A fracture mechanics approach to material testing and structural analysis of FRC beams

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ABSTRACT: The presented work has been focused on strain-softening FRC and the interrelationship between material properties and structural behaviour. The main purpose was to establish a procedure for structural analysis of flexural members with a combination of conventional reinforcement and steel fibres. A systematic approach for material testing and structural analysis, based on fracture mechanics, has been used and covers: (1) material testing; (2) inverse analysis; (3) adjustment of the σ-w relationship for fibre efficiency; and (4) cross-sectional and structural analysis. The results suggest that the approach used for the material testing provides the necessary properties to perform analyses based on non-linear fracture mechanics. The structural behaviour could be predicted with good agreement, using both FEM and an analytical model, and when comparing the peak loads obtained in the experiments with the results from the analyses, the agreement was good, with a high correlation. This demonstrates the strength of the fracture-mechanics approach for material testing and structural analysis.

1 INTRODUCTION

The number of practical applications of fibre-reinforced concrete (FRC) is increasing, as FRC offers a possibility to greatly simplify in-situ cast concrete construction, e.g. by enabling a decrease of the amount of ordinary reinforcement used for crack width reduction. However, if FRC is to be a more widely used material, general design guidelines which take into account the material properties characteristic of FRC are needed.

A problem with some of the existing test and design methods is that they have not always been consistent in treatment; for example, the tensile behaviour has been characterised by dimensionless toughness indices or by flexural strength parameters, thus failing to distinguish clearly between what is relevant to the behaviour of the material as such and what concerns the structural behaviour of the test specimen. As a consequence, determined parameters (toughness indices or flexural strength parameters) have been found to be size-dependent even though no clear explanation for the size effect has been provided (see e.g. RILEM TC 162-TDF, 2003a & 2003b). The literature gives no clear evidence for a size effect; e.g. Kooiman (2000) investigated the energy absorption for different beam sizes (ligament lengths 125, 250, and 375 mm) but found no evidence for any size effect. Furthermore, di Prisco et al. (2004) investigated size effects in thin plates and found that there was a negligible size effect for the residual strength (i.e. post-cracking) in bending, and suggested that the large scatter may instead support the Weibull theory of statistical defects. Moreover, it is not unlikely that the supposed size effect may partly be explained by the fibre distribution and the fibre efficiency factor; see Löfgren (2005).

One of the problems with the σ-ε approach of RILEM TC 162-TDF is that the basis for the approach was that it should be simple and it should be compatible with the present design regulations for reinforced and prestressed concrete, while still making optimum use of the post-cracking behaviour of fibre-reinforced concrete. Hence, the σ-ε approach of RILEM TC 162-TDF has been shown to give unreliable results and a strong size dependence (which is counteracted by introducing a size effect factor). More disturbing is that the σ-ε relationship, determined from a test, cannot be used to accurately simulate the behaviour of the test beams from which the σ-ε relationship was determined; see e.g. Barros et al. (2005) and Neocleous (2006). In addition, the approach with the σ-ε relationship is not suited for non-linear finite element analyses and, as a consequence, several modifications have been suggested; see e.g. Tlemat (2006).

On the other hand, a consistent framework for material testing and structural analysis is non-linear fracture mechanics. With non-linear fracture mechanics it is possible to accurately predict and simulate the fracture process, and this is necessary for materials like fibre-reinforced concrete – which has a significantly different cracking behaviour compared to plain concrete – and/or when design requirements for the service state are governing. Such an approach has also been suggested by RILEM TC 162-TDF (2002), i.e. the σ-w method.

The purpose of the present study was to investigate, by means of experiments and non-linear fracture mechanics analyses, the flexural behaviour of reinforced FRC members made of self-compacting fibre-reinforced concrete. In order to show the applicability of the fracture mechanics approach to different structural conditions, two separate studies are presented. The tests were carried out on beams reinforced with a combination of steel fibres and either conventional bar reinforcement or a welded mesh. The reinforcement provided a longitudinal geometric reinforcement ratio of $0.25\% < \rho < 0.45\%$, and the fibre volume fractions, V_f, used in these investigations varied from 0.25% to 0.75% (20 to 59 kg/m^3). The post-cracking behaviour of the steel fibre-reinforced concrete was determined through inverse analysis on results from wedge-splitting tests (WST).

2 EXPERIMENTAL PROGRAMME

2.1 *Materials*

The concrete used was self-compacting (with a slump flow spread of 500 to 650 mm) and had a *w/b* ratio of 0.55. The fibre content varied from 0.25 vol-% (19.6 kg/m^3) to 0.75 vol-% (58.9 kg/m^3). The mix compositions, as well as the compressive strengths for each mix, for the two investigations can be found in Gustafsson & Karlsson (2006) and in Löfgren (2005).

2.2 *Materials testing*

The tensile fracture behaviour of the fibre-reinforced concretes was determined by conducting wedge-splitting tests (WST); see Figure 1. The stress-crack opening (σ-w) relationships were obtained, for each mix, by conducting inverse analysis following a procedure presented by Löfgren et al. (2005). This approach, in previous studies by Löfgren and other researchers, has been shown to yield reliable results; see e.g. Meda et al. (2001), Löfgren (2005), and Löfgren et al. (2005). However, the fibre bridging stress is influenced by the number of fibres crossing the fracture plane and, when the stress-crack opening relationship is determined from a material test specimen, it may be necessary to consider any difference in fibre orientation between this specimen and the full-scale specimen. Thus the number of fibres was counted in all the WST specimens and an average experimental fibre efficiency factor was determined for each mix.

The experimental fibre efficiency factor, $\eta_{b.WST}$, was calculated as:

$$\eta_{b.WST} = \frac{N_{f.WST}}{V_f / A_f} \tag{1}$$

where $N_{f.WST}$ is the number of fibres per unit area, V_f is the fibre volume fraction, and A_f is the cross-sectional area of a fibre. The experimental fibre efficiency factor, $\eta_{b.WST}$, should be compared to the fibre efficiency factor, $\eta_{b.beam}$, for the tested beams, which depends on whether the fibres have a free (random) or biased orientation. For the tested beams, the fibre efficiency factor was determined theoretically using an approach suggested by Dupont & Vandewalle (2005).

To account for the differences in fibre efficiency factor between the WST specimens and the tested beams, the stress-crack opening relationship obtained from the inverse analyses was reduced with the ratio between the two fibre efficiency factors, according to:

$$\sigma_{b.beam}(w) = \sigma_{b.WST}(w) \cdot \frac{\eta_{b.beam}}{\eta_{b.WST}} \tag{2}$$

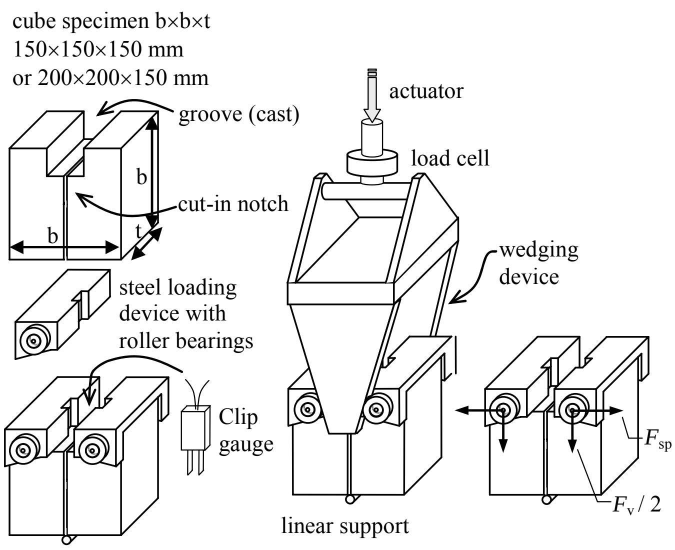


Figure 1. Principle of the wedge-splitting test method.

2.3 *Tests of beams with conventional bar reinforcement*

The tested beams had a geometry and test set-up according to Figure 2. The experimental programme consists of five series with three beams for each series, resulting in a total of fifteen beams. The programme is listed in Table 1. The beams had a geometric reinforcement ratio of $0.25\% < \rho < 0.45\%$. The compressive strength (determined at 28 days on water-cured cubes 150 mm^3) varied with the fibre content: 47 MPa for $V_f = 0$ %, 39 MPa for $V_f = 0.25$ %, ≈38 MPa for $V_f = 0.5$ %, and 37 MPa for $V_f = 0.75$ %; see Gustafsson & Karlsson (2006).
The fibres used are Dramix® RC 65/35, i.e. with aspect ratio (fibre length to fibre diameter) of 65 and with fibre length=35 mm. The σ-w relationships for the mixes, obtained with inverse analysis as discussed in Section 2.2, are presented in Figure 3.

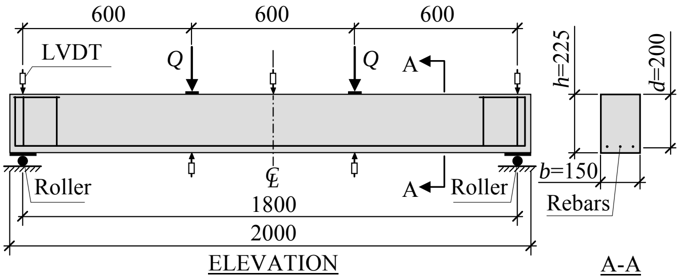


Figure 2. Test set-up for the structural beam tests.

Table 1. Experimental programme.

Series	Reinforcement	Fibre dosage [vol-%] ([kg/m^3])	Type*
V_f 0-ϕ8	3 ϕ8 (ρ=0.45%)	0% (0)	-
V_f 05-ϕ8	3 ϕ8 (ρ=0.45%)	0.5% (39.3)	RC 65/35
V_f 025-ϕ6	3 ϕ6 (ρ=0.25%)	0.25% (19.6)	RC 65/35
V_f 05-ϕ6	3 ϕ6 (ρ=0.25%)	0.5% (39.3)	RC 65/35
V_f 075-ϕ6	3 ϕ6 (ρ=0.25%)	0.75% (58.9)	RC 65/35

* Dramix® from Bekaert.

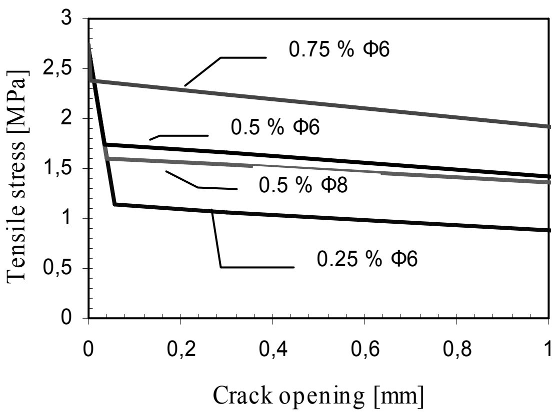


Figure 3. σ-w relationship for the full-scale elements adjusted to account for differences in the fibre efficiency factor.

2.4 *Tests on beams with welded wire mesh reinforcement*

The tested beams had a geometry and test set-up according to Figure 4. The experimental programme consists of four series (for different concrete mixes) with in total twelve beams. The beams had a geometric reinforcement ratio of $0.075\% < \rho < 0.121\%$. Different types of welded wire mesh were used, i.e. bar diameter, bar spacing, and yield strength; see Löfgren (2005) for a full description. The compressive strength of the concrete (determined at 28 days on water-cured cubes 150 mm^3) was 52 to 55 MPa for the four mixes; see Löfgren (2005). The σ-w relationships for the four mixes, obtained with inverse analysis, are presented in Figure 5.

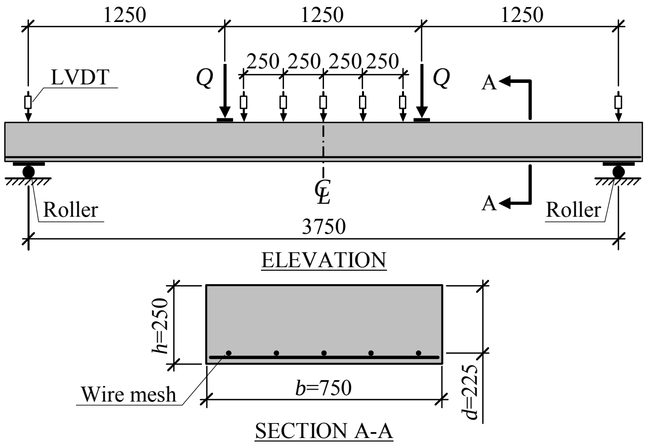


Figure 4. Test set-up for the structural beam tests.

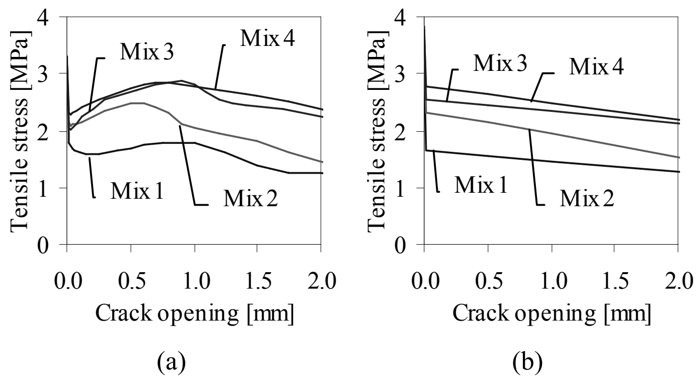


Figure 5. σ-w relationship for the full-scale elements adjusted to account for differences in the fibre efficiency factor: (a) for polylinear and (b) for bilinear σ-w relationship.

3 ANALYSES

To achieve a deeper understanding of the structural and fracture behaviour, non-linear fracture mechanics was applied, using the finite element method. The general finite element program Diana was used in all analyses; see TNO (2005). The concrete was modelled with four-node quadrilateral isoparametric plane stress elements. For the reinforcement, two different approaches were investigated: with truss elements, where the interaction between the reinforcement and the concrete was modelled by using special interface elements describing the bond-slip relation; and with the concept of ‘embedded’ reinforcement, where the reinforcement is modelled with perfect bond to the surrounding concrete with no degrees of freedom of its own – see TNO (2005). For the case where bond-slip was considered, its relationship was chosen according to CEB-FIP MC90 (1993), and confined concrete with good bond conditions was assumed.

In addition, an analytical approach was used; see Figure 6. The analytical model is based on the non-linear hinge model, as proposed by Olesen (2001a & 2001b) and described by RILEM TC 162-TDF (2002).

The non-linear hinge model is based on non-linear fracture mechanics and the fictitious crack model, originally proposed by Hillerborg et al. (1976). To be able to model the behaviour of reinforced FRC members, the model was further developed by Löfgren (2003 & 2005) to consider: (1) the non-linear stress-strain behaviour in compression; (2) a multi-linear stress-crack opening relationship; and (3) a multi-linear strain hardening relationship for the reinforcement.

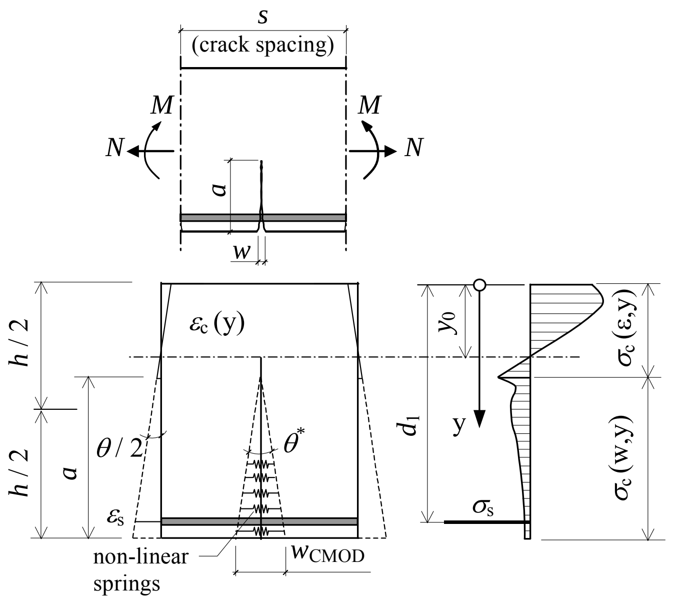


Figure 6 Non-linear hinge model.

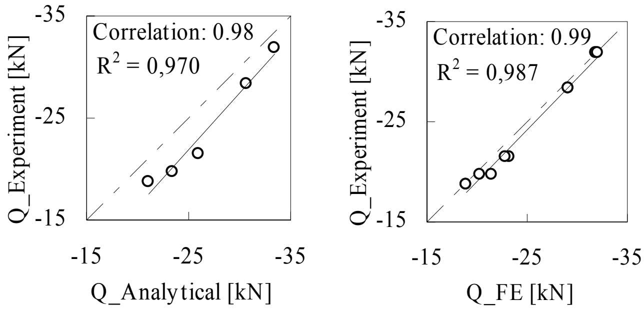
4 COMPARISON OF RESULTS FROM EXPERIMENTS AND ANALYSES

4.1 *Beams with conventional bar reinforcement*

In general, the peak load was predicted with good agreement for the FE analyses, while it was overestimated by the analytical approach; see Table 2 and Figure 7. Even though the analytical approach consequently overestimates the load-bearing capacity, the correlation is good: 0.99 for the FE analyses and 0.98 for the analytical approach. In addition, the load-deflection curves were predicted with good agreement, although the models predicted a somewhat stiffer pre-crack- and softer post-crack response, especially for the beams with V_f =0.75%; see Figures 8-12. This could possibly be explained by not having obtained the optimum parameters for the σ-w relationship, and could also be related to difficulties in determining the yield value for the reinforcement bars. Additionally, it can be said that numerical problems were encountered in the analyses of the beams with 0 % and 0.75 % fibre content.

Table 2. Maximum loads from experiments and analyses.

	Q_{max} [kN]				
	V_f0	V_f025	V_f05	V_f05	V_f075
	ϕ8	ϕ6	ϕ6	ϕ8	ϕ6
Exp	28.4	18.7	19.8	31.8	21.5
Analytic	29.8	21.2	23.4	33.4	26.1
FE embed	29.1	19	21.5	31.8	23.3
FE bond	29.1	19	20.3	32	22.9



(a.) (b.)

Figure 7. Comparison of maximum load from experiments with maximum load from (a) the analytical approach, and (b) FE analysis, using a bilinear σ-w relationship in the models.

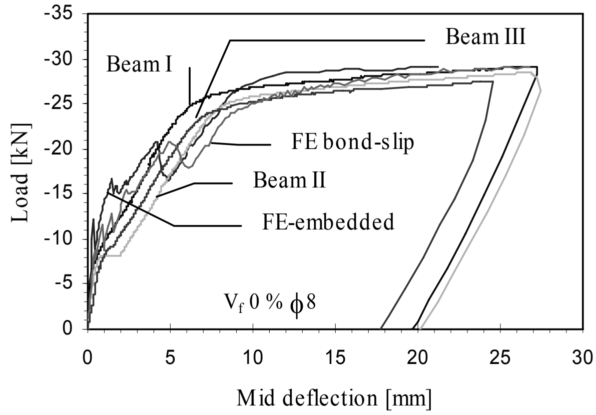


Figure 8. Comparison of load-deflection relationships for full scale elements and models; for the FE model using a bilinear σ-w relationship. No fibres added, rebar diameter 8 mm.

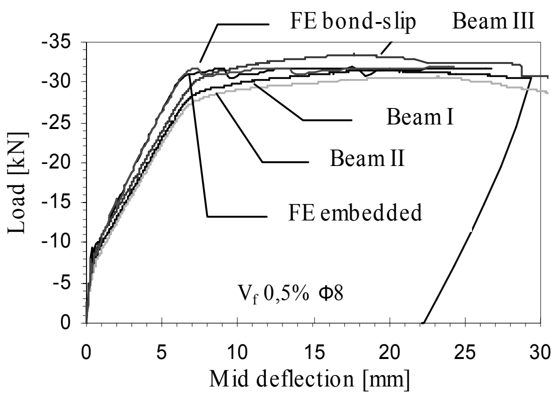


Figure 9. Comparison of load-deflection relationships for full scale elements and models; for the FE analyses using a bilinear σ-w relationship. V_f = 0.5 % and rebar diameter 8 mm.

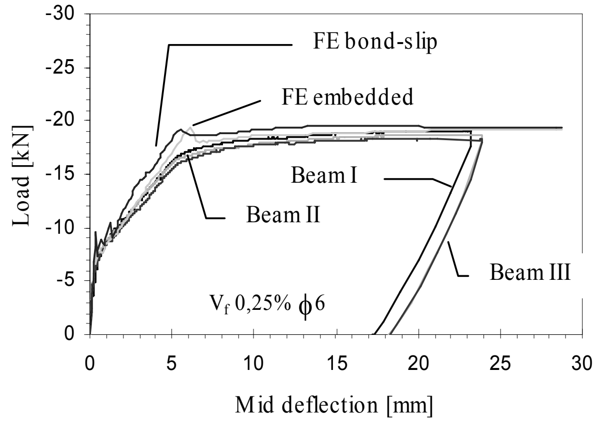


Figure 10. Comparison of load-deflection relationships for full scale elements and models; for the FE analyses using a bilinear σ-w relationship. V_f = 0.25 % and rebar diameter 6 mm.

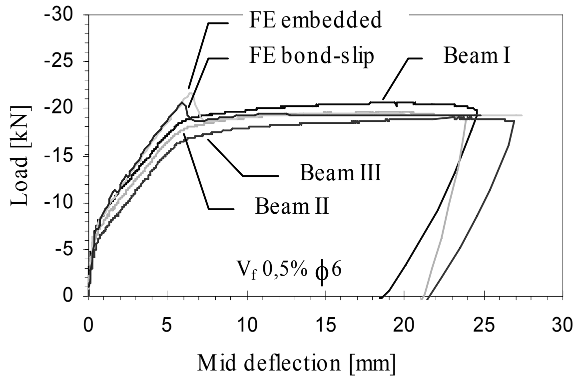


Figure 11. Comparison of load-deflection relationships for full scale elements and models; for the FE analyses using a bilinear σ-w relationship. V_f = 0.5 % and rebar diameter 6 mm.

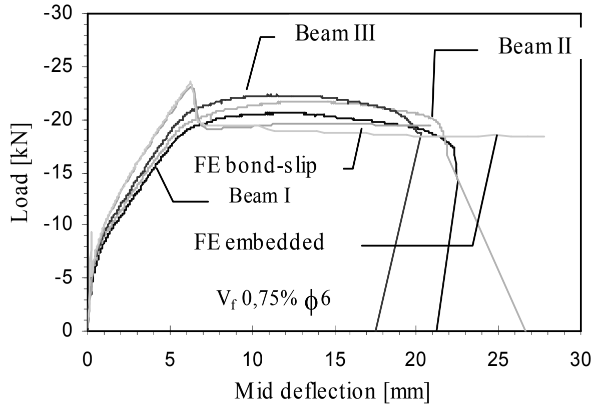


Figure 12. Comparison of load-deflection relationships for full scale elements and models; for the FE model using a bilinear σ-w relationship. Fibre volume, V_f = 0.75 % and rebar diameter 6 mm.

4.2 *Beams with welded wire mesh reinforcement*

Again, the peak load was predicted with good agreement; see Figure 13. The average ratio between analysis and experiment is close to unity (1.01 for the FE analyses and 1.00 for the analytical); moreover, the correlation is good, 0.99 for the FE analyses and 0.94 for the analytical approach. In addition, both the crack width and the load-deflection curves were predicted with good agreement (although the models predicted a somewhat stiffer response); see Löfgren (2005).

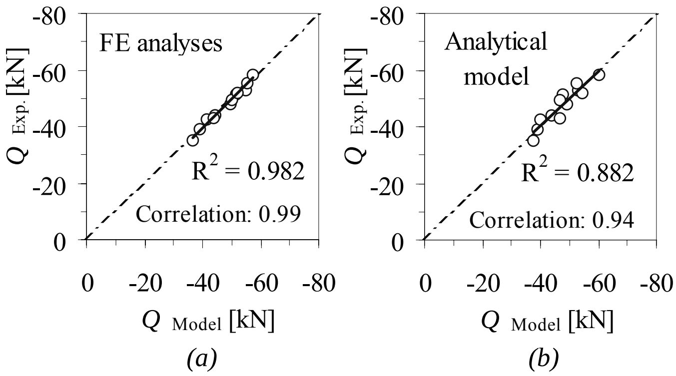


Figure 13. Comparison of maximum load from experiments and models: (a) for the FE analyses using a polylinear σ-w relationship; (b) for the analytical approach using a bilinear σ-w relationship.

5 CONCLUSIONS

To study the flexural behaviour of reinforced concrete members made of self-compacting fibre-reinforced concrete, two series of tests were carried out and non-linear fracture mechanics were used to simulate the response. The beams had a longitudinal geometric reinforcement ratio of $0.075\% < \rho < 0.45\%$ and the fibre volume fractions, V_f, used in these investigations varied from 0.25% to 0.75% (20 to 59 kg/m^3). In general, with the fracture-mechanics-based approach it was possible to determine the σ-w relationship and use this to predict the structural behaviour of reinforced FRC beams; this was done with acceptable agreement and correlation between experiments and analyses. For the finite element analyses on beams with conventional reinforcement, good agreement between experiments and analyses was found for fibre contents 0.0% to 0.5%, while for fibre content 0.75% the agreement was less satisfactory. The finite element analyses on beams with welded mesh reinforcement resulted in overall good agreement. The analytical approach is a fast and simple tool that can be used for cross-sectional analyses and the behaviour of simple structures can be determined (e.g. beams).

Based on the test results and the analyses, the following conclusions can be drawn:

- the WST method provided pertinent information regarding the post-cracking behaviour of the fibre-reinforced concrete;
- by considering the fibre distribution in the material test specimens and the full-scale elements, it was possible to adjust the stress-crack opening relationship obtained from the inverse analysis so that it could be used in the analyses of the full-scale tests with good agreement;
- in the experiments, although multiple cracking was obtained, the peak-load and post-peak behaviours were determined by a single crack.

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