Fracture Mechanics of Concrete and Concrete Structures -
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Coupled damage and creep behavior of concrete under sustained loading

D.Zheng
ChongQing Jiaotong University, Chongqing, P.R.China

W.W.Li
China Three Gorges Project Corporation, Hubei, P.R. China

ABSTRACT: The coupled damage and creep behavior of concrete under sustained loading is investigated in this paper. Based on existed experimental results, a model explaining the relationship of nonlinear creep strains and damage of concrete is presented, together with the physical mechanism and failure criterion of concrete creep failure under sustained loading. The model assumes that total strain of concrete under sustained loading consists of linear creep strain, nonlinear creep strain and damage strain, which is mainly caused by microcrack evolution. By comparing behavior of concrete under fatigue and sustained loading, the crack growth rate under sustained loading is assumed in proportion to the normalized strength intensity factor. Concrete damage and creep under sustained loading is analyzed with the proposed model. The comparison between the results by the model and those by experiments indicates a good agreement.

1 INTRODUCTION

In recent decades, several concrete high arch dams up to 300 m are built and under-constructed in China, such as arch dam of XiIuodu (278 m), Xiaowan (292 m) and the primary Jingping (305 m). The safety evaluation of high arch dams has become an important and complex research subject, where researches of concrete properties under real work condition is one of the most important subjects (Zhu B.F. 2006).

Concrete structures like dams always work for a long lifetime. However, experiments to determine concrete properties are usually done in about 2 minutes according to loading rate regulation in the test code (ASTM 0.2-0.4Mpa/s). Researches in this area have showed that concrete properties under long-term loading will differ from short-time loading. Concrete specimen will fail after a certain period under the application of the loading lower than its normal strength. The relationship between applied loading stress and failure time is shown in Table 1 (Zhu B. 2005) and the stress-strain relationship under long-term loading is shown in Figure 1 (Rüsch H. 1960).

Table 1. Concrete failure time under sustained loading.

Sustained loading (%)	95	92	90	88	78	77	73	70
Failure time	2min	10min	30min	1h	4d	1y	3y	30y

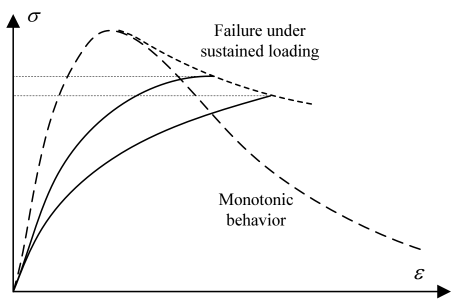


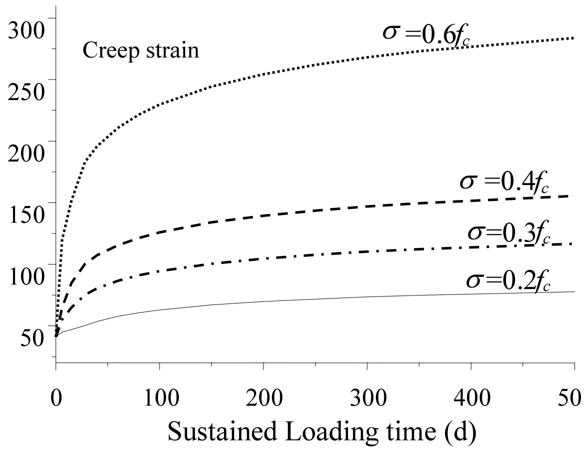
Figure 1. Sustained-loaded envelope for concrete in uniaxial compression (Rüsch, H. 1960).

It can be seen that concrete behavior under long-time sustained loading is very complicate. The present research cannot explain failure mechanism of concrete under sustained loading. In this paper, coupled damage and creep behavior of concrete under is investigated.

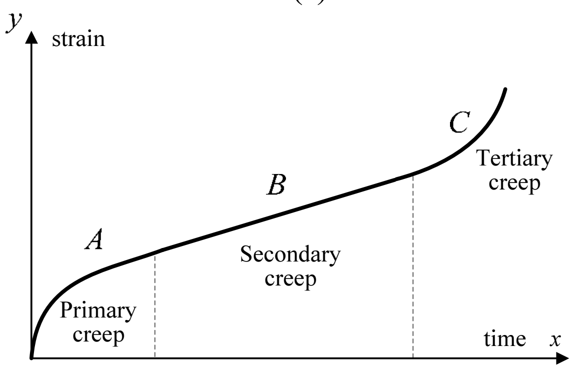
2 CONCRETE BEHAVIOR UNDER USTAINED LOADING

2.1 *Concrete deformation under sustained loading*

As we all know, concrete is a complex material with time-dependent flow properties consisting of different processes such as shrinkage and creep et.al. Shrinkage comprising the strains appears when no external loads are applied and creep is directly related to concrete stresses and microcracks evolution. For stress level below about 40% of concrete compressive strength f_c, creep strains are linearly related to the external stresses. However, at higher stress levels this linearity is lost and the creep coefficient is no longer stress-independent (Fig.2). For very high stresses (>0.7~0.8 f_c), creep strains are associated also with microcracks nucleation and growth with time, which may consequently result in concrete failure after a finite time interval (called tertiary creep). (Neville, 1973)



(a)



(b)

Figure 2. Concrete deformation under sustained loading a) Lower stress; b) Higher stress.

From the experimental results, total strain of concrete under sustained loading consists in a number of contributions, which can be written as:

$$\varepsilon_c(t,\sigma) = \varepsilon_{cl}(t,\sigma) + \varepsilon_{cn}(t,\sigma) + \varepsilon_d \tag{1}$$

where ε_{cl} is time-related linear creep strain of concrete which was mainly caused by time-dependent deformation of concrete matrix, ε_d is damage strain mainly caused by microcracks evolution under high sustained loading.

According to ACI 209(1992), linear creep strain of concrete can be expressed as:

$$\varepsilon_{c,lin}(t,\sigma) = \phi(t)\sigma \tag{2}$$

where ϕ(t) is the creep coefficient of concrete, which comprises the effect of both drying and basic creep and can be denoted as:

$$\phi(t) = \frac{(t-t_0)^{0.6}}{10+(t-t_0)^{0.6}}\phi(\infty,t_0) \tag{3}$$

where t-t_0 is loading time and $\phi(\infty,t_0)$ is ultimate creep coefficient which can be calculated by:

$$\phi(\infty,t_0) = 2.35k_1k_2k_3k_4k_5 \tag{4}$$

where k_i (i=1~5) are factors influence by humidity et al.

Based on the experiments of stress levels below 70% of concrete compressive strength f_c, the nonlinear effects of stresses on the creep coefficient can be obtained by numerical fitting as follows (Ruiz, F. 2007):

$$\varepsilon_{cn}(t,\sigma) = 2\left(\frac{\sigma}{f_c}\right)^4 \varepsilon_{cl}(t,\sigma) \tag{5}$$

2.2 *Damage evolution of concrete*

The failure mechanism of concrete under high stress level is not very clear at now. Few studies can be found where the combined effect of nonlinear creep deformations and concrete damage caused by crack evolution is considered.

As shown by the tests in the literatures (Rüsch, H. 1960, Neville, A. 1973), three different phases in can be identified for concrete under sustained loading: (1) crack formation, (2) stable crack propagation, and (3) unstable crack propagation till concrete failure. In the first and second phases, the damage strain is very small, whereas in the third phase the damage strain under sustained loading dominates the deformation of concrete due to unstable crack growth which makes the deformation increase rapidly with increasing time till final failure (Fig.2b).

In order to describe concrete behavior under sustained loading, which represents the concrete response due to micro-cracks, the results of the tests under cyclic fatigue loading are helpful. Since small-amplitude cycles will generate microcracks and produce a pseudo-plastic behavior and damage in concrete too.

Table 2 is the comparison of fatigue and creep damage deformation of concrete with time. Although there are some phenomenological differences between both processes, their physical mechanism and macroscopic behavior are very similar.

Table 2. Comparison of fatigue and creep damage deformation of concrete.

	Fatigue loading	Sustained loading
x	n/N_f	t/T_f
y	$\varepsilon/\varepsilon_{\max}$	$\varepsilon/\varepsilon_c$
A	Initial fatigue	First creep
B	Stable fatigue	Secondary creep
C	Fatigue failure	Tertiary creep

Researches showed that concrete failure time under fatigue loading is inversely proportional to the strain rate at the secondary creep stage. It is commonly known in materials science and fracture mechanics that Paris' Law relates the stress intensity factor to sub-critical crack growth rate under a fatigue stress regime. The accorded formula reads:

$$\frac{da}{dN} = C(\Delta K)^m \tag{6}$$

where a is the crack length, N is the number of load cycles, C and m are material constants, and ΔK is the range of the stress intensity factor.

Researches on concrete fatigue properties are far more than long-time behavior under sustained loading. According to the similarity of concrete behavior under sustained loading and fatigue loading, the crack propagation rate is assumed in proportion to the normalized strength intensity factor in this paper:

$$\frac{da}{dT} = C(K / K_{IC})^m \tag{7}$$

where T is sustained loading time, K is the stress intensity factor, K_{IC} is fracture toughness of concrete.

2.3 *Concrete failure under sustained compression*

To analyze concrete failure under sustained compression, the sliding crack model (Horii & Nemat-Nasser, 1986, Ashby & Hallam, 1986) is employed, which can describe properties of quasi-brittle materials under short-term compression well.

It is assumed that the concrete body is ideally composed of cement paste and aggregate. All the initial penny-shape microcracks are located on the interface between the cement paste and the aggregate facet (Fig. 1a). As the far field uniaxial compressive stress σ is applied, the shear (τ_n) and normal (σ_n) stresses are generated on the crack plane (Fig. 2a). The shear stress tends to make the crack surfaces slide, and a frictional stress $\mu\sigma_n$ opposes the sliding (μ is the friction coefficient) because the cracks are closed. The cohesive force between the cracks is neglected.

With the increase of the applied external compressive load, kinked crack growth will be initiated when the SIF K_I of the wing crack reaches the mode-I fracture toughness K_{IC}. The wing crack grows along a curved path and eventually turns parallel to the applied stress. With increasing loading, kinked crack propagated in a stable way at first and then interacted with other cracks, which finally caused concrete failure. To analyze the interaction between microcracks, a series of periodical co-linear cracks model is adopted in this paper (Fig. 3).

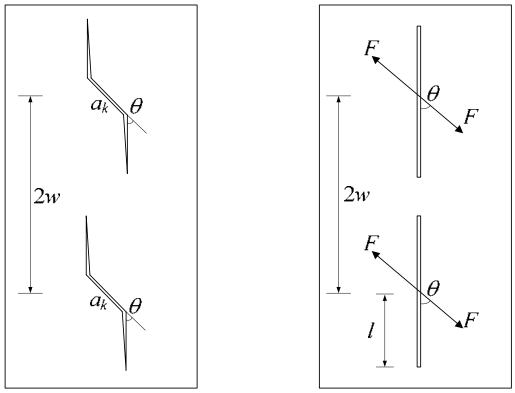


Figure 3. Periodical crack model of concrete.

In Figure 3, a_k is the crack length before kinking, W *is* the center-to-center crack spacing, θ is the angle between the crack plane and the direction of principal stress.

Thus, the equivalent splitting force $F\sin\theta$ is:

$$F\sin\theta = a_k G(\theta)\sigma \tag{8}$$

where $G(\theta) = \sin^2\theta(\cos\theta - \mu\sin\theta)$. G get its maximum value when θ satisfies:

$$\theta = \arctan\left[\left(3\mu + \sqrt{9\mu^2 + 8}\right)/4\right] \tag{9}$$

Hence, under far field stress, the stress intensity factor of crack tip can be expressed as(Horii & Nemat-Nasser, 1986):

$$K_I = \frac{\sigma a_k G_{max}(\theta)}{\sqrt{W\sin\left[\dfrac{\pi(l+l^*)}{W}\right]}} \tag{10}$$

where $l^* = 0.27a_k$ is the equivalent wing-crack length to ensure accuracy when l is very small.

It can be seen that unstable crack extension is only possible when $\partial K_I/\partial l > 0$. Hence, at any step of the short-term compressive loading process, with the fracture toughness K_{IC}, wing crack lengths can be obtained by Equation (10) until $l + l^* = W/2$. After that, since $\partial K_I/\partial l > 0$, the crack growth becomes unstable and concrete attains its compressive strength. Hence, compressive strength of concrete can be denoted as:

$$f_c = \frac{K_{IC}\sqrt{W}}{G_{max}a_k} \tag{11}$$

Therefore, when concrete was subject to sustained loading $\sigma = Sf_c$, the normalized stress intensity factor variation with kinked crack length l can be expressed as:

$$\frac{K_I}{K_{IC}} = \frac{S}{\sqrt{\sin\left[\pi(l+l^*)/W\right]}} \tag{12}$$

Define a dimensionless parameter $L=\pi(l+l^*)/W$ and substitute Equation (12) to eq.(7), crack growth rate can be simplified as:

$$\frac{dL}{dT} = BS^m (\sin L)^{-m/2} \qquad (13)$$

where $B = \pi C / W$.

Note that under short-term uniaxial compression, the response of each pre-existing crack is stable when $\partial K_I/\partial l < 0$ and a single correspondence of the crack extension length l on the compressive stress level exists. From equation (12), under sustained loading $\sigma = Sf_c$, the initial normalized crack length L_0 before sustained loading satisfies:

$$L_0 = \arcsin S \qquad (14)$$

Under sustained loading, cracks will propagated in a stable way until the SIF are greater than fracture toughness K_{IC} and $\partial K_I/\partial l > 0$. Hence, the critical crack length L_c when concrete failure happens can be expressed as:

$$L_c = \pi - \arcsin S \qquad (15)$$

Integration of Eq. (13) yields:

$$T = \frac{\int_{L_0}^{L_c} (\sin L)^{m/2} \mathrm{d}L}{BS^m} \qquad (16)$$

2.4 *Concrete strength variation with age*

Since concrete hydration process last for decades, its strength will increase with increasing time whether sustained loading is applied or not. The following equation can be used to estimate concrete strength variation with age.

$$f_c(T) = f_{28} H(t) \qquad (17)$$

where $f_c(T)$is concrete strength at age T (day), f_{28} is concrete strength at 28 days, $H(t) = e^{s(1-\sqrt{T/28})}$ is the formula presented by CEB, where s=0.25 for normal cement, and $H(t) = T/(0.85T + 0.4)$ is the formula presented by ACI. Thus, the fracture toughness variation with ages can be determined.

3 CONCRETE DAMAGE DEFORMATION UNDER SUSTAINED LOADING

After the relationship between kinked crack length and loading time under sustained compression is achieved from Equation 13, the compliance tensor of concrete can be determined by calculating the contributions of all the kinked cracks deformation to the total compliance tensor. However, this method is somehow inconvenient for the microscopic parameters such as microcracks density are hard to get. In this paper, macroscopic continuum damage mechanics are applied to calculated concrete damage strain under sustained loading.

Research showed that continuum damage mechanics can describe concrete nonlinear behavior under uniaxial compressive loading. According to Mazars (1989), the isotropic damage parameter D can be written in terms of a function of the equivalent strain:

$$\tilde{\varepsilon} = \sqrt{\sum_{i=1}^{3} \left\langle \varepsilon_i^{el,d} \right\rangle_+^2} \qquad i = 1,2,3 \qquad (18)$$

where $\left\langle \varepsilon_i^{el,d} \right\rangle_+$ is positive principal strain components. The following damage accumulation function has been adopted:

$$D = 1 - \frac{K_0(1-A)}{\tilde{\varepsilon}} - Ae^{B(\tilde{\varepsilon}-K_0)} \qquad (19)$$

where A, B, and K_0 are parameters characteristic, which can be calibrated with uniaxial experiments. This expression originally proposed by Mazars (1989) has been found in good agreement with experimental data for uniaxial tests.

Under given extent of uniaxial stress, the corresponding compliance of concrete and kinked crack length in Figure 3 can be determined from equation (10, 18 and 19). Because concrete damage states are functions of crack evolution, the relation between compliance and sustained loading time can be determined, so is the damage deformation.

4 NUMERICAL RESULTS AND DISCUSSION

4.1 *Numerical results*

Based on the above-developed theoretical framework, the influence of sustained loading to concrete strength can be investigated. The material parameters are calibrated with existed results as B=0.61, m=44.2. A=0.98, B=370, and K0=0.0001 are adopted in the numerical examples.

The variation of the sustained loading level with failure time is plotted in Figure 4 along with the existed experiments.

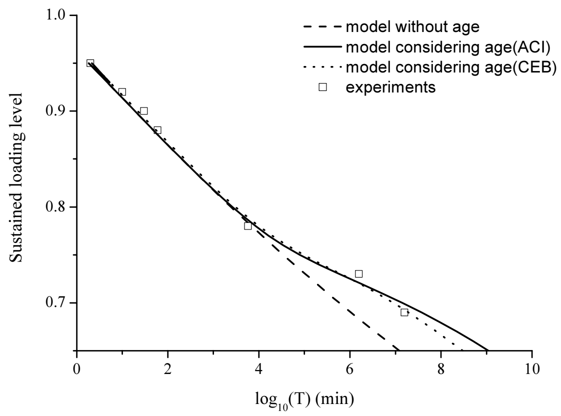


Figure 4. The relationship between loading level and failure time.

As seen from this figure, the results of the model correlate well with the experimental data. It also can be seen from Figure 4 that when concrete variation with age is considered, the failure time under sustained loading will be longer especially when the loading is lower than 80% of concrete compressive strength.

The strain-time relation is plotted in Figure 5 along with the existed experiments (Rüsch H. 1960). It can be seen from the figure that the results of the model correlate well with the experimental data.

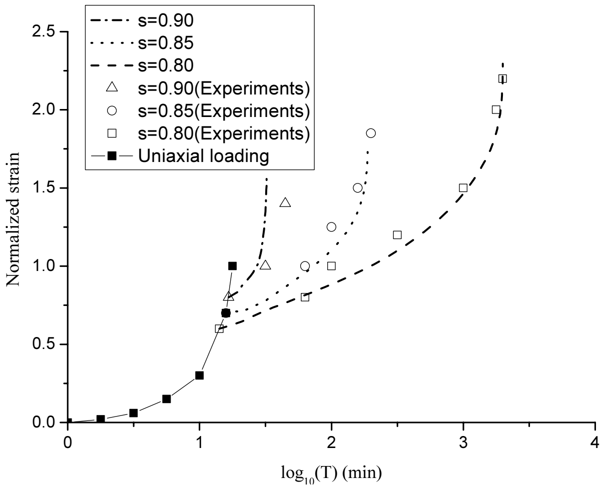


Figure 5. Strain variation with sustained loading time.

4.2 *Concrete under sustained tensile loading*

When the tensile loading exceeds concrete strength f_t, unstable crack propagation happens immediately, which makes concrete showed much more brittleness than under compression.

The failure criteria of concrete under tension in linear fracture mechanics form is:

$$K_I = \sigma\sqrt{\pi a} = K_{IC} \quad (20)$$

Therefore, crack growth rate under sustained tensile loading Sf_t are governed by:

$$\frac{da}{dT} = Cf_t^m S^m (\pi a)^{m/2} \quad (21)$$

Similar with the analysis above, the initial crack length and the critical crack length when material fail under sustained tensile loading were respectively:

$$a_0 = a_k \qquad a_c = a_k / S^2 \quad (22)$$

Then, the failure time of concrete under sustained tensile loading can be achieved by integrating Equation (22).

$$T = \frac{\pi a_k (1 - S^{m-2})}{BWS^m (m/2 - 1)} \quad (23)$$

The relationship between sustain loading and failure time are plotted in Figure 6. It can be seen that failure time of concrete under sustained tensile loading are shorter than compressive loading. For experiments of concrete under sustained tension are quite few, the difference between long-term tension and compression should be included in the future research.

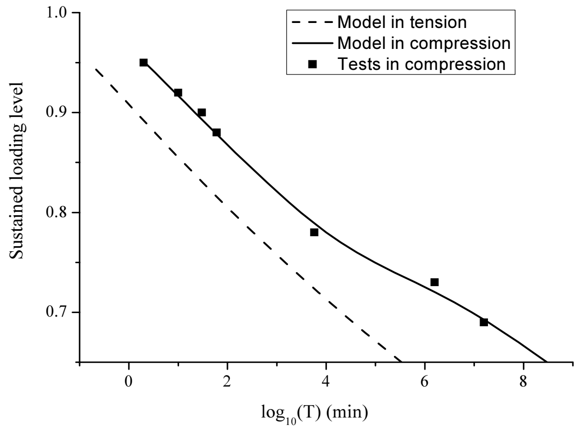


Figure 6. The relationship between tensile and compressive loading stress level and failure time.

5 CONCLUSIONS

In this study, a physical model is proposed to describe concrete long-term behavior considering stable crack growth with sustained loading time. The predicted results by this model appear reasonable comparing with the current available experiments.

Concrete failure under sustained loading is due to crack steady propagation till critical length. When the stress level is higher, stable crack propagation will be faster and concrete will fail at shorter time.

ACKNOWLEDGMENTS

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