Studies on ductility of RC beams in flexure and size effect

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ABSTRACT: This paper reports on some experimental investigations on ductility of reinforced concrete beams in flexure and evaluation of size effect. The minimum flexural reinforcement has been evaluated from experimental observations on ductility of reinforced concrete (RC) beams. Beams of depth 100mm, 200mm, 400mm at different flexural reinforcements namely 0.15, 0.30, 0.60 and 1.0 % were tested under uniform bending moment. The beams were made of 30 MPa concrete. The cracking and ultimate flexural strength, influence of beam depth on ductility and rotation capacity have been analyzed. The size of RC members has a significant influence on flexural behaviour. The variation of cracking strength is not very conclusive in small-size beams, where as it decreases as depth of beam increases beyond 200 mm. The ultimate flexural strength has been observed to decrease as the beam size increases. As the flexural reinforcement ratio in beams increased the ductility of beams was observed to increase. Ductility of RC members decreases with increase of beam size. The optimum flexural reinforcement has been obtained from an optimum ductility number, N_p, equal to 0.20. The minimum flexural reinforcement was observed to decrease as the beam depth increased, and decreased as the yield strength of steel reinforcement increased.

1 INTRODUCTION

The expressions for minimum steel ratios, both in flexure and shear, prescribed by various codes of practice are basically empirical. Due to lack of rational approach, the code provisions for design of RC members show vast variation in the design values. A minimum area of reinforcement is generally required in flexural members to prevent cracking and excessive deflections due to various loading effects. Two parameters such as tensile strength of concrete and yield strength of reinforcement are incorporated in the expressions for predicting minimum flexural reinforcement in RC beams. However, the effect of size of RC members is not considered. In some lightly reinforced beams, the cracking moment, as a plain concrete beam, may exceed the yielding moment of the beam, as an RC beam, at first cracking. This criterion has been considered for evaluating the minimum flexural steel ratio by ACI code (ACI-318-2005). The Indian standard (IS: 456-2000) specifies minimum flexural reinforcement to avoid sudden failures of RC members based on simple assumption that the yielding moment of RC beam, M_y is greater than or equal to cracking moment, M_{cr}, of plain concrete beams.

Recently, a theoretical evaluation of minimum longitudinal and transverse reinforcement ratios in beams subjected to flexure, shear and torsion, associated with ductility and minimum strength at ultimate limit state (ULS) has been reported (Shehata et al., 2003). The behavior of RC beams is useful to postulate a rational approach to estimate minimum flexural reinforcement based on rational approach. Carpenteri (1984) defined a brittleness number to determine the minimum flexural reinforcement in RC beams using fracture mechanics principles. The brittleness number is defined as "N_p"

$$N_p = \frac{f_y h^{\frac{1}{2}}}{K_{Ic}} \left[\frac{A_s}{A} \right] \qquad (1)$$

Where f_y = yield strength of steel reinforcement, K_{Ic} = concrete fracture toughness, A_s = area of steel reinforcement, A = area of c/s of beam, h = beam depth

The brittleness of structural member increases as its size increases and/or the reinforcement ratio decreases. Physically similar behaviour was revealed in some cases where the brittleness number N_p was the same. At a value of N_p equal to 0.26 using 91 MPa concrete, the yielding moment was more or less equal to the first cracking moment of the beam. The reinforcement corresponding to this condition was considered for evaluating the minimum reinforcement for flexural members. The minimum percentage reinforcement tends to be inversely proportional to the beam depth, while the values by codes are independent of the beam depth. Strain localization has been taken into account for the analysis of RC beams by Hillerborg (1990). The descending portion occurs due to crack formation within fracture process zone. The analysis of RC beams for balanced reinforcement ratio decreases with increasing beam depth.

Bosco et al. (1990) reported that the minimum flexural reinforcement corresponds to a condition at which the formation of first flexural cracking and yielding of steel reinforcement occur simultaneously. The brittleness of RC beams increases as the size of beam increases and as steel ratio decreases. An optimum value of N_p has been observed for estimating the minimum flexural reinforcement corresponding to the above condition. The minimum steel ratio is inversely proportional to depth of beam.

Bosco et al. (1990) reported that the brittleness of beams increases by increasing size-scale and/or decreasing steel area. For low N_p values in lightly reinforced beams or for small cross sections the fracture moment decreases while the crack extends. The peak or first cracking load is lower than the steel-yielding load only at high brittleness number. The size of beam seems to govern the post peak behavior, especially for low brittleness number for larger beam depth.

Baluch *et al.* (1992) proposed a criterion for minimum flexural reinforcement with which unstable crack propagation was avoided. This is achieved by ensuring that the moment corresponding to the maximum load, in a reinforced concrete beam is greater than its cracking moment as a plain concrete beam. The expression proposed to predict the minimum flexural reinforcement is

$$\rho_{\min} = \frac{1.9134 {K_{Ic}}^{0.82}}{{f_y}^{0.9922}\left(1.7 - 2.6c_s / D\right)} \tag{2}$$

Gerstle *et al.* (1992) used fictitious crack model to study tensile cracking behaviour of singly reinforced concrete beams in flexure. A theoretical analysis was performed to plot normalised moment with the normalised crack length for different values of "β" (a measure of brittleness) and "α" (a measure of steel percentages). Stable crack propagation has been associated with a continuously increasing curve and that value of "α" corresponds to minimum flexural reinforcement. An expression for minimum reinforcement ratio has been proposed defined but it does not contain f_y as shown below.

$$\rho_{\min} = \frac{E_C}{E_S}\left(\sqrt{0.0081 + 0.0148\frac{f_t D}{E_c w_c}} - 0.0900\right)^{1/2} \tag{3}$$

2 RESEARCH SIGNIFICANCE

The design of members based on strength criteria does not consider fracture mechanics theory. Failures in RC members exhibit different modes due to change of beam depth and percentage reinforcement. The ductility of RC members changes with size of member and strength of concrete. However, this is not considered in the design of structural members. In other words, failure according to strength theory should not show any size dependence, nor should the size of beam have any effect on its ductility. The effect of size of member and ductility in design of RC members can be predicted by fracture mechanics. However, there exists a controversy in the evaluation of minimum flexural reinforcement in RC members. An attempt has been made to understand size effect on ductility and minimum reinforcement of lightly reinforced beams.

3 EXPERIMENTAL PROGRAMME

3.1 *Materials*

An Ordinary Portland Cement (OPC) was used for the present study. The properties of cement are presented in Table 1. The fine aggregate was obtained from a natural river bed. The aggregate fraction passing through sieve size 1.18 mm and retained on 600 μ size was used in concrete. The specific gravity and fineness modulus of sand are given in Table 2. The machine crushed granite aggregate was used for concreting, consisting of mixture of 10 and 20mm size particles. The properties of aggregate are given in Table 3. Potable water was used for mixing of concrete and curing of specimens. The pH value of water was 7.8.

Table 1. Properties of cement

S.No.	Property	Results
1	Compressive Strength	43.0 N/mm^2
3	Fineness	3.5
4	Initial setting time	205 min
5	Final setting time	335 min
6	Specific gravity	3.12

The flexural reinforcement was high strength steel reinforcement with 415 N/mm^2 guaranteed yield strength. The diameter of the bars varied from 3 to 12mm depending on the size of beam. The nominal shear reinforcement consists of MS bars of diameter from 3mm to 6mm depending on the size of beam.

Table 2. Properties of fine aggregate

S.No	Property	Result
1	Specific gravity	2.78
2	Fineness modulus	2.82

Table 3. Properties of coarse aggregate

S. No	Property	Results
1	Specific gravity	2.70
2	Fineness Modulus	6.84

3.2 *Specimen details*

Rectangular beam specimens of different depths were adopted. In order to maintain geometric similarity, the aggregate size was varied depending on the depth. In small beams of size 50mm x 100mm x 500mm and 100mm x 200mm x 1000mm, 10mm aggregate was used, while 20mm aggregate was used in large beams of size 200mm × 400mm × 2000mm. The ratio of reinforcement cover-to-depth was 0.05 in all the beams.

Table 4: Beam dimensions and reinforcement details.

Beam	σ_y, MPa	A_{st} Provided		Stirrups Spacing, mm
		(%) mm^2	No. of bars	
A1	637	(0.30) 14	2–3mm	3mm @150
B1	637	(0.15) 28	4–3mm	3mm @ 300
C1	389	(0.15) 113	4-6 mm	6mm @ 150
A2	637	(0.3) 14	2–3mm	3mm @150
B2	389	(0.3) 56.5	2–6mm	6mm @ 140
C2	459	(0.3) 226	2-12 mm	6mm @ 300
A3	637	(0.6) 28	4–3mm	3mm @150
B3	389	(0.59) 113	4–6mm	6mm @ 140
C3	459	(0.59) 452	4-12 mm	6mm @ 130
A4	389	(1.19) 56.5	2–6mm	6mm @ 70
B4	577	(1.0) 756	6-12mm +1-10mm	8mm @ 175
C4	459	(1.0) 756	6-12 mm +1-10mm	8mm @ 175

The details of beam specimen and steel reinforcement are given in Table 4. The concrete mix proportions were 1: 2.75: 5.1 respectively cement: fine aggregate: coarse aggregate. The cement content was 250 kg/m^3 and the water cement ratio was 0.75. The compressive strength of concrete at 28 days on 100mm size cubes was 30 N/mm^2. The split tensile strength of 150mm x 300mm cylinders was 2.62 N/mm^2. The beam as well as companion cube and cylindrical specimens were cured in water for 28 days. The specimens were tested after 28 days. The steel reinforcement consisted of 3mm, 6mm, 10mm, and 12mm diameter bars as flexural reinforcement. The actual yield strength of reinforcement was used to calculate the flexural strength of beams. The yield strengths of 6mm, 10mm, and 12mm diameter bars were 577, 483 and 459 N/mm^2 respectively. The ductility number defined by Carpenteri (1984) was used for evaluation of minimum flexural reinforcement. Fracture energy, G_F of plain concrete was determined on three-point bend specimens (depth, d = 100mm, width, b = 100mm, and span, l = 500 mm). The notch-to-depth ratio was 0.5 and the notch width was 3mm. The mean value of fracture energy was 150 N/m. The critical stress intensity factor was evaluated as $K_{1C} = \sqrt{G_F}\sqrt{E}$ and was equal to 64.09 N/$mm^{3/2}$. The modulus of elasticity of concrete was 27.40GPa.

A total of 20 RC beams with depth equal to 100, 200mm and 400mm, maintaining the depth-to-width ratio 2.0 were tested. For a particular parameter, two beams in class A and B and only one beam in class C were tested. The span between supports was equal to five times beam depth, d. Therefore, the spans measured 500, 1000 and 2000 mm respectively in beams (for class A: d = 100 mm, b = 50 mm; l = 500; class B: d = 200 mm, b=100 mm; l=1000 mm; and class C: d = 400 mm, b = 200 mm; l = 2000 mm). The reinforcement was estimated from ductility numbers selected i.e. 0.091, 0.183, 0.366 and 0.732. Steel moulds were used for casting the beams of required dimensions.

3.3 *Experimental set-up and testing*

Four-point loading set-up was used for testing of RC beams as shown in Figure 1. Statically determinate system was ensured by adopting hinge and roller supports at two ends. The load was applied through a hydraulic jack at constant load increments. The load was applied symmetrically at one third points. LVDT was used to measure the deflection at the center of beams.

Figure 1. Experimental Set-up.

The load at first cracking was visualized by means of magnifying glass in the uniform bending moment region between two central loading points where the first flexural crack was formed. The strains along the depth of the beam were measured using demountable mechanical gage. The crack propagation was monitored on the beam surface and the crack width was measured by a microscope. The ultimate moments of the beam were estimated both theoretically and experimentally.

4 RESULTS AND DISCUSSION

4.1 *Load-deflections curves*

It was noticed that the deflection of beams at failure increased as the percentage steel reinforcement increased from 0.15 % to 1.0 %. It was observed that the ductility of beams increased as the flexural reinforcement increased. The beam with 0.30 % reinforcement exhibited large deflection at failure showing increased rotation capacity and ductility. Similar trend was observed in all the beams i.e. deflections increase with percentage flexural reinforcement keeping other parameters constant. At 0.30 % reinforcement the beams exhibited improved ductility. Interestingly, the nature of failure changed from ductile to brittle as the depth of beam increased. The large size beams exhibited relatively small deflection at failure. As the size of beam increased, keeping the percentage flexural reinforcement constant, the deflections at failure decreased. This shows that as the beam size increases the failure of beams turned from ductile to brittle. The ductility of beams was found to decrease with increase of depth. Further it was observed that as the percentage reinforcement increased at a given beam depth the ductility increased.

4.2 *Flexural strength*

The flexural strength is defined as the flexural capacity of the beam at the ultimate load. In this case, nominal strength of beam at the ultimate load was represented by its flexural strength. The nominal strength was calculated by dividing the ultimate load by square of depth of beams in three dimensional similarities.

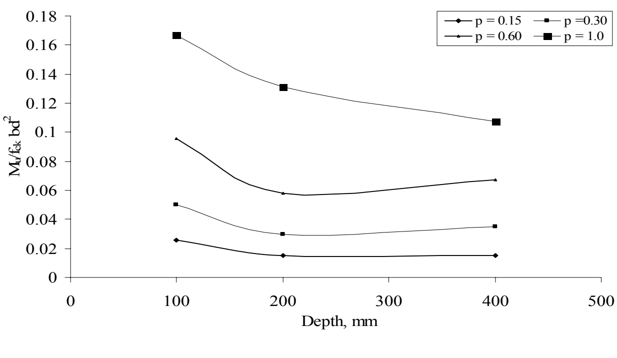

Figure 2. Moment ratio vs. Depth at different steel ratio.

Figure 2 shows the variation of flexural strength calculated as ($M_u/f_{ck}\ bd^2$) with depth of beam at different flexural reinforcements i.e. 0.15, 0.30, 0.60 and 1.0%. The flexural strength has been observed to decrease with depth at 0.15 and 0.30 % reinforcements. However, the nondimensional flexural strength has been observed to decrease as the beam depth increased at 0.60 and 1.0 % reinforcements. This shows that a general size effect law has been possible in R.C. beams in flexure at small percentage reinforcements. However, at heavy flexural reinforcements, the effect of size needs further studies. At this juncture it would not be possible to conclude the exact size effect on flexural strength of reinforced concrete beams with heavy reinforcement.

4.3 *Ductility factor*

Ductility factor may be defined as the ratio of deflection at failure to the deflection at yield or at the first crack. As there is no information on the effect of size on ductility of reinforced concrete beams, the present study was undertaken. In codes of practice, the design strength of RC members in flexure is considered to be constant. When the concepts of fracture mechanics are used, there could be an improved safety margin against failure and the prediction of failure could be possible with reasonable reliability. Figure 3 shows variation of ductility factor with size of structure at 0.15, 0.30, 0.60 and 1.0 % reinforcement. It demonstrates that the ductility factor increases as the beam size increases from 100mm to 200mm. Thereafter, the ductility factor decreases with size.

At small flexural reinforcement ratios, the ductility factor has been observed to be the highest at 200mmbeam depth. At higher percentages of reinforcement, the trend seems to be increasing with size of structure. At 100mm depth, the ductility at all percentages of reinforcement was found to be the lowest. However, as the depth of the beam increases beyond 200mm, the ductility factor has been showing size dependence. Further at small percentage of reinforcement, the ductility factor increases with increasing percentage reinforcement.

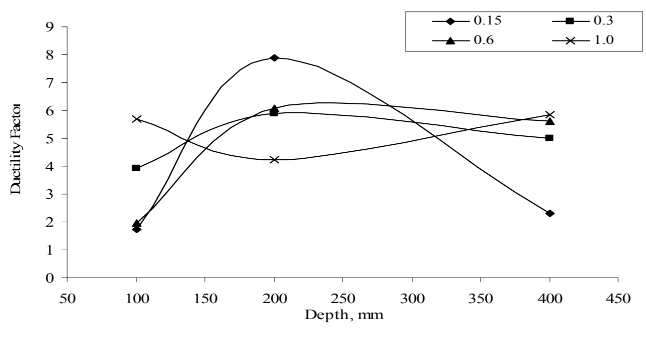

Figure 3. Ductility factor vs. beam depth.

4.4 *Ductility number*

The ductility number is a measure of brittleness of R.C. beams. The study was designed to investigate the effect of beam size and percentage reinforcement on ductility of R.C. beams. In order to evaluate the effect of beam size on ductility three different beam sizes i.e. 100mm, 200mm and 400mm were adopted by varying the flexural reinforcement at 0.15, 0.30, 0.60 and 1.0 %. Figure 4 shows the ductility number with beam depth at all percentages of reinforcement. It demonstrates that the ductility number increases with size of beam at a given percentage flexural reinforcement. The ductility number has been found to increase as the beam depth increased. Similar trend has been observed in the case of beams reinforced with 0.6 and 1.0 % flexural reinforcement. It was observed from the load-deflection curves that the beams turned brittle with increase in depth. The increase of the ductility number at 0.6 and 1.0 % was significantly higher at a given size of beams. The ductility number for 200mm deep beams was 0.198, beyond which it was found to decrease with increase in beam depth. This value of 0.198 is the optimum value for achieving minimum required ductility for evaluation of the minimum reinforcement.

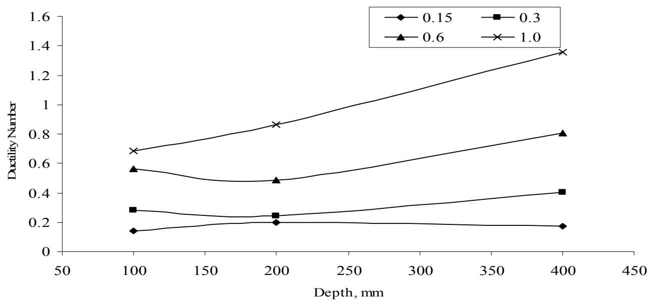

Figure 4. Ductility number vs. beam depth.

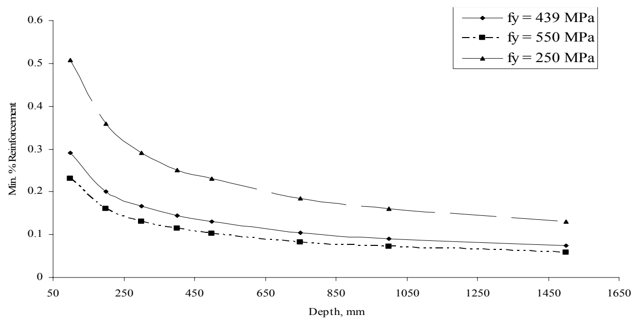

Figure 5. Min reinforcement vs. beam depth.

Figure 5 shows the minimum flexural reinforcement with beam depth at the optimum ductility, which corresponds to the value of N_p equal to 0.198 ≈ 0.20. As the strength of concrete increases the brittleness of beam increases due to material brittleness. Similar observations have been made in high strength concrete beams by Bosco et al. (1990), where the ductility number for achieving the minimum flexural reinforcement was 0.26. Therefore, in order to maintain the minimum ductility in RC members, the percentage reinforcement should be a function of strength of concrete, depth of beam and yield strength of steel reinforcement.

5 CONCLUSIONS

The following conclusions were drawn from the studies on lightly reinforced concrete beams.

1. The effect of size and percentage reinforcement on the ultimate strength of RC beams has been found to be significant. The ultimate strength is inversely proportional to the beam depth.
2. As the percentage flexural reinforcement increases, the ultimate load and the corresponding the beam deflections increase. As the depth of beam increases the ductility factor decreases.
3. The ductility number of RC beams increases with increasing beam depth and with decreasing percentage reinforcement. The optimum ductility number is 0.20 in 30 MPa concrete.
4. The minimum percentage reinforcement is inversely proportional to beam depth. It indicates that the formula for minimum steel reinforcement provided by the codes needs to be modified.

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