Use of R-Curves for Characterization of Toughening in Fiber Reinforced Concrete

Barzin Mobasher
Civil & Env. Eng., Arizona State University, Tempe, Arizona, USA

Alva Peled
Structural Engineering Dept., Ben Gurion University, Beer Sheva, Israel

Abstract: The role of fibers on the tensile stress strain response and the fracture toughness of cement based composites are studied by means of a cohesive crack approach. A model is proposed to include the interfacial debonding and pullout of fibers as closing pressure distribution which is expressed as tensile stress crack-width response. R-Curves are then used to account for increased energy dissipation and simulate the crack growth in the matrix response subjected to the closing pressure. The closing pressure, characterized as an exponentially decaying stress crack-width relationship, is integrated to compute the amount of toughening at incremental crack growth lengths. The strain energy release rate of a three point bending specimen interface are equated to the R-Curve, and solved for the critical crack extension. The R-curves are further used to compute the compliance and the load deformation response. The toughening component is due to the closing pressure of fibers which depends on the matrix crack opening. A parametric study of the effects of model parameters on the crack growth is conducted. The present model is also compared with experimental data on glass fiber composites.

Keywords: R-Curves, fiber reinforced concrete, closing pressure, stress-crack width relationship, toughness.

Introduction

Use of fibers in concrete is intended to utilize the strength and stiffness of fibers in reinforcing the brittle matrix. Reinforcing ordinary concrete materials with short randomly distributed brittle fibers such as glass has been attempted for more than 20 years [1][2]. Such brittle matrix-brittle fiber materials are superior to other FRC (Fiber Reinforced Concrete) materials for several reasons. In comparison to steel fibers, the small diameter of the individual glass fibers ensures a better and more uniform dispersion. In addition, the high surface area and relatively small size of glass fiber bundles offers significant distribution capability and crack bridging potential as compared to steel fibers. The glass fibers are randomly distributed offering efficiency in load transfer. Furthermore, the bond strength of the glass fiber is far superior to the polypropylene fibers, thus increasing the efficiency of fiber length so that there is limited debonding and fiber pullout. Finally, due to the highly compliant nature of the glass fiber bundles which bridge the matrix cracks at a random orientation, they are able to orient so as to carry the load across the crack faces.

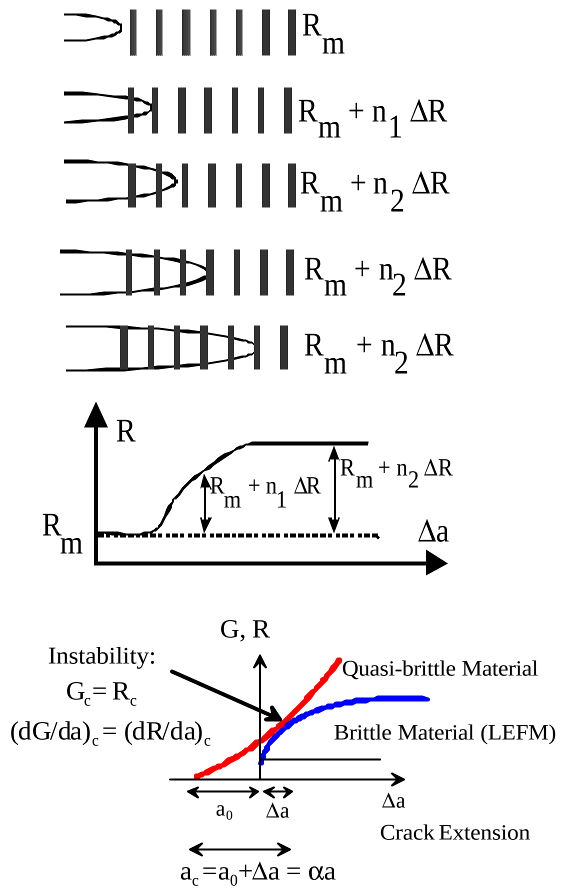


Figure 1. The schematic model of the fiber toughening resulting in R-Curve behavior, and b) the model used for use of R-Curves for crack growth and instability criterion.

Analytical Simulation of Toughening

The process of toughening can be modeled by means of *R*-curves as shown in Figure 1. *R* represents the increased resistance of the material from the base level R_m due to the growth of the crack and increases with incremental crack growth "Δa" due to the presence of bridging. It is observed that as we load a material containing a small flaw, the flaw will begin to grow (under an increasing applied stress intensity factor) until the process zone is fully developed. The crack in the process zone has a different shape because of the forces of the bridging fibers. According to a simplified approach in Figure 1a the amount of toughening due to each intersected fiber may be accounted as $n_1 \Delta R$. Once the zone has developed fully, then the whole crack may move forward with the process zone, remaining at a constant size, at an energy level of $R_m + n_2 \Delta R$. By controlling the microstructure and properties of the material to result in such an *R*-curve behavior, we can ensure that cracks are stable over certain limits of flaw size. This mechanism is thus able to explain why for many cement based composites, reduction of inter-fiber spacing results in formation and growth of significant cracking without causing catastrophic fracture.

To address the toughening due to the crack bridging of fibers at the local level, a stress intensity approach is used. The bridging force, expressed in terms of the stress intensity factor, works to reduce the applied stress intensity factor. The fiber pullout mechanism and the closing pressure are the primary parameter considered. The stress intensity factors are directly obtained from the stresses that are required to pull the fiber out of the matrix, and expressed as:

$$K_{IF} = \int_0^a P^*(U)\, g(1, \frac{x}{a})\, dx \qquad (1)$$

$$COD_f = \frac{2}{E'} \int_{a_0}^{a} \int_{a_f}^{a} P^*(U) K_{IP} \frac{\partial K_{IF}}{\partial F} d\xi\, d\eta \qquad (2)$$

Where P(u) represents the force carried by a bridging fibers as a function of crack opening. The fiber is located at distance "*x*" from the tip of a crack length "*a*". Parameter *g(1,x/a)* represents the green's function representing the stress intensity due to a unit load. The parameters obtained from equations 1 and 2 represent the contribution of a single fiber, and the collected terms of the contribution are computed in two alternative approaches. Using the Green's function Approach the contribution of a closing pressure profile is integrated over the crack length and expressed as:

$$\Delta K_b(l_b) = \int_0^{l_b} \sigma_b(x) G(a,x) dx \qquad (3)$$

where, $G(a,x)$ = green's function, "a" represents the crack length, "l_b" bridging zone length, and "σ_b" bridging stress. Alternatively, using the Potential Energy Approach, one can express it based on the crack opening profile, $u(x)$:

$$\Delta R_b = 2\int_0^{l_b} \sigma_b(u)\left(\frac{du}{dx}\right)dx \quad (4)$$

The criteria for the cracking can be defined in terms of energy balance. In an R–Curve formulation, the notch sensitivity represented as the extent of stable crack growth Δa can be normalized with respect to the specimen width:

$$\alpha = \frac{a_0 + \Delta a}{a_0} \quad (5)$$

A condition of $\alpha=1$ represents the LEFM conditions, whereas $\alpha>1$ represents the quasi-brittle materials. The energy balance criterion requires that the strain energy release rate is equal to the fracture resistance of the material at any stage of stable crack growth while the condition for crack instability is defined as the rate of strain energy release rate exceeding the rate of increasing the toughness of the materials as shown in Figure 1.b. The stable crack growth and the onset of instability are defines as equations 6 and 7.

$$R(a)=G(a)=\frac{(K_m+K_{IF})^2}{E'}, \frac{\P R}{\P a}=\frac{\P G}{\P a} \quad (6)$$

$$R(a_c)=G(a_c) \quad \frac{\partial R}{\partial a}>\frac{\partial G}{\partial a}>0 \ @\ a=a_c \quad (7)$$

Failure Conditions, Stable and Unstable Crack Growth

A closed form solution procedure for modeling of the R-curve for quasi-brittle materials has been proposed by Ouyang, Mobasher, and Shah [3]. Using this approach the R-curve representing the fracture resistance of a material is defined by two parameters "α" and "β" representing the "Δa_c" and "R". These parameters can be obtained by fitting the load-CMOD or deflection plots and expressed as:

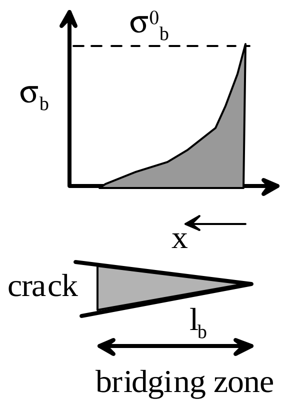
$$R(a)=ß\left[1-\frac{d_2}{d_1}\left(\frac{a a_0-a_0}{a-a_0}\right)^{d_2-d_1}\right]\left[a-a_0\right]^{d_2} \quad (8)$$

$$d_i = \frac{1}{2}+\frac{a-1}{a} \pm \sqrt{\frac{1}{4}+\left(\frac{a-1}{a}\right)^2}, \quad i=1,2$$

Since R varies with the crack length, it can not be viewed as a single valued function, and the extension of the stable cracking is determined entirely by the geometry and loading. The procedure used in the present approach is based on calculating R parameters corresponding to the load-deformation history of the specimen as suggested by Mobasher, Ouyang and Shah [4]. The procedure is based on calculation of the fit parameters which describe the effect of fibers in the context of the resistance curve, R and also the amount of critical crack length Δa_c.

A parameter that is needed in the formulation is a proper representation of the closing pressure profile. In the present study, the model proposed by Sakai and Suzuki [5] is used. Similar in nature to Foote, Mai, Cotterell, [6] Model, this approach represents the stress across the crack ligament as a function of both the crack opening and also the crack ligament length. By assuming various forms of the functional relationship, both models of increasing and decreasing stress as a function of crack opening can be represented using similar parameters. For example, the responses for both stress crack opening and crack opening vs. position can be expressed as equations 10 respectively and shown in Figure 2. Parameter l_b in this case is equivalent to the stable crack growth length Δa_c.

The proposed procedure can be used to estimate bridging tractions from the R-curve behavior using an inverse problem. The first step is to utilize a stress crack width relationship model. In the current approach we assume a generalized profile of bridging tractions (model assumption) and use Tension σ-w curve as failure criteria. The stress crack width relationship is used as the material property, and bridging tractions are derived from the stress crack width approach. In addition, using the same crack growth parameters, the magnitude of toughening in the form of stress crack opening integrals in the process zone are then calculated are converted to to elastically equivalent fracture parameters.



$$u_b(x) = u_b^0 \left(\frac{x}{l_b}\right)^n$$

$$\sigma_b = \sigma_b^0 \left[\left(\frac{x}{l_b} \right)^q \right]^{n_d} \qquad (9)$$

Figure 2. The closing pressure vs. crack opening distribution according to Sakai-Suzuki Model.

Using standard nonlinear LEFM approaches, the equivalent material parameters in terms of , G_f, and u, or K_{Ic}, or $CTOD_c$ are defined. These fracture parameters are used to obtain the parameters of the R-curve. The solution algorithm is defined by assuming the criteria for failure in terms of two parameters namely, the stable crack growth length, "a_c" and the scaling parameter in the R-Curve defined earlier as β. Using these two parameters the energy release required for growth, R(a) is constructed. At this point the crack is incrementally extended and R(a) and G are calculated and used in the equilibrium equation to solve for the parameters of the R-Curve. Newton-Raphson Algorithm for nonlinear equation solution is used. Once the parameters of the R-curve are calculated, it can be constructed, and the Load-deformation response is obtained by incrementing crack length "a", setting R(a) = G(a) to solve for P as a function of crack growth. The load deformation response is computed from the theoretically based R-curve formulation using a compliance approach. Once the load is obtained, the crack length "a" is used to get the compliance and deformation is computed. This procedure is then subjected to parameter optimization through inverse solution to fit the experimental load deformation response in terms of the parameters of stress crack width relationship. As an added extra step, one can calculate and correlate closing pressure-crack length to energy in the process zone. Parameter Optimization through inverse solution can also be accomplished by fitting the experimental data with model estimation.

Figures 3, 4, and 5 in addition to the data presented in Table 1 show the parametric study of the effect of tensile strength response on the R-curve and the resulting load deformation response. A prismatic specimen 101.6x101.6x304.2 mm in dimensions and an initial notch length of a_0= 12.75 mm was used. The material parameters for the stress strain response are listed in Table 1 in addition to constant variables used as E = 25000 MPa, maximum width of a crack opening with traction, $CTOD_c$= u = 0.06 mm. Parameters n = 0.16, n_i = 1.5 and q = 0.5 were the power coefficients of the stress-softening and stress crack length ligament response. In addition a constant of u_p = 0.004 mm was used as the displacement corresponding to the maximum stress.

F't, MPa	β_1	α_c	R, Nmm	G_f Nmm	ΔK
3	0.013	3.67	0.067	0.067	31.8
4	0.020	3.427	0.089	0.089	37.2
5	0.028	3.221	0.111	0.111	41.8
6	0.037	3.057	0.133	0.133	46

$$\Delta K_b(l_b) = \int_0^{l_b} G(a,x)\sigma_b(x)dx$$

Table 1- Parameters of the effect of tensile strength on the load deformation response. Note that as the tensile strength, and thus the

stress strain response of the specimen in the post peak region is increased, it results in an increase in the plateau value of the R-Curve and the flexural load deformation response as shown in Figure 4. According to this simulation the strength of a beam in flexure is as much as 57% with a significantly higher energy dissipation in the post peak response of the flexural curve.

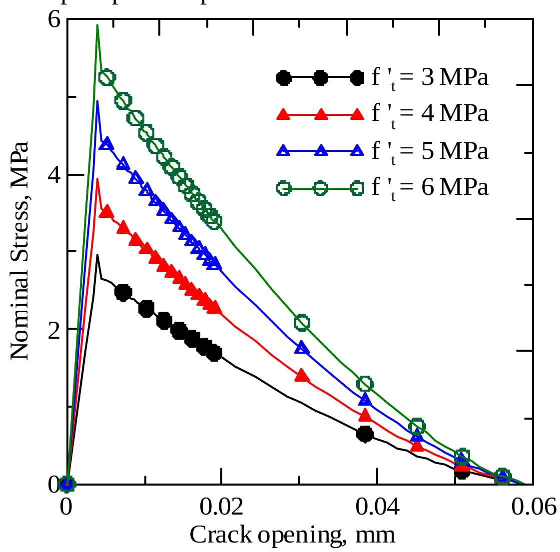


Figure 3 Parametric study of the stress crack width relationship for increasing tensile strength.

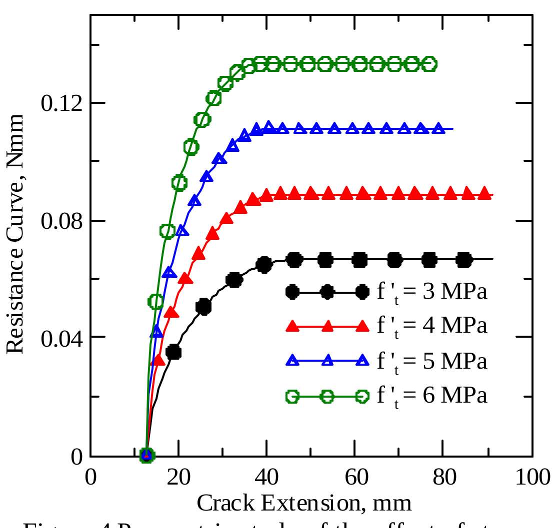


Figure 4 Parametric study of the effect of stress strain response on the R-curve

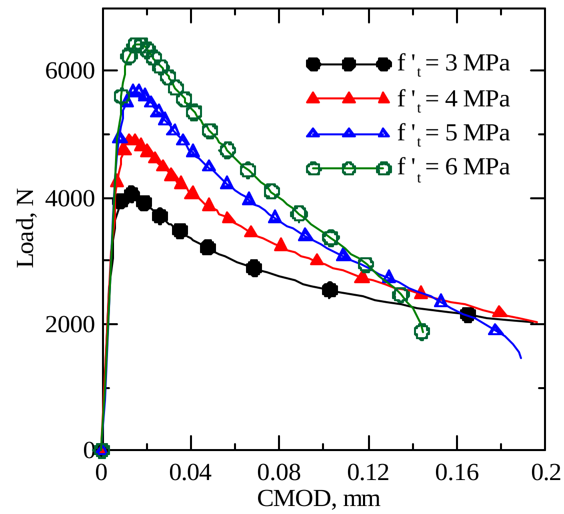


Figure 5 Parametric study of the effect of stress strain response on the load-CMOD response.

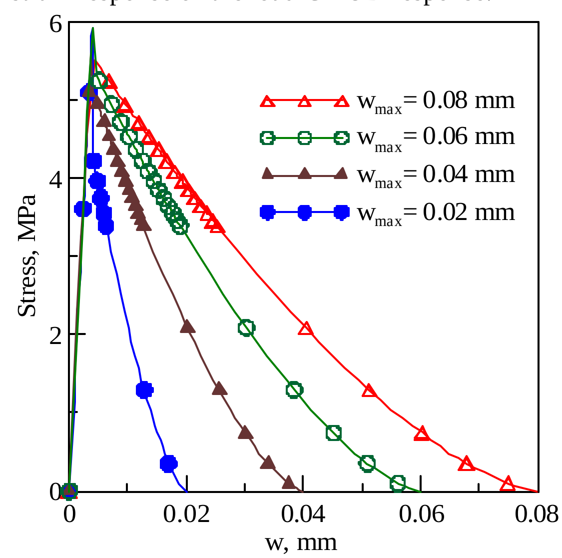


Figure 6a) Parametric study of the effect of post peak range in the stress strain response.

Figures 6a and 6b show that as the size of the stress strain response of the specimen in the post peak region is increased, it results in an increase in the flexural load deformation response. Note that according to this simulation the maximum strength of the flexural response is as much as 70% with a significantly higher energy dissipation in the post peak response of the flexural curve as the ultimate width is increased from 0.02 – 0.08 mm, a factor of four times.

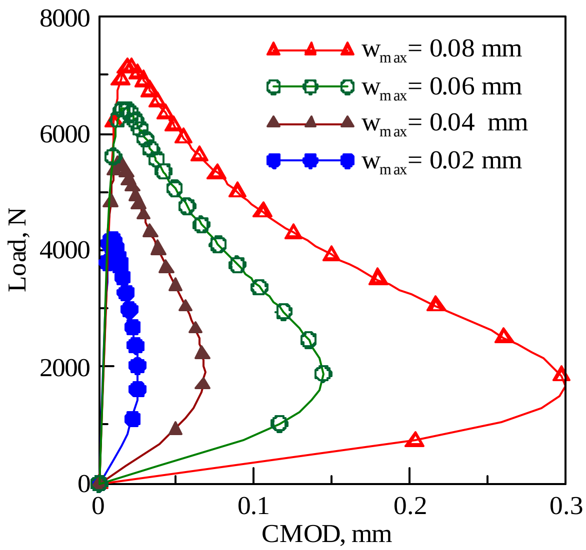
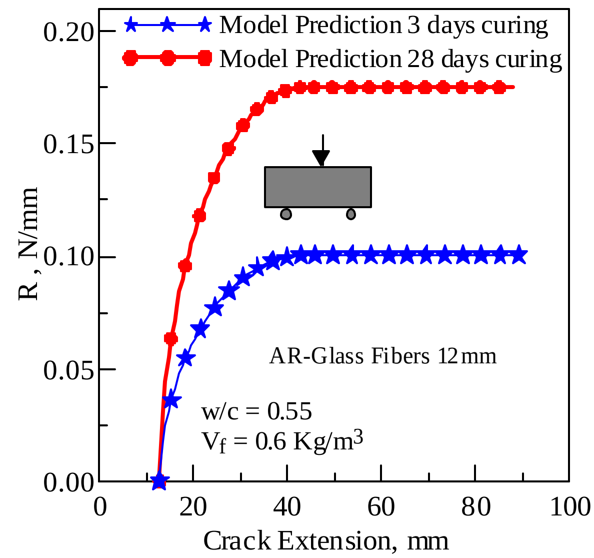


Figure 6b) Parametric study of the effect of post peak range on the resulting flexural load-CMOD response.

In order to validate the capability of the model in predicting the mechanical response of fiber reinforced concrete, the flexural load-deformation of concrete reinforced with various levels of alkali resistant (AR) glass fibers was studied. In the present work, two types of AR Glass fibers referred to as: High dispersion (HD) and High Performance (HP) obtained from Vetrotex Cemfil were considered. Several fiber lengths and contents were studied. Control specimens without fibers were prepared in both mixtures for comparison. The procedures for the mix designs and specimen fabrications in addition to comprehensive mechanical property data are provided in an earlier paper [7].

According to Figure 7 it is possible to model the effect of duration of curing on the mechanical response by developing a nonlinear curve fit model to the experimental data for the flexural load-CMOD response based on R-curves. Using these R-curves, one can calculate the contribution of fibers to toughening using Equation 3.



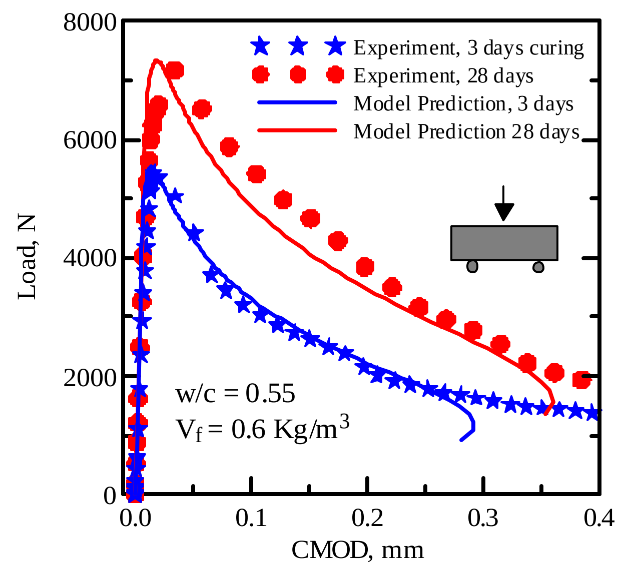
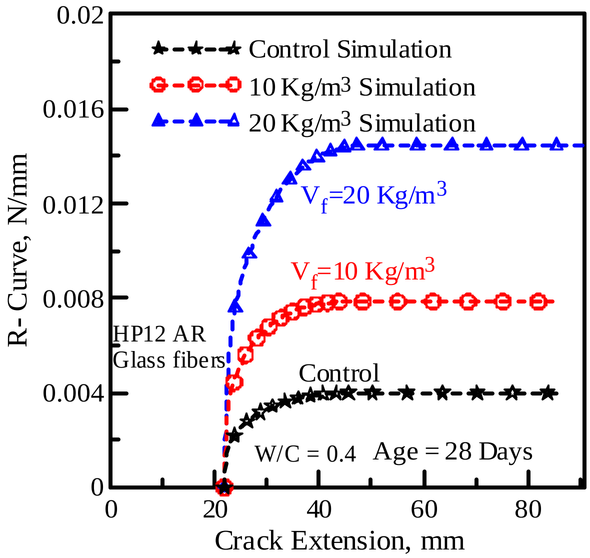


Figure 7. Modeling the effect of age on the flexural and R curves of fiber reinforced concrete a) the R-curve Response, and b) the load deformation response compared with experimental data



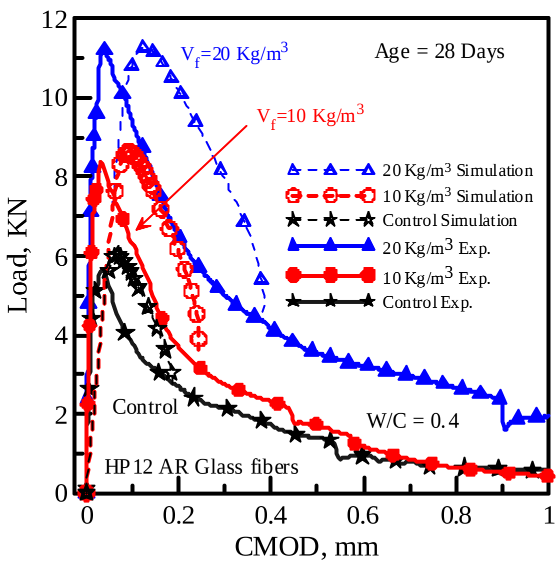


Figure 8 Modeling the fiber volume fraction using R-curves, a) the R-curve response, and b) load-CMOD plot.

Figure 8 represents the model fit parameters for the study of the effect of fiber volume fraction on the flexural response, load-CMOD plot and b) the R-curve response. the By conducting a nonlinear fit to the experimental load-CMOD responses, the two parameters, critical crack length Δa_c, and also the parameter β representing the R-curve are obtained. In this case the ranges of R values obtained are from 0.004 to 0.015 N/m and the range of critical crack extensions are in the range of 20-35 mm. Note that the predictability of the effect of fiber volume fraction in the increased load carrying capacity is significantly improved.

Conclusion

A procedure to calculate the role of fibers on the tensile stress strain response and the fracture toughness of cement based composites are studied by means of a cohesive crack approach. The tensile stress crack-width response is used as the primary material response, and the load deformation response is obtained in conjunction with experimental data. R-Curves are a convenient means of accounting for the increased energy dissipation and simulate crack growth.

References

1 Mobasher, B., and Shah, S. P.,"Test Parameters in Toughness Evaluation of Glass Fiber Reinforced Concrete Panels", *ACI Materials Journal*, Sept-Oct. 1989, pp. 448-458.

2 Mobasher, B. and Li, C. Y., "Mechanical Properties of Hybrid Cement Based Composites," *ACI Materials Journal*, Vol. 93, No.3, pp.284-293, 1996.

3 Ouyang, C. S., Mobasher, B., and Shah, S. P., *Eng. Fracture Mech.*, 37, 4, 901-913,90.

4 Mobasher, B., Ouyang, C., and Shah, S. P., *Int. J. of Fract.*. 50: 199-219, 1991.

5 Suzuki, T., Sakai, M., "A Model For Crack-Face Bridging", International Joumal Of Fracture 65 (4): 329-344 Feb, 15, 1994.

6 Foote, R.M.L., Mai Y.W, Cotterell, B., "Crack-Growth Resistance Curves In Strain-Softening Materials", J.of the Mechanics and Physics of Solids, 34 (6): 593-607 1986.

7 T. Desai , R. Shah, A. Peled, and B. Mobasher, "Mechanical Properties of Concrete Reinforced with AR-Glass Fibers, Proc. of the 7th Int. Symp. on Brittle Matrix Composites (BMC7), Warsaw, October 13-15, 2003.