Designing strengthening of structures given the uncertainty of fracture mechanics

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ABSTRACT: Strengthening of structures with carbon fibers is a new method to increase the lifetime of an existing structure. In some countries guidelines in this field are available. Numerical simulations can simulate the real behaviour of the failure mechanism of the strengthened structures. Coupling a probabilistic tool to the numerical calculation may decrease the discussion about uncertainties. The influence factor of each stochastic parameter on the reliability index shows the designer the points of emphasis.

Keywords: concrete, cracks, strengthening, nonlinear, probabilistic

1 INTRODUCTION

Strengthening of existing structures is not a new aspect in civil engineering. many structures have had a retrofit process during the lifetime of the structure. Most of these structures got extra steel plates to increase the strength of the structure.
To fix the steel plates to the structure requires a lot of time. Furthermore it reduces the headroom in the structures involved.
Strengthening with carbon fibers is a new method to strength structures. The strength of this carbon fiber is very high in comparison the steel plates, the dead weight of the material is low and they are easy to fix to the structure. The bonding material is epoxy. It is possible to prestress the carbon fiber initially in longitudinal direction.
More and more countries have seen this method and made some guidelines for strengthening structures based on the carbon fibers. Many years are needed to formalize these guidelines into the checking codes like Euro Code and Model Code.

Meanwhile some strengthened structures are being monitored. The results can be compared with the guidelines. Additional experiments and numerical simulations can be made to get a better profit of the method and the new material.

Another result of research of the last years is the possibility to decrease the elapse time of a probabilistic analysis. The most accurate analysis in this field is the well-known Monte Carlo method. An alternative method is the Directional Sampling method, which can be seen as an almost level III analysis. Adding an adaptive response surface to this directional sampling method gives the designer a new tool, which enlarges the possibilities to use new materials with some uncertain parameters.
This tool is firstly developed as a tool to calculate the reliability index of a structure. It is also possible to get the influence factors of the stochast parameters on that reliability index, which allows the designer to get a better understanding of the general material parameters.
Combining this tool with the uncertainties of fracture mechanics and the combination of strengthening with carbon fibers will be explained in this paper.

2 RELIABILITY METHODS

Safety factors like the partial safety factors in the different checking codes are well known by the designers of structures. The relation of the partial safety factor of the material and strength can be illustrated in figure 1.

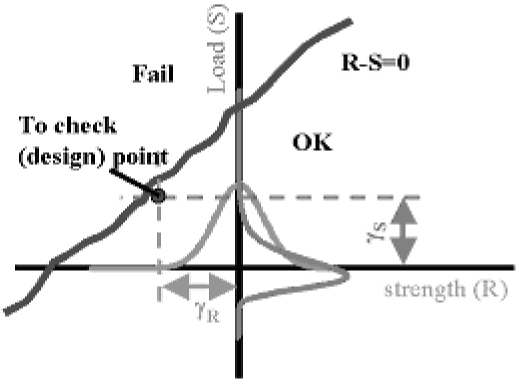

Figure 1: Relation strength-load

The strength-load relation in the above figure is a level I method and the base for the traditional design method. There is a minimum of overlap in the strength and load figure what is demonstrated in figure 2.

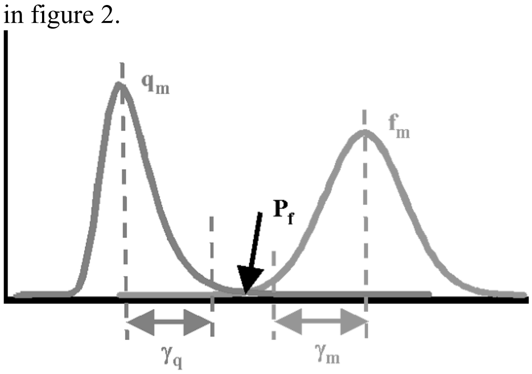

Figure 2: partial safety factors derived from the probability of failure

At a complex structure the designer needs a more accurate value for the reliability index. In that situation a level II method occurs, like the first order reliability method (FORM) or the second order reliability method (SORM). These methods results in better results but may be not accurate enough in very complex structures. Complex means in this sense nonlinear behaviour, time dependent behaviour and phased dependent results. Then the Monte Carlo method or the Directional Sampling method is the solution. However, there are a lot of calculations needed, so the elapse time needed for all these calculations will discourage the designer to use this method. Nevertheless the level III methods are the most accurate, simple understandable and never wrong when enough samples are used. This can be pointed out in the next figure.

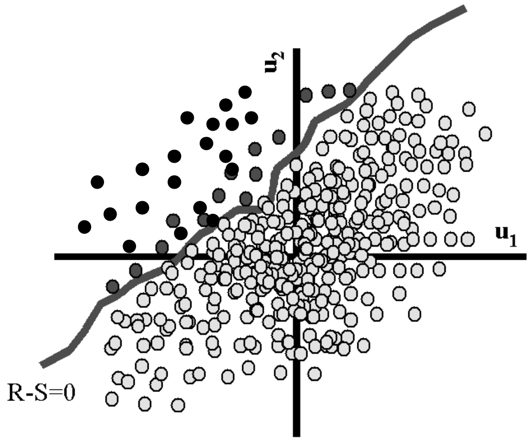

Figure 3 Monte Carlo calculations

The figure shows the failed design points with a black dot. The total number of failed design points is N_F (in the figure 3 N_F=35). The total number of the simulations is N, which must be more then 100.000 simulations to get a realistic reliability index for typical use in structural safety.
The probability of failure $P_f = N_F / N$. The reliability index is defined as: The reliability index β is defined as:

$$\beta = \Phi^{-1}(P_f)$$

where Φ is the normal distribution function

To reduce the elapse time of a level III method has been researched on many universities. On Delft University of Technology, Waarts developed a faster method suitable for complex structures. This method is called a Directional Adaptive Response surface Sampling (DARS) method. A short description will be made in the next section.

3. DARS METHOD

At Delft University of Technology, in cooperation with TNO Building Construction Research and the Dutch Ministry of Transport, Public Works and Water Management, research has been carried out to compute the structural reliability using a

combination of finite element analysis (FEA) and probabilistic methods.
The structural behaviour of a complex structure is often calculated using a Finite Element Analysis (FEA). Stresses and deformations of the structure can be computed given the (deterministic) parameters of loads, geometry and material behavior.
Structural codes require a certain level of structural reliability. The Dutch Building code, for example, demands a maximum probability of failure of 10^{-4} within a given reference period (lifetime of the structure). This probability of failure is ideally translated into partial safety factors by which variables like strength and load have to be divided or multiplied to find the so-called design values. These design values are to be used as input for a Finite Element Analysis. The outcome of the calculations is compared with the limit states (for example collapse or maximum deformation). The structure is supposed to have met the reliability requirements when the limit states are not exceeded. Reality is different. First of all the method using partial safety factors makes it only plausible that the reliability requirements are met. There is, however, no certainty. A second aspect is that safety factors are often based on experience only.
A link with the required reliability on a theoretical basis often does not exist. Third aspect is the system behavior of structures. The safety factors are often derived for components of the structure for instance single sheet piles, anchors or single failure surfaces. A structure as a whole behaves like a system of these components. As a result, depending on the system under consideration, the structure can be more or less reliable than its components. Given these problems, it would be useful to have a method to calculate an accurate (system) probability of failure of the total structure at once. Standard reliability methods compute the probability of failure given a limit state and stochastic parameters. Limit states might be for instance exceedance of yield stress in a structural member, exceedance of maximum deformation or global collapse. Well-known methods for computing the reliability are Monte Carlo simulation (MC) (Rubinstein 1981)and the First Order Reliability Method (FORM) (Hasover & Lind 1974). In this paper an unusual method is applied: an adaptive method based on Directional Sampling [4] (Bjerager 1988). For large and complex structures it is almost impossible to provide an explicit limit state function. Points of the limit state function can however be calculated using the Finite Element Analysis (FEA). Combining reliability methods and Finite Element Analysis is often referred to as Finite Element Reliability Methods (FERM). Instead of computing the structural behavior (with FEA) in terms of deformations and stresses, the behavior is computed in terms of probability of failure and uncertainty contributions. In this way the basic demands of the codes are met, i.e. meet the required probability of failure.
The problem arising is that the mentioned standard reliability methods are traditionally used for problems with only a few random variables using little time to evaluate the limit state function. In combination with FEA, the opposite occurs as there are many random variables and evaluating the limit state function takes much computational effort. The standard reliability methods in combination with FEA lead to a computational effort that is just too much. To speed up the computations, research at The Delft University of Technology has lead to the introduction of the so-called “Directional Adaptive Response surface Sampling” (DARS)(Waarts 2000). In short the improvement to the standard directional sampling lies in the use of FE for the important directions and a response surface for less important directions. In practice this means that after the response surface is constructed, only a few FE computations have to be carried out.
In the DARS procedure, for the construction of the response surface all variables are varied individually and increased or decreased until failure. A FE model with n stochastic variables gives 2n (directional) samples in the principal directions. Consequently a quadratic response surface is fitted to these results. Following this starting procedure the random directional sampling takes place. The response surface is used in case of a large distance from the origin to the response surface. FE computations are used to calculate the real distance in case of a small distance from the origin to the response surface. In that case the response surface is updated (adapted).
Influence factors give insight on the importance of stochastic variables on the limit state. After finishing the directional sampling procedure, the influence factors α are computed by means of a FORM analysis on the response surface.
In this research project the probabilistic method is implemented in an existing FE code, namely Diana (Diana 1998), release 7 of 1998.

4. EXAMPLE STRENGTHENING GIRDER

The example of the cracked girder is originally published in (Klamer2003). Klamer has simulated two delamination mechanism of failure of strengthening a girder. Results of experiments are used which have been carried out in the laboratory of the Technical University of Eindhoven, the Netherlands. Only the delamination mechanism of the shear crack in the girder will be discussed in this paper and is described in (CUR2001).

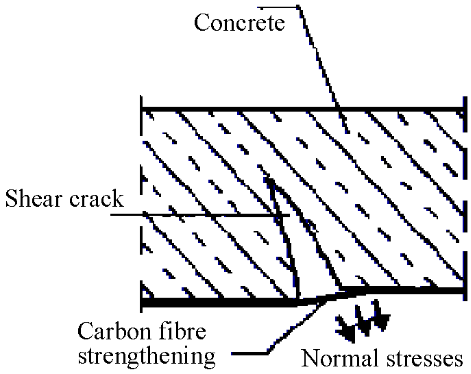

Figure 4: Delamination mechanism of the shear crack

4.1 Description of the example

The numerical FE model is a plane stress model of a half-length beam (because of symmetry considerations) including some reinforcements and stirrups. The length of the half beam is 1900 mm, the height is 450 mm and the width 200 mm.
The support point is situated 100 mm from the end of the girder.
The middle of the girder is supposed a symmetry axis. The total model is a 4-point bending example; so the load of the girder is a nodal load 650 mm with an initial value of 1 kN, situated from the symmetry axis.
De upper reinforcement counts 2ϕ8 (=101 mm^2) and the lower reinforcement 4ϕ12 (=452 mm^2).
The cover of the concrete is 33 mm.

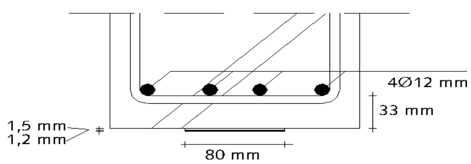

Figure 5: Lower reinforcement positions including the position of the carbon fibers

The stirrups have a diameter of 8 mm and a head to head distance of 100 mm over the total length of the girder. The thickness of the carbon fibers is 1.2 mm and the width 80 mm. The start of the carbon fibers is situated 100 mm form the support point in the direction of the middle of the girder. The thickness of the modeled epoxy between the carbon fibers and the concrete girder is 0.1 mm. The material properties are assembled in table 1.

Table 1: Material properties concrete & reinforcement & carbon & epoxy

Material	Young's modulus [N/mm^2	Density [Kg/m$^{3]}$	Poisson ratio [-]
Concrete	30600	2400	0.2
Reinforcement	210000	7800	0.3
Carbon fibers	165000	1500	0.35
Epoxy	12800	1960	0.3

In a first stage a nonlinear crack analyses of this girder is carried out without the strengthening of the carbon fibers to get the positions of the bending and shear cracks. The additional material properties of all 4 materials are given in table 2.

Table 2: Additional material properties

Material	Stresses	
	Compression [N/mm^2]	Tension [N/mm^2]
Concrete	1.74	3.8
Reinforcement	560	560
Carbon fibers	-	2800
Epoxy	-	3.8

The constant shear reduction factor after cracking of the concrete is set to β=0.20. In this research a linear softening behaviour for the concrete is included. The softening parameters of the concrete are presented in table 3 as extension to the properties given in table 1 and 2.

Table 3: Softening parameters concrete

Softening parameter	Value	Unit
Cohesion	15.4	N/mm^2
Friction angle	30°	-
Dilatancy angle	30°	-
Tension stress	3.8	N/mm^2
Ultimate crack strain	2.105x10^{-3}	-
Shear reduction factor	0.20	-

The concrete is modeled as plane stress elements and the upper and lower reinforcements (longitudinal as well as the stirrups) as embedded bars. The carbon fibers are modeled as plane stress elements and the epoxy as an interface element. The normal and shear stiffness of the interface element should be transferred from the young's modulus and the thickness of the epoxy.

4.2 Smeared crack simulation

A smooth load-displacement diagram of the middle of the girder can be achieved with the smeared crack approach. Figure 6 shows three results: the experimental, the numerical and the analytical result. In this stage we can see a good agreement of the different simulations.

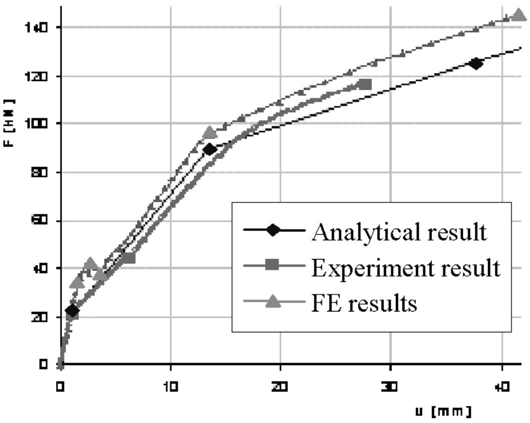

Figure 6: Load displacement diagram mid girder node

In order to simulate the shear crack delamination mechanism we need the crack pattern of the girder to get a realistic vertical displacement difference on both sides of the shear crack.
The expectation of the maximum load from the Dutch guidelines is between 110 and 120 kN. Therefore the crack pattern is first based on the 100 kN level. The result is given figure 7, for two circumstances, the experimental and numerical situation.

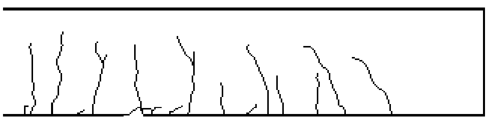

Figure 7: Crack pattern experimental situation

In figure 8 shows some shear cracks growing, as the direction of the cracks are under an angle of 45 degrees. A better view gives the figure 9 with a load of 135 kN. The developed shear cracks are lying under an angle of 40-45 degrees.
By simulating the shear cracks in a second stage by discrete cracks the girder can get a different vertical displacement near the tip of the shear crack just above the epoxy layer.

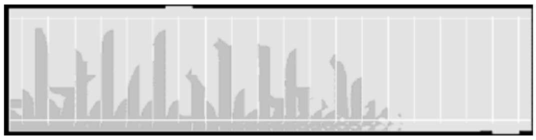

Figure. 8: Crack pattern numerical simulation with a load of 110 kN

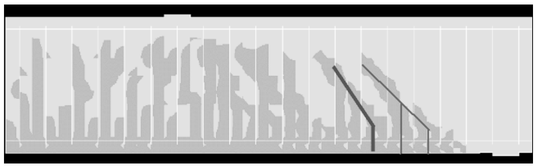

Figure. 9 Crack pattern numerical simulation with a load of 135 kN

4.3 Mixed smeared-discrete crack analysis

In the model with the shear cracks, modeled as discrete cracks the same load displacement diagram for the mid girder node can be observed (figure 6).
Intermediate conclusion is that the concrete behaviour of both models is the similar.
Now the nonlinear material behaviour of the epoxy can be added to the already mentioned material properties. It is assumed that if the tension normal stress in the epoxy of 3.8 N/mm^2 has been reached the normal stress falls down brittle to the value zero. As the epoxy is modeled as an interface element it is a stress displacement diagram.
The experiments show a very fast delamination by increasing the load to the girder. Therefore the element density near the shear crack is very high.

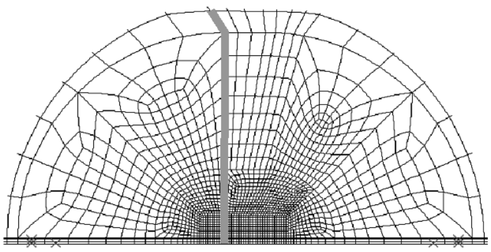

Figure 10: Detail of the shear crack tip

In figure 10 the element sides near the shear crack have a length of 1x1 mm.
The result of this nonlinear smeared-discrete crack analysis at different load steps in the strain distribution of the vertical shear crack are shown in figure 11. The figure shows at the position of the rein-forcement bar a clear drawback in the strain.

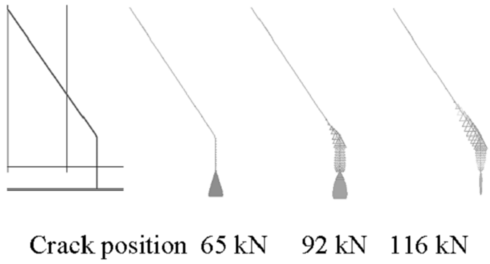

Figure 11: Strain distribution vertical shear crack

Figure 12: Draw back normal stress at a load increase

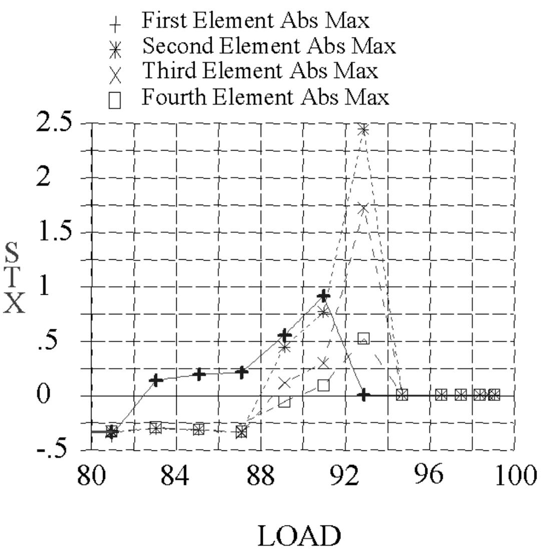

Figure 12 shows the increase of normal stress in the first four element starting from the crack. carbon fibers. The first element reaches the ulitimate stress at a loading equal to 92 kN. The other three elements delaminate in the next load step. The stress in these elements decrease to zero. In the analysis the stress does not reach the ultimate stress. In fact the load steps should have been smaller.

5. PROBABILISTIC APPROACH

The above-mentioned crack analyses types are consist of uncertainties. The typical material properties are based on a partial safety factor approach (level I), which means a characteristic value of the property parameter and partial safety factor.
Extra uncertainties originate form the smeared crack approach. Additional uncertainties originate from material parameters of the smeared-discrete crack approach. Also the general material parameters of the concrete, reinforcement, carbon fibers and the epoxy can be point of discussion.
The distribution type, the mean value and the standard deviation of the most common material parameters are listed in several engineering books, like 'Chances in civil engineering' (CUR1997).
The common used distribution types are normal, lognormal, truncated normal, shifted lognormal, exponential, Gumbel and Weibul.
Assuming that the linear and nonlinear material properties can be seen as deterministic, only the epoxy behaviour or the surrounding concrete will be treated here as a stochastic variable. This means that we get two main stochastic variables, the normal stiffness E of the epoxy and the young's modulus of the first layer of the concrete D just above the epoxy layer. Both materials are used in this example with a lognormal distribution with a relatively small standard deviation (Coefficient of variation equals 0.01), see table 4.
In this case there is no direct correlation between the two stochastic variables.
The limit state of this structure will be the actual normal strain at delamination of the carbon fibers from the concrete girder. The partial safety factor on the ultimate strain is assumed $\gamma_\varepsilon = 1.4$.

Table 4: Stochastic variables

Var	Mean	St.Dev.	Distr. type	unit
E	$3.06\ 10^4$	$3.06\ 10^2$	Logn.	N/mm^2
D	$1.28\ 10^8$	$1.28\ 10^6$	Logn.	N/mm^2

The probabilistic analysis has been performed using the DARS method. After a 73 samples the probabilistic analyses comes to a nice convergence (based on the variation of the reliability index).

Figure 13 shows this convergence process over the samples and shows a final reliability index $\beta = 3.09$

(V(β) = 0.10). The reliability index β is too small in relation to the structural safety requirements in national codes and informative annex of the Eurocode. (even with the very low standard deviation of the used stochastic variables) The reliability index β should be β=3.6, corresponding to a probability failure of 10^{-4}.

This means in this case that the allowable force on the girder must be decrease to a level in which the reliability index β can reach the value of 3.6.

The influence factors of the 2 stochastic variables are in the ratio of 1:1, which means that there is no main influence of stochastic parameters. With the choice of more stochastic parameters this might change.

In this paper it was only the idea to show the combination of the probabilistic analyses and fracture mechanics.

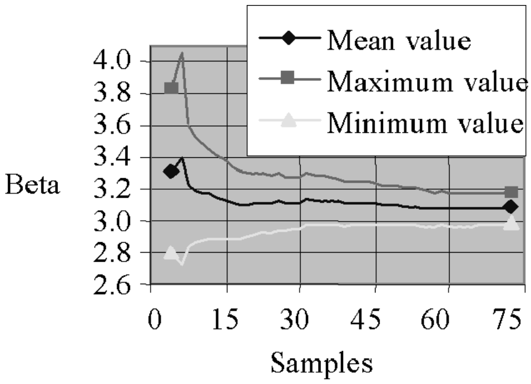

Figure. 13 Convergence process probabilistic analyses

6. CONCLUSIONS

The following conclusions can be made:

1. The verification example gives a good indication in the nonlinear analyses field
2. The step-by-step approach can calculate delamination of the strengthening of a concrete girder and gives an acceptable underlining of the existing guidelines
3. The introduction of the probabilistic analysis with this gives the designer a reliability index and the influence ratios of the chosen uncertainties.
4. Rather new and preliminary tests materials can be used earlier in construction design even if there are no specific guidelines of those new materials.

7. ACKNOWLEDGEMENTS

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