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Fiber reinforced concrete characterization through round panel test - Part II: analytical and numerical study

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ABSTRACT: The round determinate panel according to ASTM was found to be a reliable, consistent and repeatable test method for the measurement of the energy absorption in Fiber Reinforced Concrete (FRC) composites. A smaller panel was proposed and experimentally investigated by the authors in the Part I of the present paper. The results of several tests on about 100 round panels (ASTM and small panels) evidenced a lower scatter, as in the larger ASTM panels, with significant advantages. In fact, small round panels are easier to place and handle and their weight is only 40 kg, if compared to the 91 kg of the ASTM panel. Given this broad experimental database, an analytical approach is herein reported toward the definition of a suitable simplified cohesive stress-crack width constitutive law for FRC, determined from round panel tests according to the requirements of the main structural codes. To this aim, crack development was measured during the tests. In addition, a kinematic approach was proposed to determine the constitutive laws from the load point displacement of the slab.

1 INTRODUCTION

A standardized Round Determinate Panel (RDP) test is nowadays available and published by ASTM (2004) for the measurements of the energy absorption at 40 mm displacement, for characterizing Fiber Reinforced Concrete (FRC) elements with special emphasis on sprayed concrete in tunneling applications. It is a statically determinate test, with round shape slab having a diameter (ϕ) of 800 mm and a thickness of 75 mm, supported in three points at 120 degrees. The standard test is rather straightforward and only requires load and displacement measurement. The specimen weight is 91 kg.

In the first part of this investigation, the authors demonstrated that a smaller round panel with a diameter of 600 mm and a thickness of 60 mm is characterized by a similar scatter in results, which is much lower compared to the classical tests on small notched beams (Minelli & Plizzari 2007 and 2009). Moreover, the three crack measurements could be also included in the test, as the crack pattern is repeatable and predictable; therefore, the post-cracking material properties can be adequately determined. In addition, handling and placing such a specimen is easier as compared to the classical panel since its weight is only 40 kg. Finally, standard servo-controlled loading machines generally fit with the geometry of the small panel, allowing for a proper crack controlled tests with a close-loop system.

In order to use this test procedure for characterizing FRC, which means determining fracture properties such as toughness indexes, residual post cracking strengths and equivalent post-cracking stresses, one should be able to find sufficient information from the experiments. The procedures adopted in many standards for beam tests (UNI 2003, RILEM 2003 & CEN 2005) are based on the determination of a stress-crack width curve, in which:
-the stress can be derived from theory of elasticity, *i.e.* the flexural stress is just the moment over the resistance modulus. This is a rather rough simplification but it allows for the definition of suitable post-cracking strengths or toughness indexes without any non-linear analysis, which is difficult to perform;
-the crack width is measured and monitored (Crack Tip Opening Displacement – CTOD), especially while dealing with notched beams.

Stresses causing cracks (*i.e.* normal stresses along the crack line) and crack width records are therefore needed if the procedure well recognized for beams is to be transferred to round panels.

The virtual energy-based yield line method (Johansen 1972) in which the uniaxial flexural capacity of the material upon cracking is used together with an assumed pattern of failure to predict the point load capacity, might be also used to determine a simple crack width as a function of the point load displacement (Bernard 2000, 2002 and 2006). However, the yield line method was originally developed for nominally plastic materials; therefore, its application for the prediction of load capacity in structures made of lightly reinforced concrete exhibiting brittle behavior might be questionable.

The present investigation focuses on the applicability of round panel tests in defining a simplified and easy-to-use constitutive law for FRC material. Numerical elastic analyses and yield line method will be both utilized to this aim. In particular, the latter approach will be examined and evaluated as far as the prediction of cracking is concerned and compared against the experimental results.

2 ANALYTICAL STUDY

As diffusely reported in Part I of this paper, more than 100 experiments on round panels were carried out in the last 4 years at the University of Brescia. Based on this broad experimentation, an analytical model was then developed to the aim of:
-determining local stress-crack opening cohesive law from round panels, similarly to the procedure well acknowledged for beams;
-defining suitable equivalent post-cracking strength directly from panel tests, in the same way as for beam tests.

First of all, appropriate general relations, according to the thin plate theory, were determined for describing the maximum tangential stress (as well as the radial and tangential stress pattern, as shown in Fig. 1 and Fig. 2) and the displacement at the load point. An elastic finite element analysis, through the commercial software STRAUS7 (2004) with two dimensional elements (plates) was performed with the following results:

$$\sigma_t = 0.001816 \frac{P \cdot D}{t^2} \quad (1)$$

$$\eta_{RPS} = 0.000429 \frac{P \cdot D^3}{E \cdot t^3} \quad (2)$$

where P is the external load in the center (a unit load of 10 kN was applied), D is the diameter of the panel measured from the supports (550 mm in the case of small round panel), t is the panel thickness (60 mm), E and ν are Young's and Poisson's modulus of concrete, respectively. All units are defined according to SI unit system.

The elastic analysis was also able to predict the distribution of radial and tangential stresses along the line of crack formation (*i.e.*, the radial bisector between each pair of pivot supports), as depicted in Figure 3. Note that the elastic solution is an easy-to-use tool that allows a simplified calculation of the equivalent post-cracking strength being feasible for practical and design applications, in the same way as it is done for beam tests.

The results show that the peak tensile stresses occur near the loading point and a broad area of

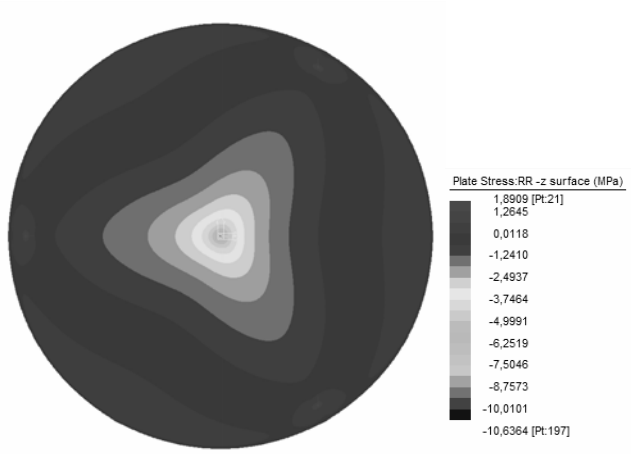

Figure 1. radial stresses σ_r on a small round panel, elastic analysis with FE program Straus7 (2004).

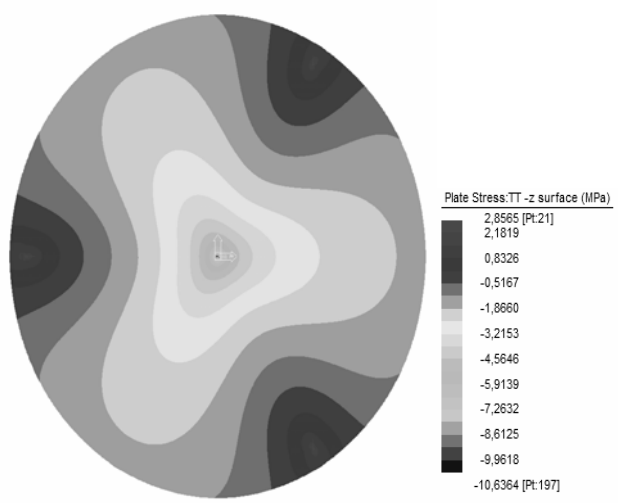

Figure 2. tangential stresses σ_t on a small round panel, elastic analysis with FE program Straus7 (2004).

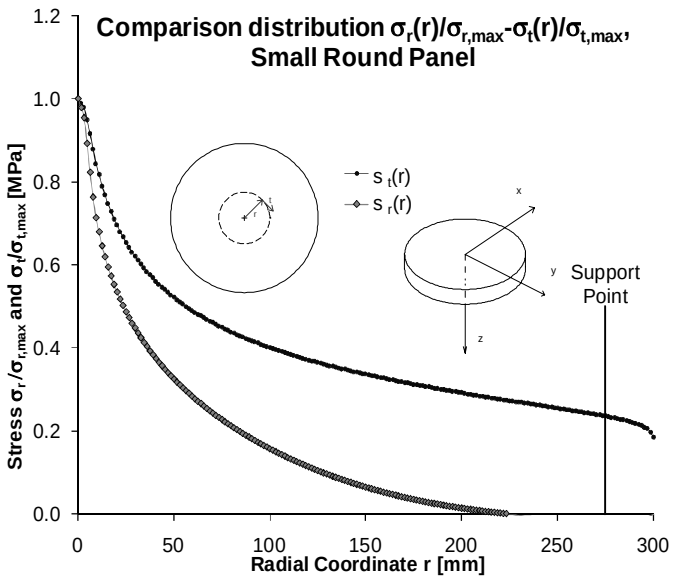

Figure 3. Elastic radial and tangential stresses along the line of crack formation (radial coordinate).

heightened tensile stress arose along the crack line. While the radial and tangential stresses have the same magnitude at the center, they diverge toward the edge of the panel along each radial bisector, the ra-

dial stress dropping to zero for meeting the boundary condition at the edge (see also Bernard 2006).

By using the above equations for the determination of the local tangential maximum tensile stress along the yield line, it is possible to come up with a σ-w cohesive law and compare it with those determined from beam tests, whereby w defines the experimental measured crack widths.

As an alternative, without direct measurement of the crack width, a kinematic approach based on the yield line theory (more properly defined as virtual energy-based yield line method, Johansen 1972) could be adopted to calculate the crack width, as also done by Bernard (2002), Tran et al. (2001) and Lambrechts (2004). In a yield line analysis of a round panel, the governing mode of failure is taken to comprise three symmetrical yield lines-cracks emanating from the center of the face opposite the point load and running radially to the edge while bisecting each sector between adjacent pivot supports.

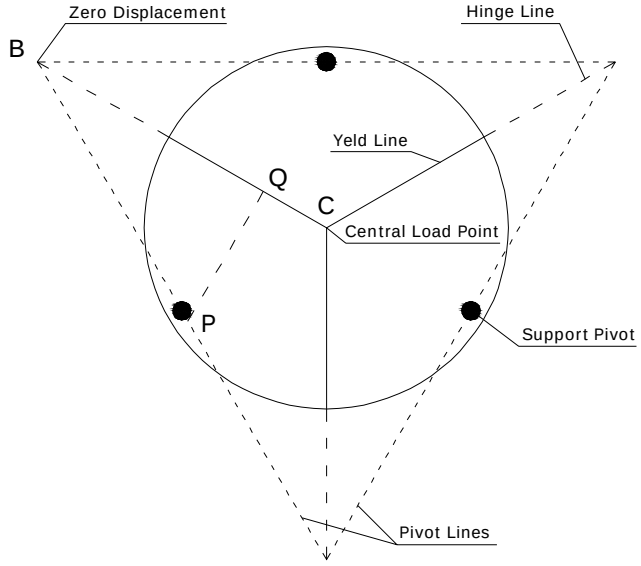

Figure 4. Yield Line Approach for the determination of crack width.

The crack width was experimentally measured at a distance from the point load of 120 mm (point Q in Fig. 4). Based on geometry consideration, it can be written that:

$$\frac{\overline{BC}}{\eta_{RPS}} = \frac{\overline{BQ}}{\delta_Q} \tag{3}$$

where CQ=120 mm (in the present experiments) whereas B is the zero displacement point, η_{RPS} and δ_Q are the vertical displacements at points C and Q, respectively. From trigonometry, one can derive that:

$$\frac{2r}{\eta_{RPS}} = \frac{\overline{BQ}}{\delta_Q} \Rightarrow \delta_Q = \overline{BQ} \cdot \left(\frac{\eta_{RPS}}{2r} \right) \tag{4}$$

with r =275 mm.

The rotation at the support α_P is equal to:

$$\delta_Q = \alpha_P \cdot \overline{PQ} \tag{5}$$

By substituting PQ and δ_q one can obtain:

$$\overline{BQ} \cdot \left(\frac{\eta_{RPS}}{2r} \right) = \alpha_P \cdot \frac{\overline{BQ}}{\sqrt{3}} \tag{6}$$

The rotation at the support P therefore yields:

$$\alpha_P = \frac{\sqrt{3}}{2} \frac{\eta_{RPS}}{r} \tag{7}$$

The rotation in Q, at the point of experimental crack measurement, can be derived as:

$$\varphi_{RPS} = 2 \cdot \alpha_P = \sqrt{3} \cdot \frac{\eta_{RPS}}{r} \tag{8}$$

The corresponding bottom crack opening (Fig. 5) is therefore given by:

$$w_m = 2t \cdot tg\left(\frac{\varphi_{RPS}}{2} \right) = 2t \cdot tg\left(\frac{\sqrt{3}}{2} \frac{\eta_{RPS}}{r} \right) \tag{9}$$

where r is the radius (r = 275 mm) and t is the panel thickness (t = 60 mm).

It is worth noticing that the crack opening obtained using Equation 9 is constant along the radius, for a given value of panel displacement. However, in the actual case, the crack width will increase from the external surface to the load point.

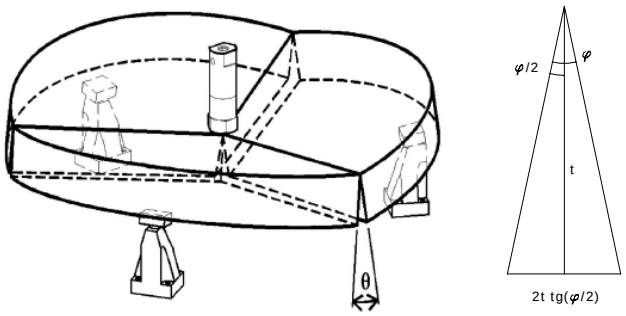

Figure 5. 3D representation of the failure mode of a round panel (a) (Bernard, 2006). Crack width, bottom surface of a round panel (b).

Figure 6 shows the comparison between experimental and analytical crack width: the experimental curve was calculated from a linear regression of all test results available. It is a bilinear curve, with a first constant line with zero crack representing the uncracked phase, in which the vertical displacement increases prior to cracking (up to a value of around η_0=0.54-0.63 mm, in which the lowest value is appropriate for normal strength concretes while the highest is more suitable for high strength concretes), and a second linear branch, showing a quite good fitting of the experiments (R^2=0.99). The difference between the two curves (with the experimental showing

larger cracks for the same vertical displacement) is probably due to the fact that a little elastic deformation is always measured by the LVDTs in the experiments (150 mm gauge length) and, most importantly, their location is at 120 mm from the load point, while the yield line theory would match better with a measurement at r/2 from the load point (*i.e.*, 137.5 mm).

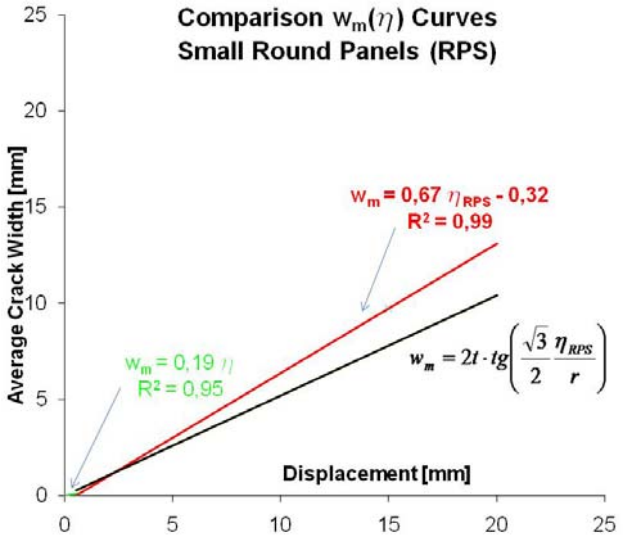

Figure 6. Crack width at bottom surface of a small round panel.

This procedure allows for the definition of a constitutive cohesive σ-w law based on round panel experiments. From a linear regression of experimental results, an analytical relation between the crack tip opening displacement (CTOD) and the vertical displacement at midspan of a beam (Fig. 7), according to UNI, was derived as follows:

$$CTOD_m = 1.03 \cdot \eta_B \tag{10}$$

From a kinematic model shown in Figure 8, it was obtained that:

$$\varphi_B = \frac{4 \cdot \eta_B}{l} \tag{11}$$

$$w_m = 2 \cdot (h - a_0) \cdot tg\left(\frac{\varphi_B}{2}\right) = 2 \cdot (h - a_0) \cdot tg\left(\frac{2 \cdot \eta_B}{l}\right) \tag{12}$$

Note that the same approach for calculating the rotation and crack width can be followed for beam according to CEN (2005), provided that a notch depth a_0=25 mm, rather than 45 mm, is used. Figure 8 would also apply for the CEN beam test (2005).

The Italian Standard UNI 11039 (2003) defines two different equivalent post-cracking strengths, $f_{eq(0-0.6)}$ and $f_{eq(0.6-3)}$, which represent the toughness given to the matrix by fibers in a conventional serviceability (crack width from 0 to 0.6 mm) and ultimate limit states (crack widths from 0.6 to 3 mm), as follows (Fig. 9):

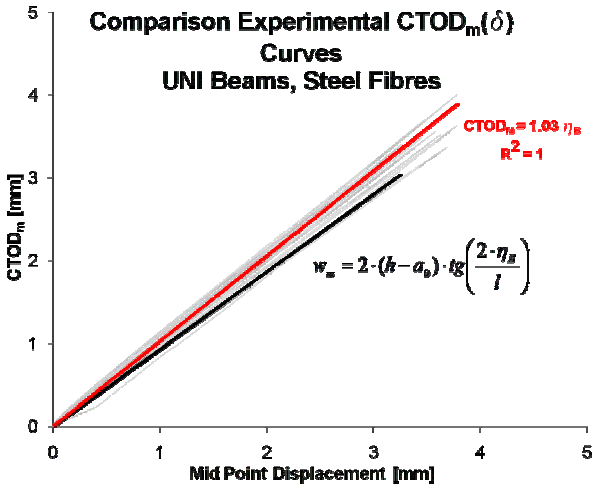

Figure 7. Crack width at bottom surface of a small round panel.

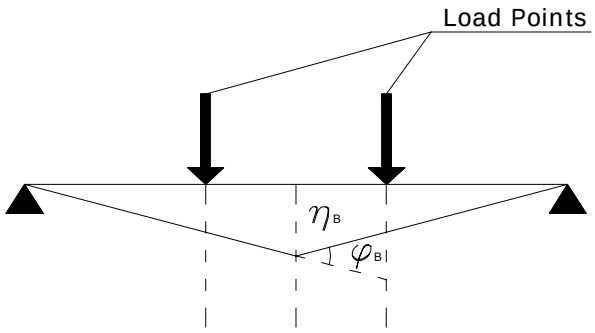

Figure 8. Kinematic model for 4 point bending tests UNI beams.

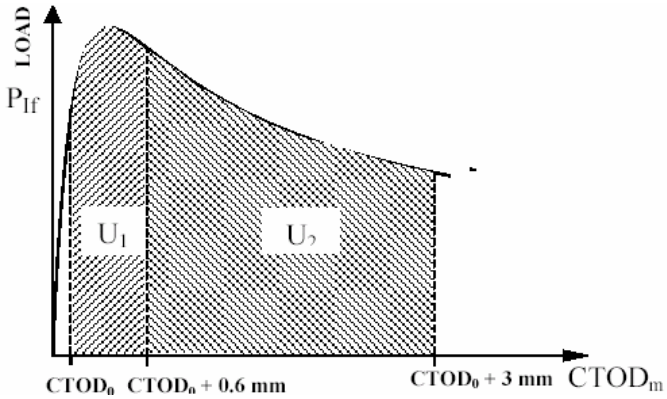

Figure 9. Definition of equivalent post-cracking stresses according to UNI Standard (2003).

$$f_{eq,(0-0.6)} = \frac{l}{b(h - a_0)^2} \cdot \frac{U_1}{0.6} \tag{13}$$

$$f_{eq,(0.6-3)} = \frac{l}{b(h - a_0)^2} \cdot \frac{U_2}{2.4} \tag{14}$$

By using Equations 11 and 12, it is possible to correlate crack ranges with vertical displacements η_B and rotations φ_B of beams.

Table 1. Midspan displacements and rotations corresponding to the $CTOD_m$ defined by the UNI standard 11039 (2003).

$CTOD_m$ [mm]	η_B [mm]	φ_B
0.6	0.58	0.052
3	2.91	0.0259

For round panels, it was experimentally found that (Fig. 6):

$$w_m = 0.67 \cdot \eta_{RPS} - 0.32 \quad (15)$$

By imposing the rotation of round panels to be equal to the one of beams, according to the yield line theory, using Equation 8 one can first determine the vertical displacement at the point load η_{RPS}, and then the crack width corresponding to the rotation given (Equation 15). The crack width values then define two ranges that allow the definition of two equivalent post-cracking strengths, relevant for serviceability and ultimate limit states, respectively. The results of this procedure are reported in Table 2.

Table 2. Vertical displacement and crack width values corresponding to the same yield line rotation between panels and beams.

$\varphi_{RPS}=\varphi_B$	η_{RPS} [mm]	w_m [mm]
0.0052	0.82	0.55
0.0259	4.11	2.75

Therefore one can define two crack width ranges and determine the corresponding equivalent post-cracking strengths, which are:
- $f_{eq,1}$ from w_0 to $w_0 + w_1$;
- $f_{eq,2}$ from $w_0 + w_1$ to $w_0 + w_2$.

being w_0 the crack width at first cracking, w_1=0.55 mm and w_2=2.75 mm (Table 2). The two equivalent post-cracking strengths can therefore be defined as:

$$f_{eq1} = \frac{k_2 \cdot D}{t^2} \cdot \frac{U_1}{w_1} \quad (16)$$

$$f_{eq2} = \frac{k_2 \cdot D}{t^2} \cdot \frac{U_2}{w_2 - w_1} \quad (17)$$

where:
- $f_{eq,1}$ is the equivalent post-cracking strength relevant for serviceability limit states, corresponding to $f_{eq(0\text{-}0.6)}$ of UNI beams;
- $f_{eq,2}$ is the equivalent post-cracking strength relevant for serviceability limit states, corresponding to $f_{eq(0.6\text{-}3)}$ of UNI beams;
- U_1 is the area under the *P-w* experimental graph calculated in the crack range from w_0 to $w_0 + w_1$ (Fig. 10);
- U_2 is the area under the *P-w* experimental graph calculated in the crack range from $w_0 + w_1$ to $w_0 + w_2$ (Fig. 10);
- k_2 = 0.001816 mm^{-1} is the constant already calculated throughout the elastic analysis previously shown.

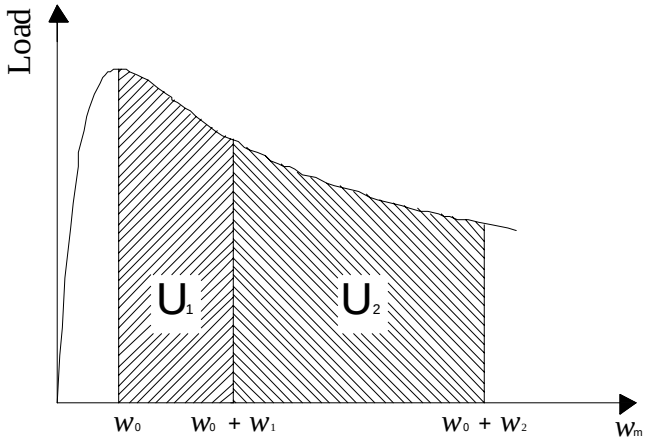

Figure 10. Load-crack width curve for small round panels depicting areas U_1 and U_2.

This approach can be used to determine average values, standard deviations and characteristic values of the toughness parameters defined, taking a significant advantage from the lower scatter of panel tests over beams (see Conti and Flelli 2009).

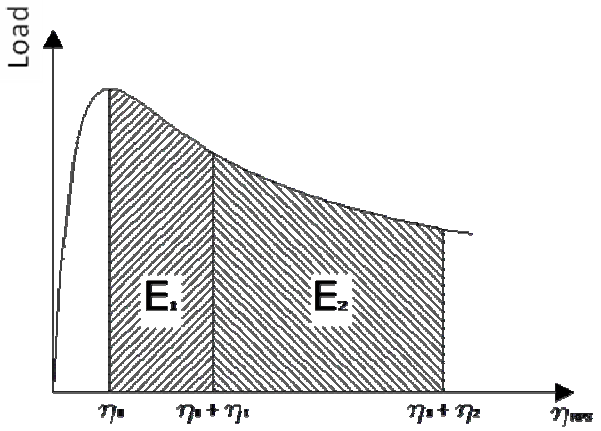

Figure 11. Typical Load-*w* curve for a small round panel defining areas E_1 and E_2.

A similar approach can be derived starting from the load-displacement curve of panels (Fig. 11), neglecting the crack width measurements (often not provided). In such circumstances, one can define suitable displacement ranges for the definition of f_{eq1} and f_{eq2}. The displacement ranges, calculated from Equation 14, are incorporated into the definition of the two equivalent post-cracking strengths as:

$$f_{eq1} = \frac{k_2 \cdot D}{t^2} \cdot \frac{E_1}{\eta_1} \quad (18)$$

$$f_{eq2} = \frac{k_2 \cdot D}{t^2} \cdot \frac{E_2}{\eta_2 - \eta_1} \quad (19)$$

being η_1=0.82 mm and η_2=4.11 mm (Table 2).

According to the Italian Guidelines for the Design, Construction and Production Control of Fibre Reinforced Concrete Structures (CNR, 2006), once suitable beam tests are performed and the corresponding values of f_{eq1} and f_{eq2} (also indicated as $f_{eq(0-0.6)}$ and $f_{eq(0.6-3)}$ if the aforementioned UNI test is considered), it is possible to define a simplified stress-crack width constitutive cohesive law as follows:

$$f_{Fts} = 0.45 \cdot f_{eq1} \tag{20}$$

$$f_{Ftu} = k \cdot \left[f_{Fts} - \frac{w_u}{w_{i2}} \cdot \left(f_{Fts} - 0.5 \cdot f_{eq2} + 0.2 \cdot f_{eq1} \right) \right] \geq 0 \tag{21}$$

where:

- k is a coefficient equal to 0.7 for uncracked sections under tension and equal to 1 in the other cases k=1 in this case;
- w_{i2} is the average of the crack width values for which f_{eq2} is calculated. In this case w_{i2}=1.8 mm (average value between 0.6 and 3 mm according to UNI 11039);
- w_u is the greatest value of the crack range for which f_{eq2} is calculated. In this case w_u=3 mm (according to UNI 11039);

Figure 12 depicts a typical linear constitutive law obtained from the equivalent post-cracking stresses calculated from beam test.

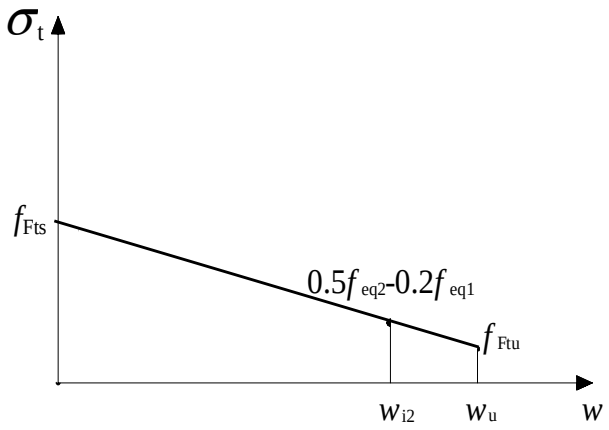

Figure 12. Typical σ-w simplified cohesive constitutive law according to CNR DT 204/2006.

One can notice that, to the aim of coming up with a simplified and easy-to-use constitutive law, the standard neglects the first branch from the tensile strength to the value f_{Fts}. Moreover, the σ-w indicated in the picture refers to a strain-softening material under tension.

Figure 13 and Figure 14 report, as an example, the characteristic values of equivalent post-cracking stresses calculated by using the procedure proposed herein for small round panels and the classical method included in UNI 11039 (2003) for beams. The exemplification refers to a normal strength concrete with two different contents of steel fibers, 30 and 60 kg/m^3, according to Conti and Flelli (2009).

The advantage concerns the significantly smaller dispersion of experimental results that means higher characteristic values and, therefore, an improved stress-crack width cohesive law, as depicted in Figure 15. The fracture energy, as the area under the σ-w diagram, is around 25% larger in the two round panel diagrams plotted, which refer to the characteristic values shown in Figure 13 and Figure 14. The constitutive law related to Beam 1, in the plot, has the only advantage to show the best performance for high crack widths, exceeding all other laws depicted. This evidence comes from the fact that the two equivalent characteristic post-cracking stresses are rather similar and then, in the calculation the subtrahend would be lower (Equation 21). This would also suggest that there is a need to further corroborate and validate the proposal included in CNR (2006) against a broader set of experimental data.

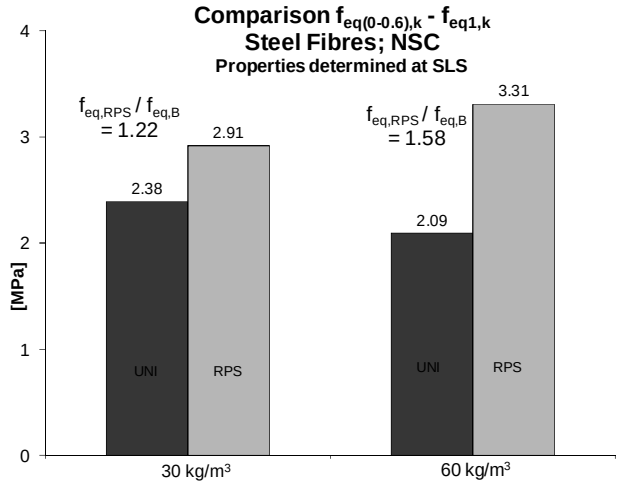

Figure 13. Analytical calculation of equivalent post-cracking stresses from experiments. *NSC* beams and panels from identical concrete batch (Conti and Flelli, 2009).

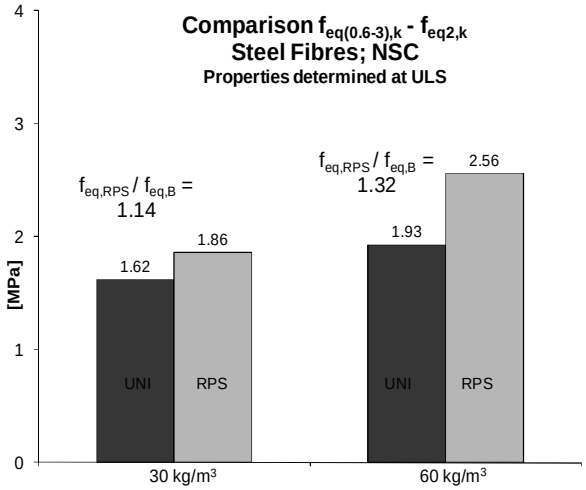

Figure 14. Analytical calculation of equivalent post-cracking stresses from experiments. *NSC* beams and panels from identical concrete batch (Conti and Flelli, 2009).

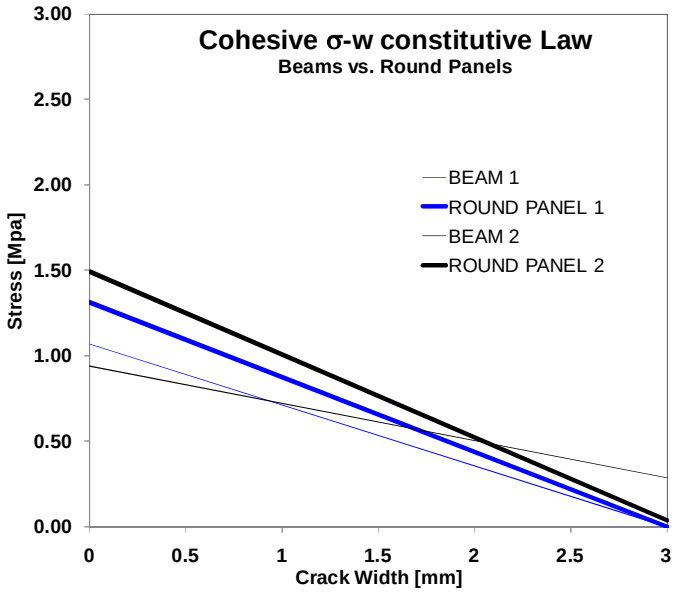

Figure 15. σ-w laws according to CNR DT 204/2006. Beam test v. Panel test. NSC beams and panels from identical concrete batch (Conti & Flelli, 2009).

3 CONCLUDING REMARKS

In the present paper, numerical elastic and the yield line approach were adopted to come up with simplified cohesive constitutive laws from round panel small tests.

Panel test is quite appropriate for representing the actual behaviour of FRC materials. It can be considered as a complete test for the characterization of FRC: suitable range of crack widths were defined and the corresponding equivalent (or residual) post-cracking strengths were derived (from σ-w plots) following the same procedure as done for beam tests.

The kinematic approach for calculating crack width turned out to be simple, effective and rather accurate if compared against experiments.

Panel test can become a very promising test method for the characterization of FRC composites, as it is relatively easy to perform and it is not expensive, it might require only one vertical displacement transducer and, last but not least, it is characterized by a very low scatter.

Further studies are actually going on at the University of Brescia for supplementary refine and validate the proposed calculation.

Among the open issues for further studies, the following key-points should be kept in mind:

- the position of the crack measurements: the crack width varies along the crack line, and changes in a different way before and after cracking. Moreover, we should also recall that tangential stresses have to be considered with radial stresses, which are non zero for a length of 2/3 of the crack line. One should think at measuring crack close to the pivot support, where radial stresses are negligible, but then the crack width would be lower and, more notably, the stress disturbance of the pivot support might affect the record.
- The 60 mm thickness of the small round panel should be well evaluated especially with longer fibers (longer than 50 mm). Suitable limitations should be defined concerning maximum allowable fiber length for which the small round panel test can be considered significant. Similar limitation, however, should be defined also for classical round panels, where the thickness is only slightly higher (75 mm).
- The thin plate geometry might cause a 2D orientation, which should be well evaluated and compared with that of beam tests and samples taken directly from the field structural application.

4 ACKNOWLEDGMENTS

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