Tension tests and structural applications of strain-hardening fiberreinforced cementitious composites

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ABSTRACT: Major indices for evaluating fracture of concrete are overviewed. The Japan Society of Civil Engineers has proposed tension test methods for SHCCs and structural design recommendations using tensile performance for ensuring safety and durability of SHCC structures. Small-size dumbbell-shaped specimens with one end fix-supported and the other pin-supported are proposed for tension tests in the recommendations. Retrofit is considered to be one of the most beneficial applications of SHCC in Japan. It is important to enrich ideas and examples of attractive SHCC applications making most of their features, and observe change of structures involving SHCCs over time.

1 INTRODUCTION

Concrete is prone to cracking. It resists compressive forces but readily suffers cracking under tensile forces. Techniques related to concrete are mostly concerned with suppression of cracking and control of crack width to small levels at the design, analysis, material, and construction stages. Concrete members or even entire structures can be destroyed unless cracks are properly controlled as found in shear failure of reinforced concrete members.

The fracture behavior of a material is generally expressed by a stress-strain curve, whereas that of a member or structure is expressed by a loaddisplacement curve. Indices characterizing the fracture properties of concrete include stress, strain, energy, and crack width. In the field of concrete fracture mechanics, the tension-softening curve, which is expressed by the relationship between crack width and tensile stress, as well as fracture energy, have attracted attention as key indices since the end of the 1970s and have been applied to evaluations of material properties and numerical analysis. Strainhardening fiber-reinforced cementitious composites (SHCCs) were developed in the 1990s. For SHCC, the tensile strain-hardening curve and crack width are important indices.

This paper overviews fracture mechanics for concrete in relation to indices used and the surrounding circumstances. The characteristics of SHCCs and test methods for their tensile performance are then described, and the current and future applications of SHCCs are discussed at the end.

2 FRACTURE MECHANICS OF CONCRETE AND CONCRETE STRUCTURES

2.1 Fracture indices and load-displacement curves of concrete

Fracture mechanics, which is based on Griffith's theory (Griffith 1921) and rapidly developed during the 1950s in the fields of mechanical engineering and metal engineering, was applied to concrete engineering in the 1960s. This was done roughly in three stages as given in Table 1.

In the 1960s, Kaplan (1961) applied fracture mechanics to concrete to determine the critical energy release rate, G_c . Such fracture toughness values as the critical stress intensity factor, K_c , the critical energy release rate, G_c , and the critical J-integral, J_c , were then used for mortar, concrete, and fiberreinforced concrete (FRC) as indices for measuring brittle fracture properties and evaluating the effect of improving these properties, followed by various other evaluation indices.

In the latter half of the 1970s, Hillerborg et al. (1976) proposed the tension-softening curve expressed by the crack width and the tensile stress

| Table 1. Fracture indices in three stages. | | | | |
|--|--|--|--|--|
| Stage 1 | $K_{\rm c}, G_{\rm c} \text{ and } J_{\rm c}$ | | | |
| | For measuring the brittle fracture properties | | | |
| | of concrete and FRC | | | |
| Stage 2 | $G_{\rm f}$ and tension-softening curve | | | |
| | For numerical analysis of fracture of concrete | | | |
| | including the size effect | | | |
| Stage 3 | Strain-hardening curve and crack width | | | |
| - | For structural design regarding safety and dura- | | | |
| | bility for SHCC | | | |
| | | | | |

transferred at cracking areas, as well as $G_{\rm f}$, expressed by the area under the curve. These indices have been incorporated in numerical analysis techniques including the finite element method, contributing to the improvement in the reality of analysis regarding fracture behavior and crack propagation. The usefulness of these indices was widely recognized when the size effect, with which the shear capacity of reinforced concrete members does not increase in proportion to its size, was explained by numerical analysis incorporating a tension-softening curve. A test method for the fracture energy, $G_{\rm f}$, using notched flexural specimens was proposed by the 50-FMC Committee of RILEM (1985), contributing to widespread use of $G_{\rm f}$. In Japan, a method of estimating the tension-softening curve from test results of flexural specimens by back analysis has been proposed by Japan Concrete Institute (JCI Standard 2003). The tension-softening curve was also adopted in the Standard Specifications for Design and Construction of Concrete Structures published by the Japan Society of Civil Engineers (JSCE 2002).

In the 1990s, Li (1993) developed strainhardening fiber-reinforced cementitious composites (SHCCs) that show strain-hardening behavior, with which both the tensile stress and tensile strain increase under tensile forces, and tendencies toward multiple fine cracks. Fracture mechanics was employed to evaluate the pullout behavior of fibers from the mortar matrix. The shape of the tensile strain-hardening curve up to the strength failure point and the crack width, which is a material property, are more important for this material than the shape of the tension-softening curve. These indices are necessary for structural design regarding the safety and durability of a concrete structure made of SHCC. This material has been applied to actual structures since 2000s. JSCE recommendations for design and construction for SHCCs (JSCE 2007) cover this material, providing a uniaxial tension test method using dumbbell-shaped specimens and method of measuring crack width.

While new fracture mechanics indices have been proposed with technical backgrounds, those that have been proven useful with standardized test methods have been widely used. In order to apply a new index to actual structures, it is important to include it in design standards.

2.2 Stable measurement of load-displacement curve after maximum load point

The fracture behavior of concrete specimens and reinforced concrete members is generally expressed as the relationship between the load and the displacement of the loading point on which the load acts. The area under the load-displacement curve up to the maximum load point represents the energy applied to the specimen, most of which is generally the stored elastic energy, $E_{\rm e}$. The area under the entire load-displacement curve including the part after the maximum load point represents the energy consumed by the fracture of the specimen, $E_{\rm d}$. While the load-displacement curve up to the maximum load point can be measured in a stable manner, the part thereafter can be difficult to measure under certain conditions, such as the case where $E_{\rm e}$ becomes greater than $E_{\rm d}$.

Such fracture toughness values as K_c , G_c , and J_c , which were used as indices to brittle fracture of concrete in the first stage of the application of fracture mechanics to concrete, can be determined from the load-displacement curve up to the maximum load point measured by tension or flexure tests on specimens. On the other hand, the tension-softening curve and fracture energy, $G_{\rm f}$, which have been used as constitutive laws for numerical analysis in the second stage of fracture mechanics application, can be determined from the entire load-displacement curve up to rupture of tension or flexure specimens. Since it is difficult to measure the entire load-displacement curve including the part after the maximum load point by uniaxial tension testing on specimens, a tension-softening curve and $G_{\rm f}$ are determined by back analysis from the entire load-displacement curve to rupture of flexure specimens. A notch on the tension edge of a flexure specimen facilitates the measurement of the entire load-displacement curve, as the reduction in $E_{\rm e}$ becomes greater than the reduction in E_{d} . In the third stage, the importance of the information on the strain-hardening curve up to the maximum load point and the crack width surpasses the significance of the load-displacement curve after the maximum load point.

In the latter half of the 1980s, the "snap back" phenomenon, in which both the load and displacement decrease after the maximum load point, was recognized during compression tests on high strength concrete specimens and specimens having an increased height while keeping the crosssectional area constant (Fig. 1). It was also reported



Figure 1. Load-displacement curve with snap back (Rokugo et al. 1986).



Fiugre 2. Snap back phenomenon of concrete specimen measured by compression testing machine.

that, when loading is not controlled, this phenomenon can cause explosive failure of specimens under test. Many studies have been conducted to control (Rokugo et al. 1986) or analyze (Carpinteri et al. 1986) the phenomenon.

As the number of reinforced concrete high-rise buildings has increased in recent years, with high strength concrete having a compressive strength of over 100 N/mm² being available in the form of ready-mixed concrete, compression testing for quality control in a safe manner has been demanded, with explosion of specimens being kept under control. A compression testing machine was developed in Japan, with which the load-displacement curve after the maximum load point can be measured under stable control of only load even when the snap back phenomenon is involved as exemplified in Figure 2. and is going to be placed on the market shortly. Studies on the control of snap back over 20 years have finally been reflected in the development of a general purpose compression testing machine. It is thus important to pursue modest studies dealing with such essential and fundamental matters as concrete fracture, even though many of them may not readily become useful.

3 FEATURES OF SHCC AND TENSION TEST METHODS

3.1 Features of SHCC

SHCC is a material that forms cracks with a width of a level of 0.1 mm or less one after another under increasing tensile forces, leading to large tensile deformation. It is characterized by the increase in the number of cracks instead of crack width. The compressive strength of SHCC ranges from 30 to 100 N/mm². Its tensile strength, flexural strength and elastic modulus are around 5 N/mm², 5 to 10 N/mm² and 15 to 20 kN/mm², respectively. SHCC is highly durable, as the widths of its cracks are smaller than those of normal concrete, preventing permeation of water and chloride ions. It is also highly resistant to freezing and thawing actions thanks to the effect of fiber reinforcement. SHCC is primarily placed by casting or shotcreting.

SHCC comprises a binder such as cement, fine aggregate, short fibers, an air-entraining and highrange water-reducing admixture, viscosity enhancing agent and water. To enhance its performance under tensile forces, coarse aggregate is not included. Since this causes large shrinkage during hardening, SHCC usually contains an adequate amount of an expansive admixture. Measures are taken for SHCC to suppress the hydration heat, which would otherwise be significantly high in thick members due to its high cement content. Polyvinyl alcohol (PVA) and polyethylene (PE) short fibers with a diameter of 0.01 to 0.04 mm and a length of around 10 mm are used at a volume ratio of 1 to 2 %. An adequate amount of viscosity enhancing agent is used to improve the fiber dispersion. The air content of SHCC is usually around 10% or more. Though the air content depends on the mixer performance and component materials, it can be adjusted by an air content adjuster.

3.2 JSCE Recommendations for SHCC

The Japan Society of Civil Engineers (JSCE 2007) published the design recommendations for SHCC in 2007 (Rokugo et al. 2009). English version of the recommendations was issued on the website in 2008. In the recommendations, SHCC is called HPFRCC with multiple fine cracks.Contents of the recommendations are shown in Table 2. The recommendations specify that structural performance, serviceability and resistance to the environmental actions have to be verified on the basis of performance verification concept. The recommendations allow cracks not only at ultimate limit state but also under service condition of members. The serviceability limit state design is required to verify the resistance to the environmental actions throughout the design service life on the basis of the calculated tensile strain or crack width in members under service conditions. The targeted materials of the recommendations are those which exhibit mean ultimate tensile strain capacity of more than 0.5 % and mean crack width of less than 0.2 mm as determined with test methods specified in the recommendations. Applicable range of the recommendations includes steel reinforced SHCC members and existing reinforced concrete structures covered with SHCC layer, but excludes monolithic use of SHCC in members.

Testing methods for measuring tensile strength, tensile strain capacity, and crack width are also specified because tensile performance is one of the most important material properties in the design of SHCC. The testing methods enable us to define Table 2. Contents of JSCE recommendations for SHCC.

| 1 General | | | | |
|--|--|--|--|--|
| 2 Design basis | | | | |
| 3 Material properties for design | | | | |
| 4 Load | | | | |
| 5 Structural analysis | | | | |
| 6 Safety verification of structures | | | | |
| 7 Serviceability verification of structures | | | | |
| 8 General structural details | | | | |
| 9 Verification for resistance to environmental actions | | | | |
| 10 Concrete work | | | | |
| 11 Shotcrete | | | | |
| Testing and evaluation methods | | | | |
| Appendix | | | | |

material properties regarding tensile yield strength, ultimate tensile strain capacity, and maximum crack width for given SHCC, which are subjected to the design verification.

3.3 Tension test methods in JSCE Recommendations

The JSCE Recommendations (JSCE 2007) contain the following four testing methods.

- Testing method 1: Preparation of specimen for strength tests

- Testing method 2: Testing method of uniaxial tensile strength

- Testing method 3: Testing method of crack width of HPFRCC -Average and maximum crack widths

- Testing method 4: Testing method of crack width of HPFRCC -Variation of crack width

The outline of Testing method 3 is introduced here. Figure 3 shows dimensions of tensile specimens whose minimum size is at least the fiber length and twice the maximum aggregate size. The specimen should be placed in the test machine with a chuck on ends, a fixed support on one end and a pin (hinge) support on the other end. As examples of chucking mechanism, test machines with a pneumatic chuck and clamp jigs are shown in Fig 4. The load should be applied at a constant specimen deformation rate of approximately 0.5 mm per minute. At least five specimens should be tested.

The test values of the tensile yield strength f_{tyi} , and the tensile strength f_{ti} are given by

$$f_{tyi} = \frac{F_{ty}}{A_0} \tag{1}$$

$$f_{ti} = \frac{F_t}{A_0} \tag{2}$$

where, F_{ty} : Load at the yielding point, F_t : Maximum load and A_0 : Sectional area of test zone of specimen.



Figure 3. Dumbbell-shaped specimen for tension tests.



(a) Pneumatic chuck (b) Clamp jig Figure 4. Examples of tension tests with different clamping mechanisms.

The strain at the softening starting point is defined as the ultimate tensile strain, whose test value ε_{tui} is given by

$$\varepsilon_{tui} = \frac{l_u - l_0}{l_0} \tag{3}$$

where, l_u : Reference point distance at the ultimate point, l_0 : Original reference point distance.

The yielding point is a point representing the minimum load between the initial cracking point and the softening starting point on the line joining convex inflection points. The point at which the load starts reducing associated with the increase in crack width is defined as the softening starting point. In the stress-strain relationship obtained in tension tests on specimens, the softening starting point is an inflection point immediately before the stress finally stops increasing. The tensile yield strength, ultimate tensile strain, maximum stress in the strain-hardening region and tensile strength are derived from the mean values of 3 or more specimens excluding those that showed the maximum and minimum ultimate tensile strain.

Table 3. Applications of SHCC in Japan (Uchida et al. 2008). (1) Direct Spraying SHCC

| Name | Vame Purpose | | Location | Amount (Thickness) | Completion | | |
|--|---|-------------------------|---|----------------------------------|------------|--|--|
| Repair of Mitaka Dam Repair | | Repair of Back of Dam | | 15 m^3 (3cm) | 2003 | | |
| Repair of Retaining W | Vall Repair of Su | ırface | Gifu | $2.5 \text{ m}^3 \text{ (7cm)}$ | 2003 | | |
| Strengthening of Tunn | nel Strengthenin | ng of Inner Linin | g Niigata | $30 \text{ m}^3 (5 \text{ cm})$ | 2004 | | |
| Repair of Railway Via | aduct Repair of Be | Repair of Beam and Slab | | $24.5 \text{ m}^3 \text{ (5cm)}$ | 2004-2008 | | |
| Repair of Building | Repair of Be | eam and Slab | Tokyo | 4 m^{3} | 2003-2004 | | |
| Waterproofing of Via | duct Waterproofi | Waterproofing Mortar | | 4 m^3 (5cm) | 2006 | | |
| Repair of Irrigation Canal Repair of Surfa | | ırface | Shiga, etc | $463 \text{ m}^3(1 \text{ cm})$ | 2006-2008 | | |
| Tunnel Lining Coating of Surface | | Gifu | $170 \text{ m}^3(1 \text{ cm})$ | 2007 | | | |
| (2) Casting SHCC | | | | | | | |
| Name Purpose | | | Location | Amount (Thick- ness) | Completion | | |
| Mihara Ohashi Bridge | e Strengthening of S | teel Deck Plate | Hokkaido | $800 \text{ m}^3 \text{ (4cm)}$ | 2005 | | |
| (3) Precast Products | | | | | | | |
| Name | Purpose | Location | Shape, Amount | | Completion | | |
| Precast Joint Panel | recast Joint Panel Jointless of girders Tokyo | | Shape: 1,960×600×t 30, 1,700×1400×t 30, 1,400× 847×t 30 Number of panels: 165 | | 2005-2008 | | |

3.4 Comments on tension test methods for SHCC

The tensile performance of SHCC, i.e., strainhardening and multiple fine crack behavior, can be estimated to a certain extent from flexure test data using beam specimens (Uchida 2007), for which a test method has already been established (JCI Standard 2007). However, it is hoped that a tension test method specifically for these cementitious composites having unique tensile performance is established, for the sake of future development in the concrete engineering field as well. The establishment of a tension test method will not only accelerate the spread of this material but also lead to the development of new materials showing strainhardening behavior.

The so-called "wall effect," in which fibers are oriented along the mold surfaces and finishing surfaces, is inevitable for fiber-reinforced cementitious composites. However, this effect can be relatively reduced by sufficiently enlarging the specimen size with respect to the fiber size. The performance determined from larger specimens is definitely closer to the tensile performance of SHCC in an actual structure. Nevertheless, a larger specimen is heavier with greater difficulty in handling. Meanwhile, five specimens are necessary for a set of test conditions in consideration of the scatter of performance obtained by tension testing. The JSCE Recommendations recommend the use of small dumbbell-shaped specimens as shown in Figure 3, because, at the current stage, SHCCs are mostly used as a surface repair material to a thickness of 10 to 30 mm. It is desired that better tension test methods for SHCC will be developed in the near future.

When planning a uniaxial tension test for a cementitious composite, how to transmit tensile forces to the specimen is a key factor requiring ingenuity. The JSCE Recommendations provide two methods: to pneumatically secure the ends of dumbbell specimens and to clutch the shoulders of each specimen with clamp jigs.

It is very difficult to conduct a tension test with both ends of specimens being fix-supported. With pin (hinge) supports at both ends, cracking tends to lead to excessive flexural deformation. In view of



Figure 5. Dumbbell-shaped specimen fabricated using pieces.

this, the JSCE Recommendations require that one of the ends of each specimen be fix-supported and the other pin-supported.



Figure 6. Cracks of SHCC and scale showing position.

In addition to usual casting fabrication, dumbbellshaped specimens can be fabricated using barshaped pieces as follows (Fig. 5): Place a bar-shaped piece, which may be molded or sawed from blocks or boards, in a dumbbell-shaped mold and fill SHCC in the enlarged parts of the mold to form shoulders. The method in which the shoulders of specimens are clutched using clamp jigs as shown in Figure 4 (b) is suitable for tension tests on specimens fabricated in this manner (Rokugo et al. 2007).

The crack width of SHCC can be accurately measured using a microscope. When measuring cracks in each specimen in multiple stages, it is advisable to glue a sheet of paper marked with a scale and positions on the specimen so as to serve as a measuring rule as shown in Figure 6.

4 APPLICATIONS OF SHCC

4.1 Applications of SHCC in Japan (Uchida et al. 2008)

The applications of SHCC in Japan are tabulated in Table 3. Retrofit is considered to be one of the most beneficial applications of SHCC in Japan, in which SHCC was used as a surface protection layer to recover and improve the function of existing concrete structures.

Figure 7 shows a retrofit project to protect deteriorated concrete surfaces of an aged irrigation channel. Wet type direct sprayed SHCC was constructed as the protection layer on the concrete surfaces. Waterjet was used in advance for substrate treatment to remove deteriorated mortar. Many irrigation channels suffer deterioration, having been in service for several decades. SHCC is regarded as a promising cementitious substitute for such a lining material.

Figure 8 shows another example of surface protection, where ASR damaged retaining walls are covered by SHCC layer. ASR may be delayed due to limiting water penetration to the existing damaged retaining wall structure. This protection layer has



Figure 8. Retaining wall retrofitting application.



Figure 9. ECC joints in expressway

been frequently observed to monitor cracking behave-ior of SHCC since completion. This observation demonstrates that multiple fine cracking was maintained after five years of environmental exposure.

To reduce noise and deterioration of expansion joints in bridges, deformable ECC (one kind of SHCC) joints were adopted in an existing urban expressway consisting of simply supported PC girders as shown in Figure 9. Both ends of an ECC plate with welded-wire mesh were fixed to the floor slabs with anchor bolts. The movement in the gap between girders due to temperature effects was absorbed through the deformation of ECC having narrow and dispersed cracks and then a gapless smooth asphalt



Figure 7. Waterway retrofitting application.



Figure 10. Tunnel lining with SHCC.

pavement surface was achieved. Plastic plates were inserted under the ECC plates as separators. It took eight hours to remove the damaged existing joints and to install new ECC joints including curing.

A sprayed multilayered FRC tunnel lining system including SHCC as the top layer was adopted instead of a conventional cast-concrete lining at emergency parking zones (more than 20 zones) in the Hida Tunnel, which is 10.7 km in length, the second longest among road tunnels in Japan (Fig. 10). The prevention performance against carbonation and concrete spalling was added to a sprayed SFRC layer through embedded carbon fiber grids inside and SHCC layer on the top. The SHCC layer increased the finishability with a trowel and water tightness of the lining. This method remarkably reduced the construction time and cost and therefore is expected as a quick repair method for damage caused by fire accidents or earthquakes.

4.2 For wider use of SHCC

To widen the use of SHCCs having new and excellent performances, the following points are important:

- Carry out reliable studies on the behavior and performance of SHCCs extensively and intensively.

- Establish methods of evaluating their tensile and cracking performances.

- Recommendations and guidelines for design and construction to make the most of their features.

- Establish methods of stable production and supply and try to reduce the costs.

- Calculate their economic competitiveness regarding weight, durability, etc., and make data available in a quantitative manner.

- Enrich ideas and examples of attractive applications making the most of their features.

- Inform the public of their features and attractiveness through seminars and reports.

- Activate cooperation among owners, contractors, designers, and researchers.



Figure 11. SHCC block showing large deformation at failure.

- Exercise care that new applications can pose new problems.

In regard to SHCCs, most of the above points have been rapidly accomplished in recent years. From now on, emphasis should be put on enriching ideas and examples of attractive applications to make the most of their features.

Concrete is a composite material consisting of various component materials. The potential and utility of SHCCs can also be enhanced by combining with other materials and substituting component materials. Inclusion of a large amount of fine air bubbles in SHCC, for instance, makes a material with a light weight and high deformability. A combination with net-shaped fibers instead of steel is also effective in regard to load-bearing capacity and ductility. Coating the surfaces of SHCC with a repellent as a surface impregnation compound enhances its waterproofing effect at cracks. Cracks in SHCCs are so fine that it is easy to close them proactively. It may be interesting to fabricate members using SHCC so that they would undergo large deformation to failure in a desirable manner when subjected to strong forces as shown in Figure 11. In order to materialize a new idea, it is particularly important to examine not only the short-term but also long-term and wideranging performance of the resulting material, as well as to pay attention to cost reduction.

In Japan, more SHCCs have been applied to civil structures than to buildings, because a technical system emphasizing the performance of SHCCs has been established with JSCE recommendations for SHCC. It is therefore easy for civil engineers in Japan to introduce new technology at their discretion. It is hoped that these materials are used domestically and worldwide, boldly and carefully, keeping in mind that new applications can pose new problems and advantages can turn to disadvantages. Care should also be exercised regarding the following:

- As SHCCs are a fiber-reinforced material, the continuity of fiber reinforcement should be ensured when applied to actual structures.

- Unification of joints between members or between steel and SHCC should be ensured.

- Structures involving SHCCs should be observed over many years, investigating the validity of element technologies to reflect the results to future application.

5 CONCLUSIONS

Major indices for evaluating fracture proposed so far in the field of fracture mechanics for concrete and concrete structures have been overviewed. In regard to evaluation indices for concrete in fracture mechanics, those proven to be useful with standardized test methods have been widely used, and their use has been promoted by inclusion in design standards. While the values of the achievement of engineering studies are generally enhanced by their utility in industrial practice, research into fracture of concrete is so essential and fundamental that it may not immediately be found useful. Nevertheless, it is important to make persistent efforts in pursuing studies in these fields.

SHCCs are unique cementitious composite materials showing strain-hardening and multiple cracking behavior under tensile forces. They have been increasingly applied to actual structures, primarily as surface repair for existing concrete structures such as waterways. The Japan Society of Civil Engineers has proposed tension test methods for SHCCs and structural design recommendations using tensile performance for ensuring safety and durability of structures made using SHCCs. The publication of these recommendations facilitated the application of SHCCs to actual structures in Japan. In these recommendations, small-size dumbbell-shaped specimens with one end fix-supported and the other pinsupported are proposed for measuring tensile yield strength, tensile strength, tensile ultimate strain, and maximum crack width.

It is important from now on to enrich ideas and examples of attractive SHCC applications making most of their features, observe change of structures involving SHCCs over time, and verify the validity of element technologies employed. It is hoped that SHCCs are used boldly and carefully, keeping in mind that their new application can pose new problems and their advantages can turn to disadvantages.

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