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Experimental study on the double-k fracture parameters and brittleness of concrete with different strengths

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ABSTRACT: The double-K fracture parameters and the brittleness of concrete with a compressive strength varying from 30 MPa to 150 MPa were studied through both three-point bending tests (TPB) and wedge splitting (WS) tests. A total of 84 notched concrete specimens were tested. The fracture parameters were determined following a double-K fracture model. The non-dimensional brittleness index of concrete was subsequently evaluated according to the obtained double-K fracture parameters. Results indicated that the initial fracture toughness K_{Ic}^{ini}, unstable fracture toughness K_{Ic}^{un}, and cohesive toughness K_{Ic}^{c} increase as the compressive strength of concrete increases. An empirical relationship between the non-dimensional brittleness index and the concrete strength was obtained. It was also found that both TPB and WS tests lead to consistent results on the double-K fracture parameters of concrete regardless of its strength.

1 INTRODUCTION

High strength concrete (HSC) has been widely used for construction nowadays due to its lower porosity, higher load-bearing capacity and excellent durability. Compared to normal strength concrete (NSC), a significant feature of HSC is its brittleness. As a consequence, increasing attention has been paid to the brittleness and fracture properties of HSC regarding the effects of the type and size of coarse aggregates (e.g. Jenq & Shah 1985a), the change of water to binder ratio (e.g. Barr et al. 1998), the addition of silica fume (e.g. Zhou et al. 1995), and the addition of fly ash etc. (e.g. Bharatkumar et al. 2005). However, there is relatively limited data available on the fracture parameters and brittleness of concrete with an ultra-high strength (e.g. more than 100 MPa). In addition, the influence of strength on the fracture parameters of concrete is quite controversial (Einsfeld & Velasco, 2006)

There are several fracture mechanics models, which can be used to evaluate the fracture properties of concrete, including the fictitious crack model (Hillerborg et al. 1976), the crack band model (Bazant & Oh 1983), the two-parameter fracture model (Jenq & Shah 1985a,b), the effective crack model (Swartz & Refai 1984, 1987, Karihaloo & Nallathambi 1989, 1990), the size effect model (Bazant & Pfeiffer 1990) and the double-K fracture model (Xu & Reinhardt 1998,1999,2000). Based upon the above fracture mechanics models, some indexes have been proposed for the evaluation of the brittleness of concrete, such as the characteristic length l0 = (KIcun/fu)2 (Irwin 1965, Hillerborg et al. 1976), a length parameter Q = (E·CTODc/KIc)2 (Jenq & Shah 1985b), and the brittleness number β=d/d0 (Bazant & Pfeiffer 1987). In this paper, fracture tests were conducted to investigate how the initial fracture toughness KIcini, the unstable fracture toughness KIcun, and the cohesive toughness KIcc change with the compressive strength of concrete, which varied from 30 to 150 MPa. The non-dimensional brittleness index, which was defined as the ratio of KIcini to KIcun (Kumar & Barai 2009), was adopted to quantitatively evaluate the relationship between the brittleness and the strength of concrete.

To evaluate the K_{Ic}^{ini}, K_{Ic}^{un}, and K_{Ic}^{c} of concrete in case of mode I crack, different types of test methods can be used, including the three-point bending (TPB) test, the eccentric compression edge-notched beam (ECENB) test, the uni-axial tensile (UT) test, the compact tension (CT) test and the wedge-splitting (WS) test. In this paper, two most popularly used tests, TPB and WS tests, were applied to investigate the geometry effect on the double-K fracture parameters.

2 EXPERIMENTAL PROGRAM

2.1 *Experimental materials*

Ordinary Portland cement, high-quality silica fume, sand and coarse aggregates were used for concrete casting. The maximum size of the coarse aggregate was 20 mm. Seven different types of mixing proportions of concrete were applied to evaluate their fracture properties. Some concrete cubes (150 × 150 × 150 mm) were cast for testing the compressive strength of concrete strength. The mixing proportion, cube compressive strength f_{cu}, and elastic modulus of concrete at the testing time are given in Table 1.

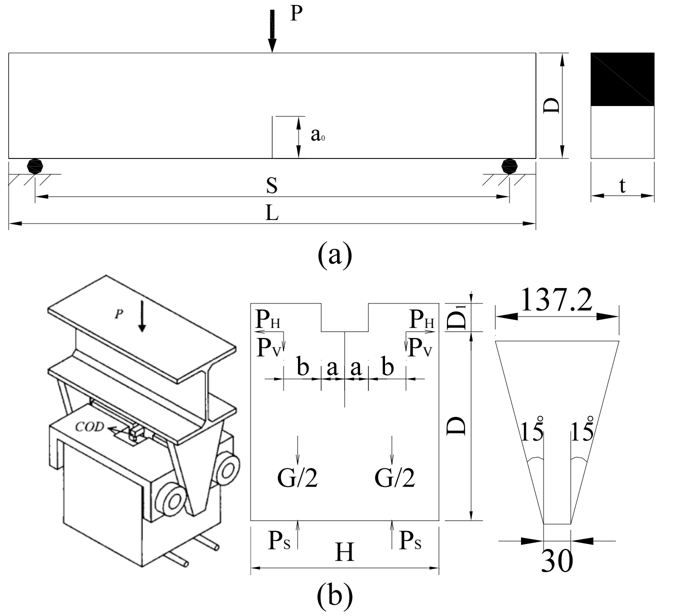

Figure 1. (a) The configuration of a three-point bending notched beam (b) The configuration of a wedge splitting specimen.

2.2 *TPB and WS specimens*

Two types of specimens: TPB specimens and WS specimens were tested. The test configurations are shown in Figures 1a, b, respectively. A total of 84 specimens included in 14 combinations (6 identical specimens for each combination) were prepared for the tests (refer to Table 2). The notch in the specimens was formed using a greased steel plate of 3 mm thick. The ratio of the initial notch length to the whole depth of the specimens was 0.4 for both types of tests. In the WS tests, the influence of the additional bending moment on the stress field around the crack tip should be removed through an appropriate design of the tests. Details of the two types of specimens can be found in another contribution of the authors in the same proceedings (Wang et al. 2010). Two strains were symmetrically attached at the two sides of crack tip to form a full-bridge electro circuit to monitor the occurrence of initial cracking and the monitoring of crack propagation (see Figs 2a,b). Figure 3 present the measured values of the strain gauges taking two specimens (one TPB test and one WS test) as examples. The value of the strain at the crack tip increased first with the load because of the energy absorption of concrete around the crack tip. Once the initiation of crack occurred, the energy stored at the crack tip was released and subsequently the value of the strain at the crack tip started to reduce. Therefore, the load corresponding to the turning point of the strain in Figure 3 can be defined as the initial cracking load.

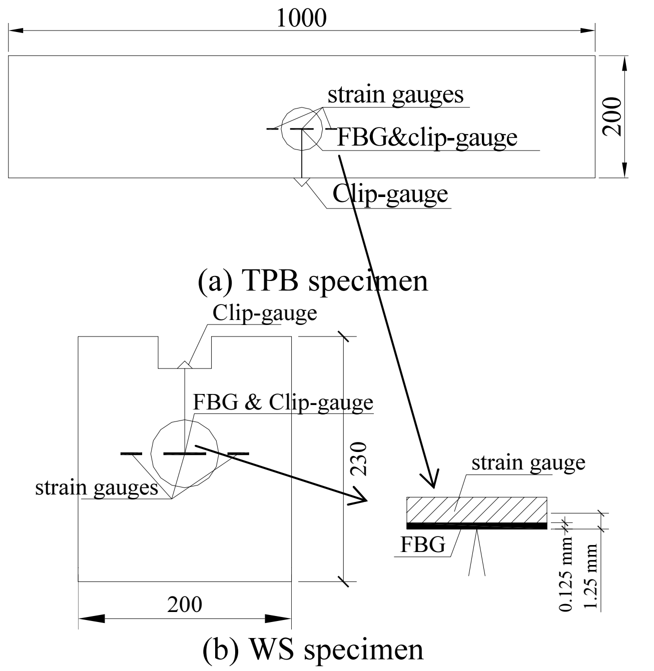

Figure 2. Illustrations of strain gauges, a clip gauge and a FBG sensor in TPB and WS specimens.

Table 1. Summary of specimens.

Type		Cement	Silica fume	Water	Water to binder ratio	Sand	Coarse aggregate	f_{cu} (MPa)		E_c (MPa)	
								TPB	WS	TPB	WS
TPB1	WS1	1.00	0.00	0.83	0.83	2.86	4.29	37.80	37.87	31.90	31.99
TPB2	WS2	1.00	0.00	0.73	0.73	2.47	3.71	48.01	48.61	34.07	34.17
TPB3	WS3	1.00	0.00	0.48	0.48	1.33	2.47	60.06	60.40	36.70	36.98
TPB4	WS4	0.87	0.13	0.45	0.45	1.27	2.35	74.15	75.44	39.12	39.32
TPB5	WS5	0.80	0.20	0.40	0.40	1.52	2.28	86.72	89.28	41.34	41.65
TPB6	WS6	0.79	0.24	0.32	0.32	1.54	2.40	103.56	103.73	44.30	44.26
TPB7	WS7	0.70	0.30	0.20	0.20	0.88	1.51	144.11	145.76	49.20	49.39

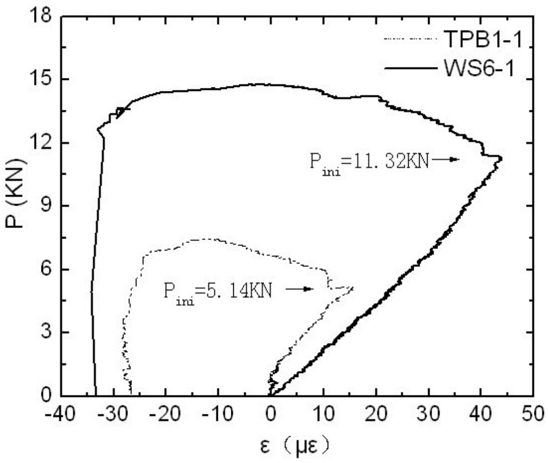

Figure 3. P-ε relationship.

All the tests were carried out according to "China norm for fracture test of hydraulic concrete (DT/T 5332-2005) (DT/T 5332-2005)" under a closed-loop servo-controlled MTS structural loading system of 30 tons capacity. The loading speed was 0.02 mm/min. The crack mouth opening displacement ($CMOD_c$) was recorded using a clip gauge and the crack tip opening displacement ($CTOD_c$) was recorded with a strain gauge and a fiber Bragg grating (FBG) sensor (see Figs. 2a-2b). The details of installation can be found in Wang et al. (2010).

3 DETERMINATION OF DOUBLE-K FRACTURE PARAMETERS AND BRITTLENESS INDEX

3.1 *Double-K fracture parameters*

There are three different stages in the crack propagation of quasi-brittle materials: crack initiation, stable crack propagation, and unstable fracture. The Double-K fracture model was proposed to evaluate the entire fracture process of concrete materials (Xu 1988). In the model, two fracture controlling parameters are employed. One is the unstable fracture toughness K_{Ic}^{un} and the other is the initial fracture toughness K_{Ic}^{ini}.

For TPB specimens, the value of K_{Ic}^{un} can be evaluated as follows (Xu & Reinhardt 1999a):

$$K_{Ic}^{un} = \frac{1.5 P_{\max} S \sqrt{a_c}}{D^2 B} f(\alpha_c) \tag{1}$$

where P_{max} is the maximum load; S, D and B are the span, depth, and width, respectively, of the testing beam; $f(\alpha_c)$ is a geometry factor, which depends on the ratio of the critical crack length, a_c, to the depth, D, of the beam. In case of S = 4D as applied in the current study, $f(\alpha_c)$ can be written as follows (Tada et al. 1985):

$$f(\alpha_c) = \frac{1.99 - \alpha_c(1-\alpha_c)(2.15 - 3.93\alpha_c + 2.7\alpha_c^2)}{(1+2\alpha_c)(1-\alpha_c)^{3/2}} \tag{1.1}$$

$$\alpha_c = \frac{a_c}{D} \tag{1.2}$$

On the other hand, the value of K_{Ic}^{ini} can be evaluated using the following Equation (Xu & Reinhardt 1999a):

$$K_{Ic}^{ini} = \frac{1.5 P_{ini} S \sqrt{a_0}}{D^2 B} f(\alpha_0) \tag{2}$$

where P_{ini} is the initial cracking load; $f(\alpha_0)$ is a geometry factor, which depends on the ratio of initial crack length a_0 to depth D of the beam:

$$f(\alpha_0) = \frac{1.99 - \alpha_0(1-\alpha_0)(2.15 - 3.93\alpha_0 + 2.7\alpha_0^2)}{(1+2\alpha_0)(1-\alpha_0)^{3/2}} \tag{2.1}$$

$$\alpha_0 = \frac{a_0}{h} \tag{2.2}$$

For WS specimens, the value of K_{Ic}^{un} can be evaluated according to the following expressions (Xu & Reinhardt 1999b):

$$K_{Ic}^{un} = \frac{P_{\mathrm{H}\max} \times 10^{-3}}{B D^{1/2}} f(\alpha_c) \tag{3}$$

where P_{max} is the horizontal component of maximum load; D and B are depth and width of WS specimens, respectively; $f(\alpha_c)$ is geometry factor, which depends on the ratio of critical crack length a_c to depth D of the beam. For $S=4D$, $f(\alpha_c)$ is given as follows (Xu & Reinhardt 1999b):

$$f(\alpha_c) = \frac{3.675\left[1 - 0.12(\alpha_c - 0.45)\right]}{(1-\alpha_c)^{3/2}} \tag{3.1}$$

$$\alpha_c = \frac{a_c}{D} \tag{3.2}$$

The value of K_{Ic}^{ini} can be evaluated according to the following expression (Xu & Reinhardt 1999b):

$$K_{Ic}^{ini} = \frac{P_{\mathrm{Hini}} \times 10^{-3}}{B D^{1/2}} f(\alpha_0) \tag{4}$$

where P_{ini} is the horizontal component of initial cracking load; $f(\alpha_c)$ is a geometry factor, which depends on the ratio of initial crack length a_0 to depth D of the beam:

$$f(\alpha_0) = \frac{3.675\left[1 - 0.12(\alpha_0 - 0.45)\right]}{(1-\alpha_0)^{3/2}} \tag{4.1}$$

$$\alpha_0 = \frac{a_0}{h} \tag{4.2}$$

Table 2. The geometry effect of double-K fracture parameters.

Types	K_{Ic}^{ini} (MPam$^{1/2}$)	$K_{Ic}^{ini}{}_{(TPB)}/K_{Ic}^{ini}{}_{(WS)}$	K_{Ic}^{un} (Mpam$^{1/2}$)	$K_{Ic}^{un}{}_{(TPB)}/K_{Ic}^{un}{}_{(WS)}$
TPB1	0.722	1.221	1.655	1.168
WS1	0.591		1.417	
TPB2	0.834	1.243	1.823	1.151
WS2	0.671		1.585	
TPB3	0.934	1.246	1.938	1.152
WS3	0.749		1.682	
TPB4	0.981	1.136	2.021	1.091
WS4	0.863		1.852	
TPB5	1.103	1.150	2.118	1.067
WS5	0.959		1.985	
TPB6	1.177	1.087	2.276	1.030
WS6	1.083		2.210	
TPB7	1.352	1.162	2.566	1.120
WS7	1.163		2.292	
Ave.		1.178		1.111
S.D.		0.060		0.051
C.V.		0.051		0.046

For both TPB and WS specimens, the cohesion toughness K_{Ic}^{c}, which is defined as the energy absorbed in the progressive extension of a fictitious crack zone, can be obtained by the following expression:

$$K_{Ic}^{c} = K_{Ic}^{un} - K_{Ic}^{ini} \tag{5}$$

3.2 *Brittleness Index*

The non-dimensional brittleness index, β, proposed by Kumar & Barai (2009), was applied to evaluate the brittleness of the concrete by using the ratio of K_{Ic}^{ini} to K_{Ic}^{un}:

$$\beta = \frac{K_{Ic}^{ini}}{K_{Ic}^{un}} \tag{6}$$

4 TEST RESULTS AND DISCUSSIONS

4.1 *Double-K fracture parameters*

Table 2 summarizes the values of P_{ini} and P_{max}, which were obtained from the tests, and K_{Ic}^{ini}, K_{Ic}^{c}, K_{Ic}^{un} and β, which were calculated through the formula presented in the previous section. The average values of P_{ini} , P_{max} and P_{ini}/P_{max} for TPB specimens changed from 4.885 kN to 9.152 kN, from 6.644 kN to 10.942 kN and from 0.741 to 0.838, respectively, when concrete strength increased from 30 MPa to 150 MPa. Since P_{ini} is linearly proportional to the tensile strength of concrete, it can be known that the tensile strength increased by 0.87 times only when the concrete strength increased by 4 times. So the ratio of tensile strength to compressive strength decreased significantly when concrete strength increased. The increase of P_{ini}/P_{max} implies that the linear portion of the uni-axial tensile stress-strain curve becomes more significant. Both the above two observations indicate the more brittle behavior of the HSC. For WS specimens, the similar results were obtained: the average values of P_{ini}, P_{max} and P_{ini}/P_{max} increased from 6.647 kN to 13.079 kN, from 9.876 kN to 16.424 kN and from 0.678 to 0.796, respectively.

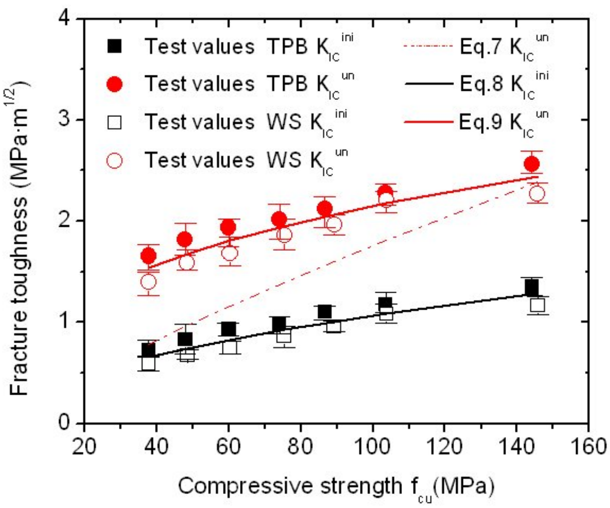

Figure 4. Double fracture toughness vs. concrete strength.

Figure 4 present the relationships between the initial fracture toughness K_{Ic}^{ini}, the unstable fracture toughness K_{Ic}^{un} and the compressive strength of concrete. The increase of the average values of K_{Ic}^{ini} and K_{Ic}^{un} with the concrete strength is apparent in both

TPB and WS tests. A larger fracture toughness of HSC means that it is more difficult for cracks to initiate and propagate in HSC as compared NSC. Based on fracture tests on cement mortar with different strengths but with the same maximum size (4.8 mm) of fine aggregates, John and Shah (1989) proposed a following empirical equation on the relationship between the compressive strength and the unstable fracture toughness:

$$K_{Ic}^{un} = 0.06(f_c)^{0.75} \tag{7}$$

where f_c is the compressive strength of cylinder in MPa, K_{Ic}^{un} is in MPa·m$^{1/2}$. Figure 4 compares the predicted unstable toughness by Equation 7 and the test results. It is shown that Equation 7 generally leads to a significant underestimation. This large gap is understood to be mainly attributed to the difference in material composition between concrete and mortar. Compared to the compressive strength (water to binder ratio), the maximum size and properties of coarse aggregates seem to be more dominant factors influencing the fracture toughness of concrete.

Based upon regression analysis on the testing data, two empirical relationships between the compressive strength of concrete (30 MPa ~150 MPa) and fracture parameters K_{Ic}^{ini} and K_{Ic}^{un} can be obtained as follows:

$$K_{Ic}^{ini} = 0.108(f_{cu})^{0.497} \tag{8}$$

$$K_{Ic}^{un} = 0.447(f_{cu})^{0.341} \tag{9}$$

where f_{cu} (MPa) is the cube compressive strength of concrete. It should be noted that the above equations are obtained based on concrete with the use of coarse aggregates with a maximum size of 20 mm. The equations may reflect mainly the effects of water to binder ratio on the fracture toughness but the effects of aggregate size remain to be further studied.

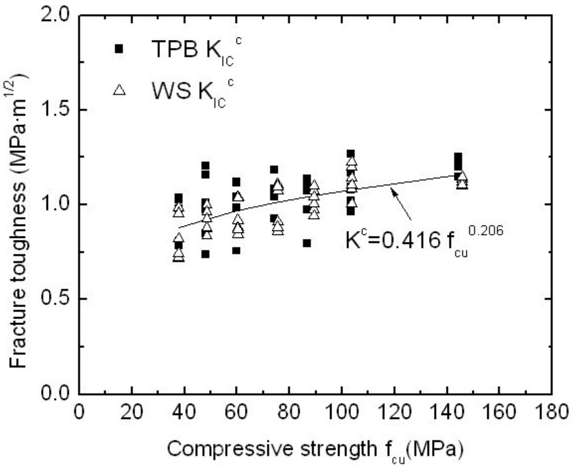

Figure 5. Cohesive toughness K_{Ic}^c vs. concrete strength.

4.2 *Cohesive toughness and brittleness*

Figure 5 present the values of cohesive toughness K_{Ic}^c, which were calculated from the aforementioned double-K fracture parameters. For TPB specimens, the average value of K_{Ic}^c increased by 30% only (from 0.933 MPa·m$^{1/2}$ to 1.214 MPa·m$^{1/2}$) when the compressive strength increased by 4 times (from 30 MPa to 150 MPa), while for the WS specimens, the average values of K_{Ic}^c increased by 36.5% only (from 0.826 MPa·m$^{1/2}$ to 1.128 MPa·m$^{1/2}$). This result can be interpreted that the cohesive fracture toughness is mainly governed by the maximum size and properties of coarse aggregates. The compressive strength itself may not have strong effects on the cohesive fracture toughness. An empirical relationship between the cohesive toughness K_{Ic}^c and the compressive strength of concrete (30 MPa ~150 MPa) can be obtained as follows:

$$K_{Ic}^{c} = 0.416(f_{cu})^{0.206} \tag{10}$$

Figure 6 presents the relationship between the non-dimensional brittleness indexes, *β,* and the concrete strength. It is seen that the former increased gradually with the latter. The larger the value of *β* is, the more brittle fracture behavior the concrete possesses. Although the resistance of concrete to the crack initiation increased in case of HSC due to stronger bonding action between the aggregate particles and the cementitious matrix and high matrix strength, the propagation of cracks seemed to faster than NSC.

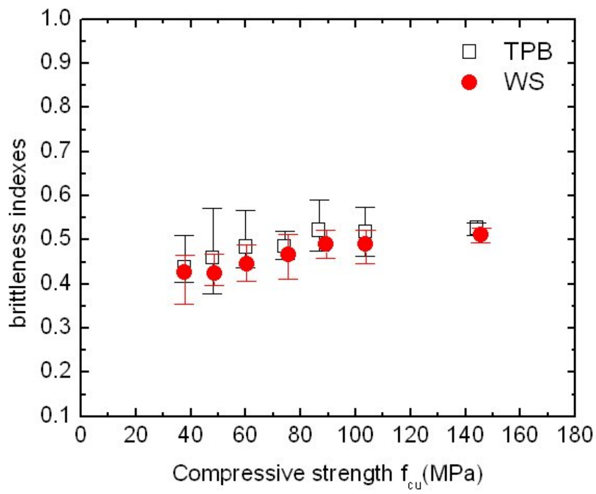

Figure 6. Concrete brittleness vs. concrete strength.

4.3 *Comparison between TPB and WS tests*

Figure 4 presents a comparison of Double-K fracture toughness of concrete between the TPB and WS tests. The ratios of $K_{Ic}^{ini}{}_{(TPB)}$ to $K_{Ic}^{ini}{}_{(WS)}$ and $K_{Ic}^{un}{}_{(TPB)}$ to $K_{Ic}^{un}{}_{(WS)}$ are in between 1.087 and 1.246 with an average value of 1.178 and between 1.03 and 1.168

with an average value of 1.111, respectively. Basically, two test methods lead to rather consistent results on the double-K fracture parameters without obvious significant geometry effects, while the values of K_{Ic}^{ini} and K_{Ic}^{un} in WS specimens are slightly smaller than those in TPB specimens provided the same concrete strength.

5 CONCLUSIONS

A series of tests on TPB and WS specimens were carried out to investigate the fracture properties and brittleness of concrete with different strengths. Based upon the testing results, the following conclusions can be drawn up:

a) The initial cracking load, ultimate failure load, initial fracture toughness K_{Ic}^{ini}, unstable fracture toughness K_{Ic}^{un}, and the cohesive toughness K_{Ic}^{c} exhibited an increasing tendency with the increase of the compressive strength of concrete.
b) A higher compressive strength of concrete materials led to a larger value of non-dimensional brittleness index, β, in other words, more brittle fracture of concrete.
c) Empirical formula on the initial fracture toughness K_{Ic}^{ini} and unstable fracture toughness K_{Ic}^{un} as a function of compressive strength of concrete are obtained and can be used to evaluate the fracture properties of concretes with different strengths.
d) The double-K fracture parameters and the brittleness index are independent of test geometry and hence are good indicators for assessment of the fracture behavior of concrete materials with different strengths.

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