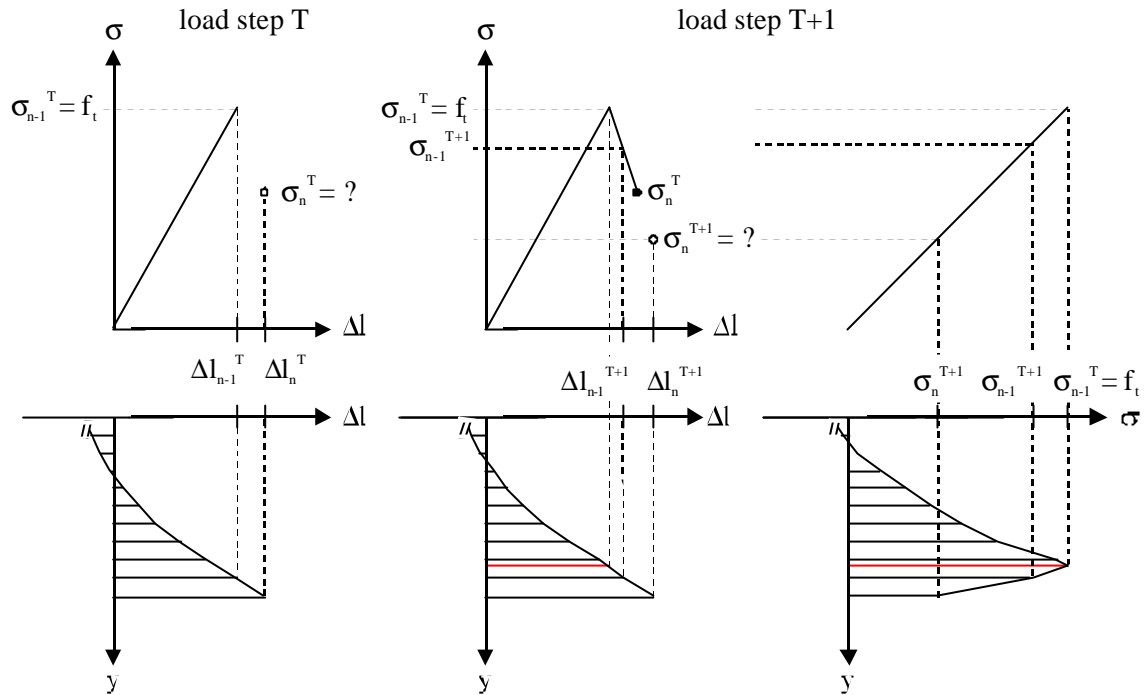
Figure 4. Scheme of the proposed model for load steps T and T+1

3 MATERIALS AND TEST METHODS

For the 3-point bending tests the reference mixture (PZ-0899-01) developed within the research project SFB 532 was used (Brameshuber & Brockmann 2001). See the mixture proportions in Table 1. At a testing age of 28 days the tensile strength of the fine grained concrete was determined as $f_t = 4.8\,N/mm^2$ and the Young´s modulus as $E = 33,000\,N/mm^2$ (Brockmann 2001).

Table 1. Composition of the reference mixture

Materials	kg/m³	
CEM I 52.5 (c)	490	
Fly ash (f)	175	
Silica fume (s)	35	
Water (w)	280	
w/b*		0.4
Plasticizer		1.0 % b. m. of binder
Siliceous fines	500	
Siliceous sand	714	

* w/b = w/(c + f + s)

3.1 3-point bending test

Specimens with the dimensions 40 x 40 x 240 mm³ and a notch depth a = 10 mm as shown in Figure 5 were used for the 3-point bending test. The specimens were cured for 24 hours at a temperature of 20 °C and 95 % R. H., and then until the testing age of 28 days a sealed storage at a temperature of 20 °C was chosen. Before testing the notches were sawn using a diamond saw with a thickness of 2 mm. The displacement controlled 3-point bending tests were carried out at a rate of 0.015 mm/min (Brameshuber & Brockmann 2002).

Additionally to the recording of the applied load P_{exp} and the resulting mid-section displacement u measured by LVDT the displacements $\Delta l(y_i)$ in x-direction at 11 measuring levels assembled over the ligament height of the specimen (Figure 6) were measured by a video extensometer.
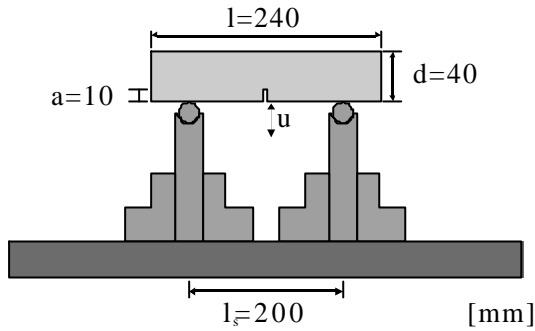
Figure 5. 3-point-bending test (notched) for fine grained concrete beam specimens

Figure 6. Measuring levels y_i

The system of the video extensometer exists of two main parts: a video camera and a video processing part, which is stored in a PC containing a frame-grabber interface card and appropriate software to analyse the data. The frame-grabber card converts the PAL video signal into an 8-bit digital format whilst simultaneously generating a 800 x 600 pixel image. The interface is capable of resolving the grey scale level of each pixel in 256 shades. The video extensometer operates directly as a non-contact displacement measurement device by determination of the change in distance between two markers of one measuring level, the so called targets. For the video measuring technique frames are set as shown in Figure 6 to locate these targets. Within these frames the targets produce rapid contrast changes in grey scale and thus allow the evaluation of the change in displacement ($\Delta l(y_i)$) between two targets by tracking the specific grey scale distributions in the x-direction in the sequence of the pictures taken (12.5 pictures per second). Also, the initial distance between two targets of one measuring level in x-direction (i.e. measuring length $l_m(y_i)$) is determined.

The measured displacements $\Delta l(y_i)$ imply elastic and plastic deformations. In the case of the linear-elastic state ($f(\varepsilon)$) only the elastic fraction is considered, while for the post-cracked state, i.e. when a FPZ is formed, only the plastic fraction is considered. In this case the elastic fraction of $\Delta l(y_i) = l_{elast.} + l_{plastic}$ is assumed to be negligible. For the experiment the measuring length l_m was chosen as $l_m = 25$ mm. Ideally the measuring length l_m should be as small as possible in order to minimise the fraction of elastic deformations on $\Delta l(y_i)$ for the softening region, but on the other hand a certain length of l_m has to be chosen to assure that the crack will be within the considered display window.
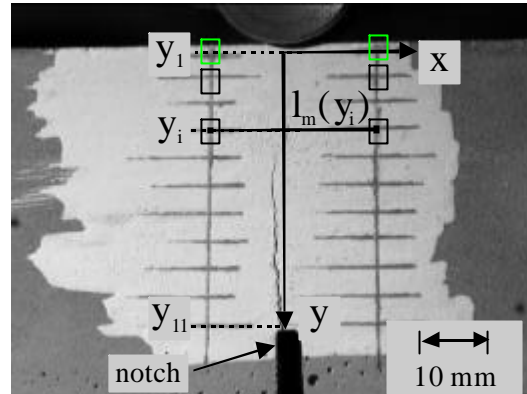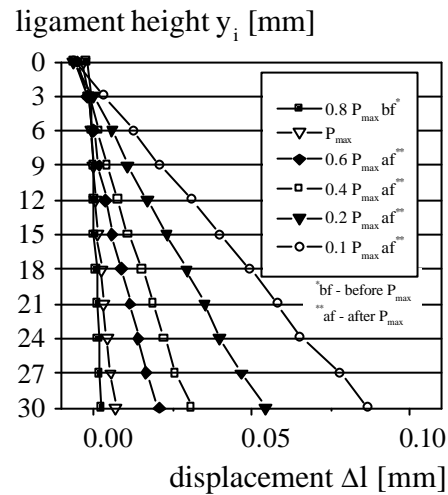

Figure 7. Distribution of $\Delta l(y_i)$ over ligament height y_n shown exemplary for different load steps

The maximum resolution of the video measuring technique depends on the field of view. In the presented case the accuracy is determined for a display window of about 40×60 mm², which is used for testing, as about $0.3\,\mu$m. An important aspect of this non-contact displacement measuring technique is that the set frames which mark the targets, i.e. the contrast changes within this frame, follow the movements of the concrete beam and hence of the targets in y-direction within the display window. Hence the actual displacements in x-direction are measured with a negligible impact due to the bending of the concrete beam.

4 RESULTS AND DISCUSSION

Figure 9 shows the complete load-displacement curve (P-u-curve) for a single test on a notched beam to exemplify typical results of the 3-point bending tests carried out within the presented study. Figure 7 shows the distribution of displacements Δl_i over the ligament height as measured by means of the video extensometer for selected load steps of the 3-point bending test. These load steps are expressed in relation to the normalised maximum load $P_{max} = 1$ and are distinguished as bf = before, and af = after the maximum load P_{max} is reached. The measured displacements Δl_i in the region of compression strain are in the upper section of the beam while they are in the other sections for strain or crack propagation. With ongoing bending of the concrete beam there is a shifting of the compression and tension zone and hence of the neutral axis y_s, and an increase of the measured displacements Δl_i.

As described before knowing such distribution of displacements Δl_i for each load step of the 3-point bending test, the corresponding stress distribution is determined. Subsequently by analysing the data with the proposed analytical model the softening function of the concrete is derived, where the tensile strength $f = 4.8\,N/mm^2$ and the Young´s modulus $E = 33,000\,N/mm^2$ are used as input parameters. In Figure 8 the so determined multi-linear σ-w-curves and the corresponding average-value curve are shown for the fine grained concrete derived from three different load-displacement curves one of which is shown in Figure 9.

The results presented in Figure 8 show that there already is a good agreement between the different deduced softening curves within a certain scatter. At the moment there is some limitation to the accuracy of the proposed calculation method due to impacts of the measurement technique as e.g.

- the accuracy of the video measurement technique which is about 0.3 µm
- in the case of development of a FPZ the measured displacements Δl_i include an elastic fraction while only the plastic fraction should be considered for the calculations
- the limitation of measuring levels y_i assembled over the ligament height of the concrete beam (here 11 measuring levels are chosen)
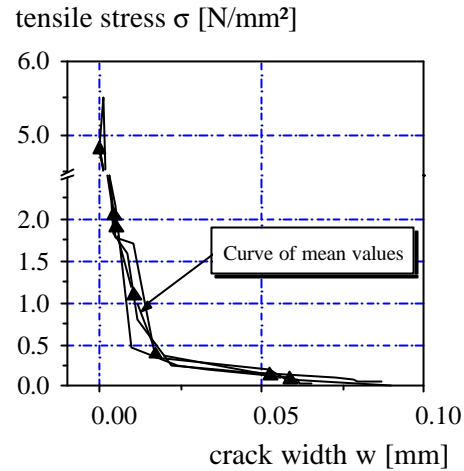
tensile stress σ [N/mm²]

Figure 8. σ-w-curves derived with proposed model – average value curve

As shown in Figure 8 for one σ-w-curve these inaccuracies may lead to an overestimation of the given tensile stress, which is used as input parameter to determine the load step T where $f_t = 4.8\,N/mm^2$ is exceeded. Still for the next load step T+1 the tensile stress σ_n^{T+1} which is determined by equilibrium of forces (moments) may exceed this given f_t due to inaccuracies of the measured displacements Δl_i. Furthermore an error propagation can be observed within the calculation procedure i.e. that for example at load step T the tensile stress σ_n is determined for a measured displacement Δl_n^T which means that for the following load steps $\geq$ T+1 errors possibly made at a foregoing load step will propagate through the calculation process, sum up or even potentially grow and lead to so called oscillation effects or non converging calculations. This phenomenon was observed in many cases for inverse calculations, see e.g. Tanaka (1998 & 2000). To minimise the influence of this error propagation certain features and regularisation methods can be implemented in the analytical solution routine developed during this study, e.g. averaging techniques and conditions of monotony.

To validate the proposed analytical model the results are compared with a multi-linear σ-w-relation which has been derived by numerical simulation of the 3-point bending test as described before using the FE program DIANA. Figure 9 shows the good agreement of the numerically

determined load-displacement curve with the experimental results of the 3-point bending test.
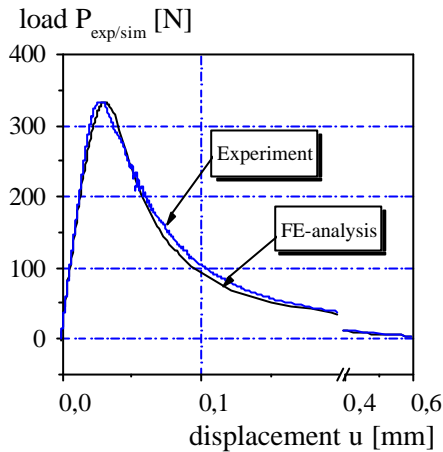
load $P_{exp/sim}$ [N]

Figure 9. Load-displacement curve of 3-point bending test – experiment and FE-analysis

The corresponding multi-linear σ-w-curve is shown in Figure 10. The quality of the so determined softening behaviour has been approved of by further numerical simulations of 3-point bending tests using the same fine grained concrete mixture but different specimen geometry and notch depths.
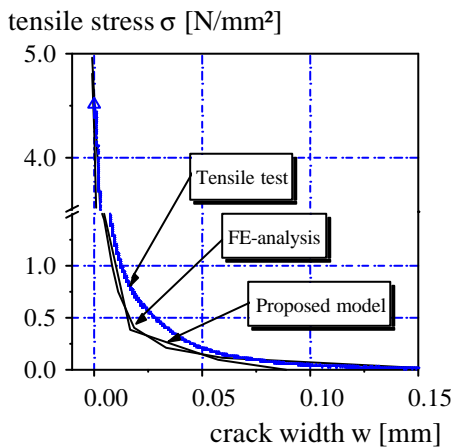
tensile stress σ [N/mm²]

Figure 10. σ-w curve derived by (i) uniaxial tensile test, (ii) FE-analysis, (iii) proposed model

Figure 10 shows the softening behaviour obtained by different approaches in comparison. The chosen methods are conventional FE-analysis, the proposed analytical model, and a stable uniaxial tensile test carried out within the research project SFB 532 where a σ-w-relation was derived according to Akita (2003). Comparing these curves approve that the results of the model show a quite acceptable agreement with known methods to derive the tension-softening behaviour of concrete.

5 SUMMARY

The advantages of the proposed model are a free determination of the σ-w-relation as there is no mathematical limitation for a multi-linear approach. Similar to most of the known models deriving the softening behaviour by either an analytical or numerical approach the proposed model uses the Young´s modulus E and the tensile strength f_t as input parameters and also is based on the results of a 3-point bending test. The displacement Δl_i over the ligament height of the concrete beam is determined directly with continuous video measurements during the experiment which allow the direct calculation of the corresponding stress distributions by means of equilibrium conditions. This is in contrast to known models, where the stress distributions within the ligament always are derived indirectly by complex analytical or numerical simulation of measured load-displacement curves of e.g. 3-point bending tests. In most of the cases FEM methods which require special knowledge and usually are time consuming are implemented in some way in the models.

The new approach offers a tool for an analytical derivation of the softening behaviour of concrete from 3-point bending tests without the need of using FEM. The experimental complexity might be higher implementing the video extensometer, but is compensated by the relatively simple and less time consuming analysis of the results.

To validate these presented first results the proposed model will be advanced and used for future investigations. Further mixtures at different testing ages and also the size effect will be considered.

6 PREFERENCES

Akita, H. et al. 2003. A practical method for uniaxial tension tests of concrete. *Materials and Structures* (RILEM) 36 (2003), Nr. 260, S. 365-371

Brameshuber, W. & Brockmann, T. 2001. Development and optimisation of cementitious matrices for textile reinforced elements. London : Concrete Society, 2001. *Proceedings of the 12th International Congress of the International Glassfibre Reinforced Concrete Association*, Dublin, 14-16 May 2001, S. 37-249

Brameshuber, W. & Brockmann, T. 2002. Betonformulierung - Methodik und Stoffgesetze. Aachen : Rheinisch-Westfälische Technische Hochschule, 2002. - In: SFB 532: Textilbewehrter Beton - Grundlagen für die Entwicklung einer neuartigen Technologie, Arbeits- und Ergebnisbericht 2. Hj '99-'01, 2.Hj.'02, S. 195-233

Brockmann, T. 2001. Anforderungen und Eigenschaften zementgebundener Feinbetone. – Development and optimisation of cementitious matrices for textile reinforced elements. Aachen: Lehrstuhl und Institut für Massivbau, 2001. *Textilbeton. 1. Fachkolloquium der Sonderforschungsbereiche 528 und 532, 15. und 16. Februar 2001 in Aachen (Hegger, J. (Hrsg.))*, S. 82-98

Chuang, T.J. & Mai, Y.W. 1989. Flexural behaviour of strain-softening solids. *International Journal of Solids Structures* 25 (1989), Nr. 12, S. 1427-1443

Duda, H. 1991. Bruchmechanisches Verhalten von Beton unter monotoner und zyklischer Zugbeanspruchung. *Schriftenreihe des Deutschen Ausschusses für Stahlbeton*, Heft 419, Beuth Verlag GmbH, Berlin, 1991

Gustafsson, P.J. 1985. *Fracture mechanics studies of non-yielding materials like concrete.* TVBM-1007, Division of Building Materials, University of Lund, 1985

Hillerborg, A. 1983. Analysis of one single crack. Amsterdam : Elsevier Science Publishers, 1983. - *Fracture Mechanics of Concrete* (Wittmann, F.H.(Ed.)), S. 43-74 (Developments in Civil Engineering 7)

Hillerborg, A. 1985. Dimensionless presentation and sensitivity analysis in fracture mechanics. Amsterdam : Elsevier, 1986. - *Fracture Toughness and Fracture Energy of Concrete : Proceedings of the International Conference on Fracture Mechanics of Concrete*, CH-Lausanne, 1985, (Wittmann, F.H. (Ed.)), S. 413-421

Petersson, P.-E. 1981. *Crack growth and development of fracture zones in plain concrete and similar materials.* Lund, Sweden : Division of Building Materials, University of Lund, 1981. - Report No. TVBM-1006, *Dissertation*

Planas, J. & Elices, M. 1987. Asymptotic analysis of the development of a cohesive crack zone in Mode I loading, for arbitrary softening curves. Houston, Tx : SEM; RILEM, 1987. - *Proceedings of the SEM-RILEM International Conference on Fracture of Concrete and Rock*, June 17-19, 1987, Houston Texas, (Shah, S.P. ; Swartz, St.E. (Ed.)), S. 384

Sundara, K.T. et al. 1998. Analytical study of fictitious crack propagation in concrete beams using a bilinear σ-w relation. Freiburg : AEDIFICATIO, 1998. - *Proceedings of the Third International Conference on "Fracture Mechanics of Concrete Structures" (FRAMCOS-3)*, Gifu, Japan, 12-16 Oct. 1998, (Mihashi, H.; Rokugo, K. (Ed.)), Vol. 1, S. 315-324

Tanaka, M. & Dulikravich. 1998. ISIP 1998: Inverse Problems in Engineering Mechanics. *International Symposium on Inverse Problems in Engineering Mechanics 1998* (ISIP '98), Nagano, Japan. Amsterdam u.a. : Elsevier, 1998

Tanaka, M. & Dulikravich. 2000. ISIP 2000: Inverse Problems in Engineering Mechanics II. *International Symposium on Inverse Problems in Engineering Mechanics 2000* (ISIP 2000), Nagano, Japan. Amsterdam u.a. : Elsevier, 2000

Uchida, Y. 2001. Determination of tension softening diagram of concrete (Draft, 2001), *Japan Concrete Institute*, JCI-TC992 Test Method for Fracture Property of Concrete

Ulfkjaer, J.P. et al. 1990. Analytical model for complete moment-rotation curves of concrete beams in bending. Warley West Midland, UK : EMAS, 1990. *Proceedings 8th European Conference on Fracture Behaviour and Design of Materials and Structures*, Turin, 1.-5.10.1990, (Firrao, D. (Ed)), Vol. 2, S. 612-617

Ulfkjaer, J.P. et al. 1995. Analytical model for fictitious crack propagation in concrete beams. *Journal of Engineering Mechanics* 121 (1995), Nr. 1, S. 7-14