Fracture Mechanics of Concrete and Concrete Structures -
Recent Advances in Fracture Mechanics of Concrete - B. H. Oh, et al.(eds)

Damage-fracture coupling analysis of mode I crack in the concrete under high rate of loading

H.L. Wang, W.L. Jin & X.Y. Sun
Zhejiang University, Hangzhou, PR China

Q.B. Li
Tsinghua University, Beijing, PR China

ABSTRACT: Based on the similarity of stress-strain relationship between the static and the dynamic experiments, dynamic constitutive equation of concrete is putted forward in this paper extended from Mazars static damage model. In terms of the relationship between damage and strain field, dynamic damage is introduced into the analysis of stress/strain field near the crack tip of Mode I crack. And the dynamic damage factor of concrete is derived from dynamic fracture mechanics coupled with damage mechanics. The coupling analysis of fracture and damage supplies a theoretical basis for the dynamic failure mechanism of concrete. Iteration method is adopted to decouple the equation and compute the dynamic and static damage factors of concrete. The theoretical results are in a good agreement with the experiments, which indicates that the analysis of damage-fracture coupling for concrete material is valid.

1 INTRODUCTION

As a kind of composite material, concrete contains many cracks in the interface between aggregate and matrix due to the shrinkage and other reasons. The failure of concrete is always caused by the linkages among cracks (Wang et al. 2006). The damage of concrete relates closely with the development of cracks. The damage mechanics and the fracture mechanics are two effective methods to study the mechanical properties of cracked concrete. Damage mechanics focus their research emphases on the evolution process of original defects, and great progresses had been made on the static damage model up to now. Loland damage model (Loland 1980), Mazars damage model (Mazars 1982) and Sidoroff damage model (Sidoroff 1985) are three extensive-applied models in the current investigations. However, few of dynamic damage models have been established in the concrete researches. Fracture mechanics focus their studies on the regularity of macrocrack developments in the solid. The damage before macrocracks' formation and damage around macrocrack are often neglected in the fracture researches. Generally, the microcracks and microdefects in the concrete material can't be simplified into macrocracks, therefore fracture mechanics are failure to study the behavior of concrete in present state. Fracture models applied extensively to concrete material include the linear elastic fracture model (Yu et al. 1991), fictitious crack model (Hillerborg 1983) and blunt crack band model et al. (Bažant 1985). Hence the failure of concrete is the interaction of damage and fracture. So damage-fracture coupling can reflect the failure processes of concrete better.

Besides the static loading, concrete structures always suffer dynamic loadings such as earthquake, impact and explosion. Compared with the static performance, concrete under dynamic loading generally shows different mechanical behavior which is sensitive to the loading rate (Sukontasukkul et al. 2004). Investigations on the dynamic damage of concrete material were relative shortage compared with that on the static damage. Brooks (Brook et al. 1989) used the concept of high stress volume to study the dynamic damage behavior of concrete under uniaxial tension, but many parameters are gained by curve fitting method. Based on static damage kinematical law, a dynamic damage constitutive relationship for concrete under uniaxial tension was established by LI (Li et al. 1993), and a good conformity with experiment was achieved in his study. Dynamic fracture mechanics were the frontier in fracture mechanics. The dynamic damage-fracture coupling analysis on the concrete material was unwonted. Therefore, an effort is made in this paper to obtain some beneficial discussions on this topic. Firstly, dynamic damage factor model is given based on the similarity of stress-strain relationships under static and dynamic loadings. Secondly, dynamic damage model is introduced into dynamic fracture mechanics of concrete to analyze the stress/strain fields near the tip of Mode I crack, and the coupling analysis of damage and fracture is carried out in the same time. Finally, iteration method is applied to simulate the distribution of static and dynamic damage in the concrete.

2 DYNAMIC DAMAGE OF CONCRETE

Many kinds of static concrete damage models had been established in the past centuries. Mazars model was famous in the static damage researches based on the isotropic hypotheses on the concrete material (Mazars 1982):

$$D_s = \left(\frac{\varepsilon_e - \varepsilon_s^0}{k_s} \right)^n \tag{1}$$

where D_s is the static damage factor of concrete; n and k_s are material constants calculated by:

$$n = \frac{\sigma_s^u}{\varepsilon_s^u} \left[\frac{\varepsilon_s^u - \varepsilon_s^0}{E_s \varepsilon_s^u - \sigma_s^u} \right] \tag{2}$$

$$k_s = \left(\varepsilon_s^u - \varepsilon_s^0 \right) \left[1 - \frac{\sigma_s^u}{E_s \varepsilon_s^u} \right]^{-\frac{1}{n}} \tag{3}$$

In which, σ_s^u and ε_s^u are the peak stress and the peak strain respectively; E_s is the Young's modulus under static loading; ε_s^0 is the threshold strain of static damage of concrete; ε_e is the equivalent strain and can be defined as:

$$\varepsilon_e = \sqrt{\langle \varepsilon_1 \rangle^2 + \langle \varepsilon_2 \rangle^2 + \langle \varepsilon_3 \rangle^2} \tag{4}$$

where ε_1, ε_2 and ε_3 are principal strains in three directions; $\langle x \rangle$ is defined as:

$$\langle x \rangle = \begin{cases} x, & x > 0 \\ 0, & x \le 0 \end{cases} \tag{5}$$

The similarity between static and dynamic stress-strain relationships is achieved in many experiments (as shown in Fig. 1). Therefore, the effective stress and the damage under dynamic loading can be defined as follows:

$$\sigma' = \sigma / (1 - D_d) \tag{6}$$

$$D_d = \left(\frac{\varepsilon_e - \varepsilon_d^0}{k_d} \right)^{n_d} \tag{7}$$

where σ' is the effective stress; σ is the dynamic macroscopic stress; D_d is the dynamic damage factor of concrete; ε_d^0 is the threshold strain of dynamic damage; n_d and k_d are two material constants, which can be defined as follows according to the corresponding definitions under static loading:

$$n_d = \frac{\sigma_d^u}{\varepsilon_d^u} \left[\frac{\varepsilon_d^u - \varepsilon_d^0}{E_d \varepsilon_d^u - \sigma_d^u} \right] \tag{8}$$

$$k_d = \left(\varepsilon_d^u - \varepsilon_d^0 \right) \left[1 - \frac{\sigma_d^u}{E_d \varepsilon_d^u} \right]^{-\frac{1}{n_d}} \tag{9}$$

In which, σ_d^u and ε_d^u are the peak stress and the peak strain of concrete material respectively under dynamic loading; E_d is the dynamic Young's modulus of concrete.

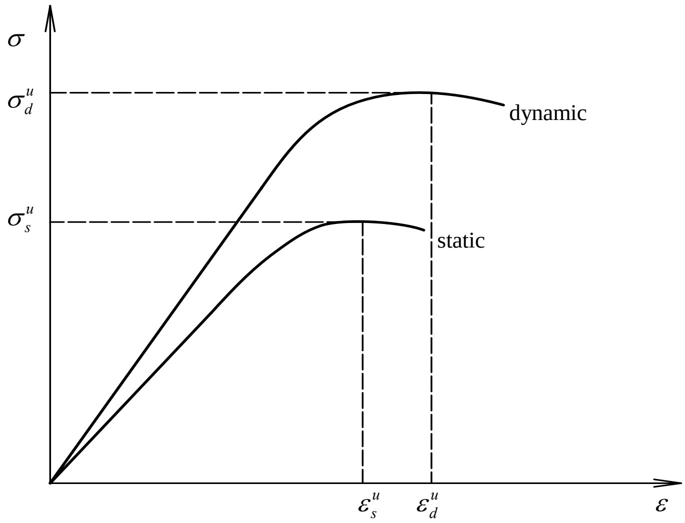

Figure 1. Stress-strain relationships under dynamic and static loadings.

According to the reference (Li et al. 1996), the dynamic and static parameters of concrete satisfy the following relationships:

$$\begin{cases} \sigma_d^u = K_\sigma(\dot{\varepsilon}) \sigma_s^u \\ \varepsilon_d^u = K_\varepsilon(\dot{\varepsilon}) \varepsilon_s^u \\ E_d = K_E(\dot{\varepsilon}) E_s \end{cases} \tag{10}$$

where $K_\sigma(\dot{\varepsilon})$, $K_\varepsilon(\dot{\varepsilon})$ and $K_E(\dot{\varepsilon})$ vary with strain rate $\dot{\varepsilon}$ and strain ε, and can be obtained from experimental curve of concrete under dynamic and static loadings.

The damage thresholds under dynamic and static loadings satisfy:

$$\varepsilon_d^0 = K_\varepsilon(\dot{\varepsilon}) \varepsilon_s^0 \tag{11}$$

Substituting Equation (10) and Equation (11) into Equation (7) yields the relationship between dynamic and static damage factors. Accordingly, referring to the static relationship, the dynamic damage constitutive Equation can be written as:

$$\varepsilon_{ij} = \left(\frac{1+\upsilon}{E}\sigma_{ij} - \frac{\upsilon}{E}\sigma_{kk}\delta_{ij} \right)(1-D_d)^{-1} \qquad (12)$$

where δ_{ij} is Kroneker Delta function; υ is the Poisson's ratio of concrete.

3 FRACTURE AND DAMAGE ANALYSIS AROUND MODE I CRACK UNDER DYNAMIC LOADING

In the case of cracks developing steadily while dynamic loading changing with time, the stress field near the tip of Mode I crack can be expressed as follows (Freund 1990):

$$\begin{cases} \sigma_{xx} = \dfrac{K_{\mathrm{I}}^d}{\sqrt{2\pi r}} \cos\dfrac{\theta}{2}\left(1 - \sin\dfrac{\theta}{2}\sin\dfrac{3\theta}{2}\right) \\ \sigma_{yy} = \dfrac{K_{\mathrm{I}}^d}{\sqrt{2\pi r}} \cos\dfrac{\theta}{2}\left(1 + \sin\dfrac{\theta}{2}\sin\dfrac{3\theta}{2}\right) \\ \sigma_{xy} = \dfrac{K_{\mathrm{I}}^d}{\sqrt{2\pi r}} \cos\dfrac{\theta}{2}\sin\dfrac{\theta}{2}\cos\dfrac{3\theta}{2} \end{cases} \qquad (13)$$

where K_{I}^d is the dynamic stress intensity factor of crack; r and θ are two parameters in the polar coordinate (expressed detailedly in Fig. 2). Under impact loading, the solution of K_{I}^d can be achieved as:

$$K_{\mathrm{I}}^d = f(c_1 t/(2a))\sigma\sqrt{\pi a} \qquad (14)$$

in which, $2a$ is the length of crack in the infinite body; c_1 is the velocity longitudinal wave.

$$c_1 = \sqrt{\frac{\lambda + 2\mu}{\rho}} \qquad (15)$$

Making $f(c_1 t/(2a)) = m$, the stress field near the Mode I crack can be gotten as:

$$\begin{cases} \sigma_{xx} = m\sigma\sqrt{\dfrac{a}{2r}} \cos\dfrac{\theta}{2}\left(1 - \sin\dfrac{\theta}{2}\sin\dfrac{3\theta}{2}\right) \\ \sigma_{yy} = m\sigma\sqrt{\dfrac{a}{2r}} \cos\dfrac{\theta}{2}\left(1 + \sin\dfrac{\theta}{2}\sin\dfrac{3\theta}{2}\right) \\ \sigma_{xy} = m\sigma\sqrt{\dfrac{a}{2r}} \cos\dfrac{\theta}{2}\sin\dfrac{\theta}{2}\cos\dfrac{3\theta}{2} \end{cases} \qquad (16)$$

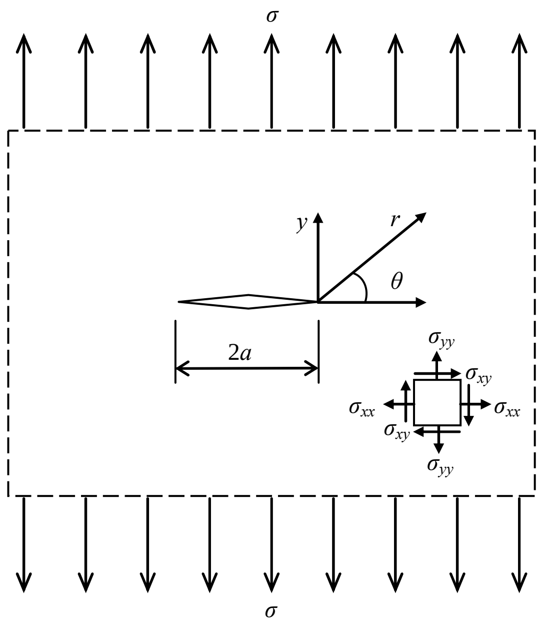

Figure 2. Mode I crack in the concrete material.

In terms of the relationships between principal and component stresses, the principal stress field near the crack tip can be obtained as:

$$\begin{cases} \sigma_1 = m\sigma\sqrt{\dfrac{a}{2r}} \cos\dfrac{\theta}{2}\left(1 + \sin\dfrac{\theta}{2}\right) \\ \sigma_2 = m\sigma\sqrt{\dfrac{a}{2r}} \cos\dfrac{\theta}{2}\left(1 - \sin\dfrac{\theta}{2}\right) \end{cases} \qquad (17)$$

For plane stress state, ε_3 is less than zero. In terms of the definition of Equation (4), it has no contribution to the equivalent strain ε_e, and can be removed from the Equation. So the principle strain field near the tip of Mode I crack is:

$$\begin{cases} \varepsilon_1 = \dfrac{m\sigma}{E}\sqrt{\dfrac{a}{2r}} \cos\dfrac{\theta}{2}\left[(1-\upsilon) + (1+\upsilon)\sin\dfrac{\theta}{2}\right] \\ \varepsilon_2 = \dfrac{m\sigma}{E}\sqrt{\dfrac{a}{2r}} \cos\dfrac{\theta}{2}\left[(1-\upsilon) - (1+\upsilon)\sin\dfrac{\theta}{2}\right] \end{cases} \qquad (18)$$

According to the equivalent strain principle in the damage mechanics, the effective stress field near the crack tip in damaged concrete is:

$$\begin{cases} \sigma'_{xx} = \dfrac{\sigma_{xx}}{1-D_d} \\ \sigma'_{yy} = \dfrac{\sigma_{yy}}{1-D_d} \\ \sigma'_{xy} = \dfrac{\sigma_{xy}}{1-D_d} \end{cases} \qquad (19)$$

Substituting Equation (18) and Equation (19) into Equation (4) yields:

$$\varepsilon_e = \frac{m\sigma}{E}\sqrt{\frac{a}{r}}\cos\frac{\theta}{2}\sqrt{(1-\upsilon)^2+(1+\upsilon)^2\sin^2\frac{\theta}{2}}(1-D_d)^{-1} \quad (20)$$

From Equation (20) and Equation (1), the damage and the strain are two coupling variances. In order to get the solution of dynamic damage field, rational computing method is needed to decouple the strain and damage.

4 DISCUSSION AND EXAMPLES

Iteration method is adopted to decouple the Equation and get the solution of damage field. By numerical computation, we find that the value of damage factor approaches to a constant after six times iteration. The dynamic damage factor after 1-time and 6-times iteration can be expressed as:

$$D_{d1} = \left(\frac{\beta}{\beta-\varepsilon_e^*}\cdot\frac{\varepsilon_e^*-\varepsilon_d^0}{k_d}\right)^{n_d} \quad (21)$$

$$D_{d6} = \left(\frac{\zeta}{\vartheta}\right)\cdot\left(\frac{\varepsilon_e^*-\varepsilon_d^0}{k_d}\right)^{n_d} \quad (22)$$

in which, $\zeta = \beta^6 - 5\varepsilon_e^* k_d \beta^4 + 6\left(\varepsilon_e^* k_d \beta\right)^2 - \left(\varepsilon_e^* k_d\right)^3$;
$\vartheta = \beta^6 - \varepsilon_e^* \beta^5 - 5\varepsilon_e^* k_d \beta^4 + 4\left(\varepsilon_e^*\right)^2 k_d \beta^3 + 6\left(\varepsilon_e^* k_d \beta\right)^2 -$
$3\left(\varepsilon_e^*\right)^2 k_d{}^3 \beta - \left(\varepsilon_e^* k_d\right)^3$; $\beta = k_d + \varepsilon_d^0$;
ε_e^* is the dynamic equivalent strain of undamaged concrete and can be defined as:

$$\varepsilon_e = \frac{m\sigma}{E}\sqrt{\frac{a}{r}}\cos\frac{\theta}{2}\sqrt{(1-\upsilon)^2+(1+\upsilon)^2\sin^2\frac{\theta}{2}} \quad (23)$$

Letting m=1 and some dynamic parameters like ε_d^u, ε_d^0 and E_d being replaced with ε_s^u, ε_s^0 and E_s accordingly, the static damage field near the tip of crack with consideration of damage-facture coupling is achieved.

Referring to the experimental data (Li et al. 1993, Xu et al. 1991), the static and dynamic damage fields of concrete near the tip of Mode I crack are calculated. The parameters for example computing are listed in the Table 1.

Table 1 Parameters for example.

a m	σ MPa	E_s GPa	υ	ε_s^0 $\mu\varepsilon$	n_s	k_s $\mu\varepsilon$
1.2	1.05	32.4	0.197	60	0.979	212

As stated in the reference (Li et al. 1996), K_σ and K_ε were given in the form of strain rate and fitted as:

$$K_\sigma(\dot{\varepsilon}) = 1.0 + 0.1948\lg\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_s}\right) + 0.03583\left[\lg\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_s}\right)\right]^2 \quad (24)$$

$$K_\varepsilon(\dot{\varepsilon}) = 1.0 + 0.1612\lg\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_s}\right) + 0.02117\left[\lg\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_s}\right)\right]^2 \quad (25)$$

where $\dot{\varepsilon}_s$ is the quasi-static strain rate; $\dot{\varepsilon}$ is the dynamic strain rate. When $\dot{\varepsilon}=100\dot{\varepsilon}_s$, the parameters for dynamic calculation are achieved by above Equations and listed in the Table 2.

Table 2. Parameters for dynamic calculation.

$K_\sigma(\dot{\varepsilon})$	$K_\varepsilon(\dot{\varepsilon})$	$K_E(\dot{\varepsilon})$	m	n_d	$k_d/\mu\varepsilon$
1.533	1.407	1.089	0.99	0.979	298.3

In terms of the parameters listed in the Table 1 and 2, the static and the dynamic damage fields near the tip of Mode I crack after six-times iteration are presented in the following figures (from Figs. 3-5). Figure 3 is the distribution of damage field under static loading corresponding to fracture toughness. Figure 4 is the distribution of damage field under dynamic loading corresponding to fracture toughness. And figure 5 is the distribution of damage under dynamic loading when it increases to 1.05MPa immediately. Form inside to outside, the isolines of damage factors are 1, 0.6, 0.1 and 0 sequentially.

The zone is damaged entirely when the damage factor in this zone equals to 1 (shadow zone in Figs. 3-5). The length of entirely damaged zone is 18.7 cm when θ equals to zero based on the theory in this paper, which means that the crack spreads forward steadily for 18.7 cm which is accordant with the test result in the reference (Xu et al. 1991)—the steady developing length of crack is 20 cm. Therefore, the fracture-damage coupling analysis is effective to describe the distribution of damage and steady development of crack in the concrete material.

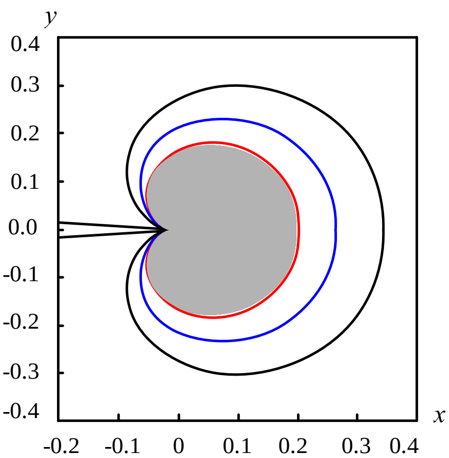

Figure 3. Damage distribution under static loading corresponding to fracture toughness (1.05MPa).

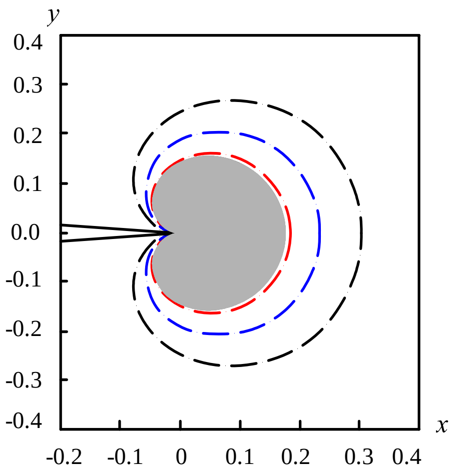

Figure 4. Damage distribution under dynamic loading corresponding to fracture toughness.

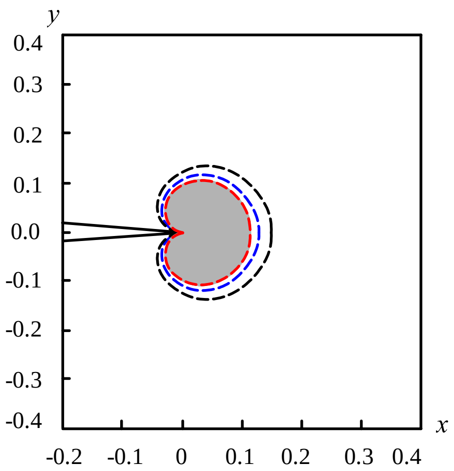

Figure 5. Damage distribution under dynamic loading (1.05MPa).

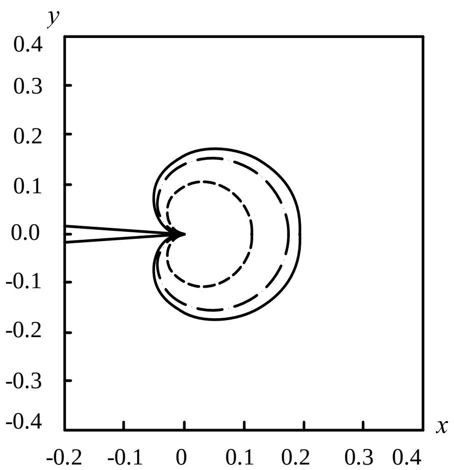

Figure 6. Entirely damaged zones under 3 kinds of loading rates.

The entirely damaged zones under 3 kinds of loading rates mentioned above are compared in Figure 6. Form this figure, the lagging character of damage is shown under dynamic loading compared with the static state, which accords well with the experimental results presented in references (George et al. 2001, Bichoff et al. 1991). The strength of concrete under dynamic loading increases due to the less damage under the same static loading based on the damage mechanics.

5 CONCLUSIONS

Some conclusions can be drawn as follows by the theoretical and the computing analyses in this paper:

(1) The stress-strain curves under dynamic and static loadings are similar. Based on this similarity and Mazars damage definition under static loading, the concrete constitutive Equation under dynamic damage is achieved.

(2) Fracture and damage are coupled when introducing damage principle into fracture analysis of concrete. This coupling method is valid by the damage field analysis near the tip of Mode I crack, and the theoretical results in this paper accord well with the experiments.

(3) The coupling model in this paper shows that the dynamic damage of concrete has some lagging character compared with the static one, which leads to the increase of strength under dynamic loading. The model shows great agreement with the existing experiments.

ACKNOWLEDGEMENT

This work was partly supported by the National Science Foundation of China (No.50809061, 50808158), the National High Technology Research Program of China (No.2007AA04Z437), the Natural Science Foundation of Zhejiang Province (Y107049), China Education Ministry (200803351117) and the Department of Hangzhou Science and Technology.

REFERENCES

Bažant, Z.P., 1985. Mechanics of fracture and progressive cracking in concrete structures. *Fracture Mechanics of Concrete: structural Application and Numerical Calculation*, edited by George C Sih and A Ditommaso, Martinus Nijhoff Publishers.

Bichoff, P.H., Perry, S.H., 1991. Compression behavior of concrete at high strain-rates. *Mater. Struct*., 24: 425-450.

Brook, J.J, Samaraie, N.H., 1989. Effects of strain rates on concrete constitutive relationship, *Fracture of Concrete and Rock*, Elsevier Applied Science Publisher, London, 397-408.

Freund, L.B., 1990. *Dynamic fracture mechanics*. Cambridge University Press, UK.

George, H.J., Dimittri, B., Dimitrios, T., 2001. A simple concrete damage model for dynamic FEM applications. International *Journal of Computational Engineering Science*, 2(2):267-286.

Hillerborg, A., 1983. Analysis of one single crack. *Fracture Mechanics of Concrete*, edited by F H Wittmann, Elsevier Scientific Publishing Company, London, p212.

Li Q.B, Zhang C.H., Wang G.L., 1996. Dynamic damage constitutive model of concrete in uniaxial tension. *Engineering Fracture Mechanics*, 53(3):449-455.

Li Q.B., Zhang C.H., Wang G.L., 1993. Static and dynamic damage fracture analysis of model I crack in concrete. *Journal of Civil Engineering*, 12: 20-27 (in Chinese).

Loland, K.E., 1980. Continuous damage model for load-response estimation of concrete. *Cement and Concrete Research*, 10:395-402.

Mazars, J., 1982. Mechanical damage and fracture of concrete structure. *Advances in Fracture Research*, *ICF5*, Pergamon Press , 4:1499-1506.

Sidoroff, F., 1985. Damage mechanics and its application to composite materials. *Proceedings of the European Mechanics Colloquium 182*, London: Elsevier Applied Science Publisher, p21-35.

Sukontasukkul, P., Nimityongsul, P., Mindess, S., 2004. Effect of loading rate on damage of concrete. *Cement and Concrete Research*, 34:2127-2134.

Wang H.L., Li Q.B., 2006. Mesomechanics analysis of compressive strength and constitutive equation of wet concrete. *Chinese Journal of Rock Mechanics and Engineering*, 25(8):1531-1536 (in Chinese).

Xu S.L, Zhao G.F., 1991. Concrete fracture toughness of huge specimens and criterion of fracture toughness for judging cracks in high concrete dam. *Journal of Civil Engineering*, 5: 1-9.

Yu R.Z, Qiao C.X, Zhou Q.L., 1991. Fracture mechanics of rock and concrete. *Central South University of Technology Press*, Hunan, China, p175 (in Chinese).