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DEVELOPMENTS OF PREDICTION MODEL FOR CRACK WIDTH DUE TO REBAR CORROSION

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Abstract: Deterioration of concrete structures has been drawing a greater social attention. The strength of concrete structures decreases due to corrosion of reinforcement. The relationship between the crack width and the amount of reinforcement corrosion (corrosion ratio) is an important parameter in maintenance and management activities of the concrete structures. This paper develops a model of relationship between the crack width and the corrosion ratio. Basic calculation formulation of the expansion pressure, caused by corrosion, was derived using cylinder model. Experimental comparison indicated the proposed model is available to estimate the crack width based on the corrosion ratio of reinforcement due to corrosion.

1 INTRODUCTION

Cracks of concrete due to reinforcement corrosion caused by carbonation and chloride attack have a significant effect on structural performance. Hence, it is urgent to establish a method that can evaluate the behavior of corroded reinforcement from deformation of concrete surface, such as the amount of uplift and crack width, on concrete surface. However, there is no any available method that can quantitatively predict the corrosion properties of the rebar embedded in the concrete based on the properties of the corrosion cracks on concrete surface. Therefore, the structural performance has been evaluated by the semi-quantitative method in the range of a certain degree of safety margin.

Furthermore, little is known about what the condition of the occurring of the corrosion crack is, when it will reach the concrete surface, and how it proceeds. Hence, it is difficult to predict the corrosion ratio from the crack width on concrete surface.

Over the past few decades, a considerable number of studies have been conducted on the behaviors of the corroded rebar. Previous studies estimated the expansion pressure of corrosion by cylinder theory, and estimated the crack width by principle of virtual work. In fact the expansion pressure is reduced when the crack reaches the concrete surface. Therefore, crack width was calculated largely because this effect was not taken into account in the previous studies.

The purpose of this study is to examine the effect of the reduced expansion pressure of the corrosion and the effect of the increased crack width due to the increase of the corrosion ratio of the rebar.

In this study, the predicted model for the corrosion crack width is developed, based on the theory and the result of the experiments.

2 PREDICTION MODEL FOR THE CORROSION CRACK WIDTH

2.1 An Overview of the Model

Firstly, there will be a brief model overview for the corrosion crack width.

Figure 1 shows the expansion pressure caused by the corrosion products deposited on the surface of the rebar. The corrosion crack is caused by this expansion pressure of corrosion.

The process of cracking is divided into the following steps in order to model the corrosion crack. These are the sequence of behavior from occurrence of crack for proceeding. Figure 2 shows the condition of the crack at each step.

Step 1: Crack occurs, and then proceeds to the concrete surface (Figure 2(a)).

Step 2: Crack opens and the expansion pressure is reduced as the crack reaches the concrete surface. This crack is called as Primary crack (Figure 2(b)).

Step 3: Primary crack width grows gradually as corrosion ratio increases (Figure 2(c)).

Step 4: Stress in the circumferential direction of the upper edge of the rebar reaches the tensile stress of the concrete, and then cracks occur in the other area. This crack is called as Secondary crack (Figure 2(d)).

Step 5: As Primary crack and Secondary crack grows, the crack width on the concrete surface grows gradually (Figure 2(e)).

Secondly, the model for the occurrence and growth of cracks will be described at each step.

In both Step 1 and Step 2, crack does not occur on the concrete surface. The expansion pressure of corrosion is calculated with the cylinder theory, assuming the concrete cover as cylindrical shape. And then, corrosion crack width is calculated as its equivalent.

In Step 3, the width of the crack on the concrete surface grows gradually with the increase of the corrosion ratio. Hence, the concrete surrounding the reinforcing bar transforms to the beam shape from the cylindrical shape. Then part of the expansion pressure of corrosion is released. The crack width is calculated by the remaining pressure equivalently.

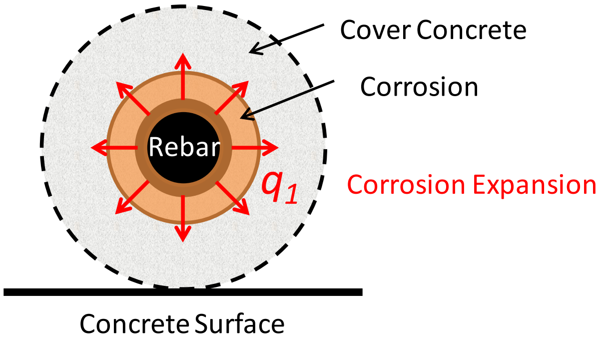


Figure 1 : Diagram of corrosion expansion.

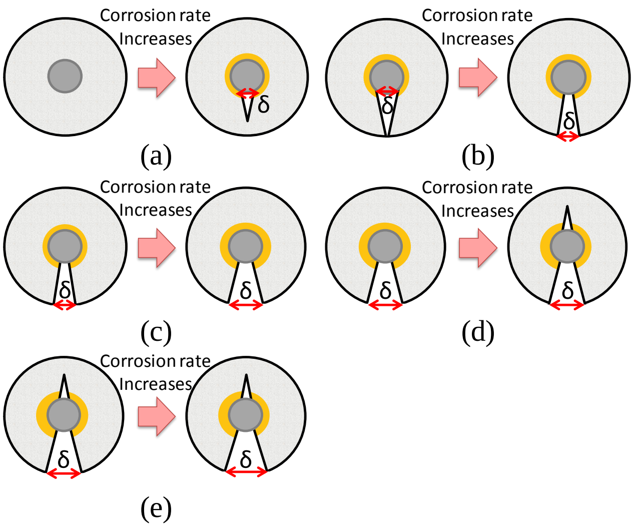


Figure 2 : Condition of cracking at each step. –(a) Crack proceeds to the concrete surface (b) Crack reachs the concrete surface (c)Primary crack width grows (d)Secondary crack width grows (e) Both of Primary crack and Secondary crack grow

In Step 4, tensile stress in the area other than the area of cover concrete surrounding the rebar reaches the tensile strength of the concrete, the Secondary crack appears in the upper area and side area of the rebar.

In Step 5, crack width on concrete surface grows as defined in Step 3 and Step 4.

Here, the Step2, the Step3, and the Step 4 are concerned.

2.2 Modeling for Corrosion Expansion

Corrosion expansion is discussed in this chapter. It is suggested to divide it into layers which compose RC. The point is to consider each layer have different physical properties.

Firstly, in the early stage such as rebar starts to corrode in Step 1 and Step 2, the member of RC is composed of three materials with rebar, concrete, and corrosion products (Three-layer Model). Secondly, as the corrosion ratio increases in Step 3, the corrosion can be divided into existing (old) corrosion and new corrosion, so member of RC is composed of four materials (Four-layer Model). Finally, in Step 4 and Step5, the stiffness of concrete is reduced greatly due to presence of Secondary crack. The concrete can be divided into the uncracked concrete and cracked concrete, and then the member of RC is composed of five materials (Five-layer Model).

Next, the external pressure and the internal pressure acting on each layer are calculated based on the displacement of the layers and the Compatibility condition of the displacement between the layers. Figure 3 shows the conceptual diagram of Five-layer Model.

(1) Displacement of Each Layer

A) Cover Concrete Layer

Assuming the concrete cover as thick cylinder subjected to internal pressure q_{ci} and it is plane strain, the displacement expansion of the inner diameter of the concrete u_{ci3} can be given by Equation (1)

$$u_{ci3} = \frac{\phi_s(1+\nu_c)}{2E_c({K_1}^2-1)}\{(1-2\nu_c)+{K_1}^2\}q_{ci} \quad (1)$$

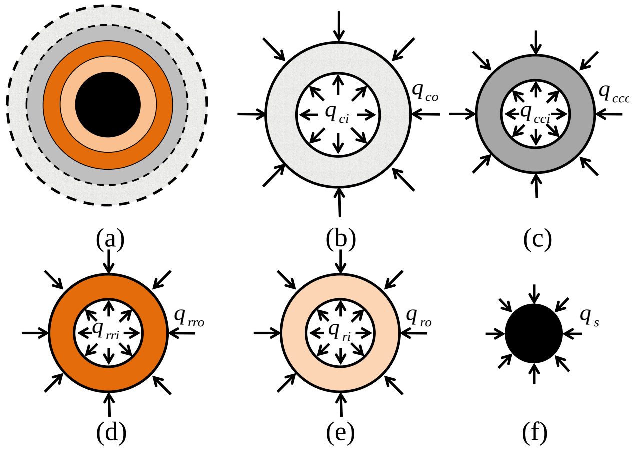


Figure 3 : Diagram of Five-Layer Model. (a) Five-Layer Model (b) Uncracked Concrete (c)Crack Concrete (d)Old-corrosion (e)New-corrosion (f)Rebar

where K_1 is given ϕ_a/ϕ_s, and ϕ_a is the two times the distance from the center of rebar to the concrete surface. ν_c is Poisson's ratio of the concrete and E_c is the modulus of elasticity of the concrete.

B) Rebar Layer

Assuming corrosion produced on the rebar has uniform thickness, its diameter is ϕ_s, so the diameter reduced due to corrosion ϕ_{si} can be given by Equation (2)

$$\phi_{si} = \sqrt{{\phi_s}^2 - 4A_w/\pi} \quad (2)$$

where the cross-sectional area corroded A_w, is given by Equation (3)

$$A_w = A_s \times \alpha/100 \quad (3)$$

where A_s is a cross-sectional area of the rebar, α is the corrosion ratio of rebar (mass change after corroded) given by Equation(4)

$$\alpha = (W_b - W_a)/W_b \times 100 \quad (4)$$

where W_b is weight of rebar before corroded and W_a is weight of rebar after corroded.

Rebar is subjected to external pressure q_s as compressive stress, so the displacement u_s is given by Equation (5)

$$u_s = \frac{\phi_{si}(1+\nu_s)(1-2\nu_s)}{2E_s}q_s \quad (5)$$

where ν_s is Poisson's ratio of the rebar, and E_s is the modulus of elasticity of the rebar.

C) New Corrosion Layer

Corrosion acts a free expansion when it is produced. Assuming inside diameter is ϕ_{ri} and outside diameter is ϕ_{ro}, they are given by Equation (6) and (7)

$$\phi_{ri} = \phi_{si} \tag{6}$$

$$\phi_{ro} = \sqrt{{\phi_s}^2 - 4(n-1) \times A_w/\pi} \tag{7}$$

where n (= 7850/5300) is the ratio of volume expansion of corrosion, which is the unit of volume weight ratio of iron and corrosion products, obtained in previous study.

Corrosion is subjected to internal pressure q_{ri} and external pressure q_{ro}, so the displacement expanded on the inner diameter u_{ri} and the displacement restrained in the outer diameter u_{ro} are given by Equation (8) and (9)

$$u_{ri} = \frac{\phi_{ri}(1+\nu_r)}{2E_r({K_2}^2-1)}\{(1-2\nu_r) \times (q_{ri} - q_{ro}{K_2}^2) + (q_{ri} - q_{ro}){K_2}^2\} \tag{8}$$

$$u_{ro} = \frac{\phi_{ro}(1+\nu_r)}{2E_r({K_2}^2-1)}\{(1-2\nu_r) \times (q_{ri} - q_{ro}{K_2}^2) + (q_{ri} - q_{ro})\} \tag{9}$$

where K_2 is ϕ_{ri}/ϕ_{ro}, ν_r is Poisson's ratio of the new corrosion and E_r is the modulus of elasticity of the new corrosion.

D) Old Corrosion Layer

Corrosion existed already is subjected to internal pressure q_{rri} and external pressure q_{rro}, so the displacement expanded on the inner diameter u_{rri} and the displacement restrained in the outer diameter u_{rro} are given by Equation (10) and (11)

$$u_{rri} = \frac{\phi_{rri}(1+\nu_{rr})}{2E_{rr}({K_3}^2-1)}\{(1-2\nu_{rr}) \times (q_{rri} - q_{rro}{K_3}^2) + (q_{rri} - q_{rro}){K_3}^2\} \tag{10}$$

$$u_{rro} = \frac{\phi_{rro}(1+\nu_{rr})}{2E_{rr}({K_3}^2-1)}\{(1-2\nu_{rr}) \times (q_{rri} - q_{rro}{K_3}^2) + (q_{rri} - q_{rro})\} \tag{11}$$

where K_3 is ϕ_{rri}/ϕ_{rro}, ν_{rr} is Poisson's ratio of the Old corrosion and E_{rr} is the modulus of elasticity of the Old-corrosion.

E) Cracked Concrete Layer

Cracked Concrete is subjected to internal pressure q_{cci} and external pressure q_{cco}, so the displacement expanded on the inner diameter u_{cci} and the displacement restrained in the outer diameter u_{cco} are given by Equation (12) and (13)

$$u_{\mathrm{cci}} = \frac{\phi_{cci}(1+\nu_{cc})}{2E_{cc}({K_4}^2-1)}\{(1-2\nu_{cc}) \times (q_{cci} - q_{cco}{K_4}^2) + (q_{cci} - q_{cco}){K_4}^2\} \tag{12}$$

$$u_{\mathrm{cco}} = \frac{\phi_{cci}(1+\nu_{cc})}{2E_{cc}({K_4}^2-1)}\{(1-2\nu_{cc}) \times (q_{cci} - q_{cco}{K_4}^2) + (q_{cci} - q_{cco})\} \tag{13}$$

where K_4 is $\phi_{\mathrm{cci}}/\phi_{cco}$, ν_{cc} is Poisson's ratio of the Crack-concrete and E_{cc} is the modulus of elasticity of the Crack-concrete.

(2) Acting Forces and Compatibility Conditions of Displacement of Each Layer

A) Three-layer Model

The compatibility condition of the displacement between uncracked concrete and new corrosion, and between new corrosion and rebar are given by Equation (14). And, the pressures acting each layer are given by Equation (15).

$$\begin{cases} u_{ci3} + \phi_s/2 = u_{ro} + \phi_{ro}/2 \\ u_s = u_{ri} \end{cases} \tag{14}$$

$$\begin{cases} q_{ci} = q_{ro} \\ q_{ri} = q_s \end{cases} \tag{15}$$

B) Four-layer Model

The compatibility condition of the displacement between uncracked concrete and old corrosion, between old corrosion and new corrosion and between new corrosion and rebar are given by Equation (16). And, the pressures acting each layer are given by Equation (17).

$$\begin{cases} u_{ci4} = u_{rro} \\ u_{ro} + \phi_{ro}/2 = u_{rri} + \phi_s/2 \\ u_s = u_{ri} \end{cases} \tag{16}$$

$$\begin{cases} q_{ci} = q_{rro} \\ q_{rri} = q_{ro} \\ q_{ri} = q_s \end{cases} \tag{17}$$

C) Five-layer Model

The compatibility condition of the displacement between uncracked concrete and cracked concrete, between cracked concrete and old corrosion, between old corrosion and new corrosion and between new corrosion and rebar are given Equation (18). And, the pressures acting each layer are given by Equation (19).

$$\begin{cases} u_{ci5} = u_{cco} \\ u_{ro} + \phi_{ro}/2 = u_{rri} + \phi_r/2 \\ u_{cci} = u_{rro} \\ u_s = u_{ri} \end{cases} \tag{18}$$

$$\begin{cases} q_{ci} = q_{cco} \\ q_{cci} = q_{rro} \\ q_{rri} = q_{ro} \\ q_{ri} = q_s \end{cases} \tag{19}$$

The expansion pressure of corrosion and the stress acting on the inner diameter concrete q_{ci} are calculated with solving a system of equations by substituting a variety of material properties in the above equation.

2.3 Prediction of Crack Width on the Concrete surface

(1) The Case Crack Reached the Concrete Surface

First, assuming the concrete cover as a cylindrical shape, crack width is calculated as crack reaches the concrete surface. As shown in Figure 4, we provided the virtual cracked section A-A' on the cylindrical with a thickness of concrete cover. The circumferential direction stress σ_θ with cylindrical theory acts in this cross-section is given by Equation (20). Also, total force P is given by Equation (21)

$$\sigma_\theta = -\frac{a^2 b^2 (q_2 - q_1)}{r^2 (b^2 - a^2)} + \frac{a^2 q_1 - b^2 q_2}{b^2 - a^2} \tag{20}$$

$$P = \int_a^b \sigma_\theta dr \tag{21}$$

where a is rebar radius, including corrosion products $(= \phi_r/2)$, b is the distance from the center of the rebar to concrete surface. Internal pressure q_1 is the corrosion expansion calculated by the model shown in the previous section. On the other hand external pressure q_2 is decided by mutual expansion pressure if there are some rebars. However, this is not our present concern because there is only a rebar.

The force on the virtual cracked section will be zero because crack occurs. Therefore, it is equivalent to act total force P in the opposite direction. The crack width is assumed to be the displacement as the opposite force acts and it given by Equation (22)

$$\delta_0 = \frac{4\pi D}{E} \tag{22}$$

where D is constant of integration applied to the solution of a nearly complete ring for the elementary theory of bending rod curved [H.Golovin] and it is given by Equation (23)

$$\begin{aligned} D &= -\frac{P}{N}(a^2 + b^2) \\ N &= a^2 - b^2 + (a^2 + b^2) log\,(b/a) \end{aligned} \tag{23}$$

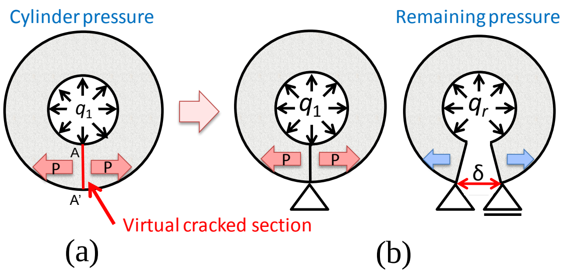


Figure 4 : Transform configuration from cylinder to beam.—(a)Cylinderical shape (b)Beam shape

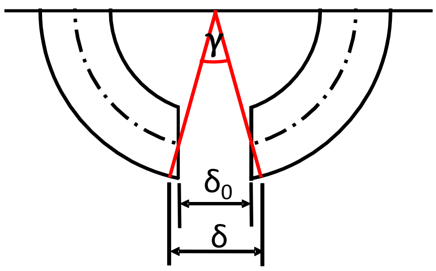


Figure 5 : Corrosion crack and crack angle.

Since concrete cover bended deformation has a central angle γ as shown in Figure 5, crack width on concrete surface δ and the crack angle γ are given by Equation (24) and (25)

$$\gamma = \delta_0 \frac{2}{a+b} \tag{24}$$

$$\delta = \gamma \times b \tag{25}$$

(2) Primary Crack Width

Next, the following describes the method of the calculating crack width after crack occurred on the concrete surface.

Concrete surrounding the rebar transforms to beam from cylinder at the same time as crack occurs on the concrete surface. Then, the internal pressure acting on the concrete significantly reduces. The remaining internal pressure is calculated by assuming the shape of the concrete transforms to the beam from the cylinder. The crack width is calculated by the remaining internal pressure q_r .The released internal pressure q_k is given by Equation (26) from the force acting on the beam.

$$q_k = \frac{P}{\left(a + \frac{\gamma a}{2\pi}\right)(1 - cos\pi)} \tag{26}$$

The remaining pressure q_r can be determined by subtracting the released pressure q_k from the pressure assuming it as a cylinder q_1, and it is given by Equation (27).

$$q_r = q_1 - q_k \tag{27}$$

At this time, the crack width δ is given by Equation (28)

$$\delta = \frac{3\pi a^4}{E_c I} q_{remain} \tag{28}$$

where I is geometrical moment of inertia as a beam given by Equation(29)

$$\mathrm{I} = 1 \times (b-a)^3/12 \tag{29}$$

3 TEST OF CRACK PROCEEDING

3.1 Parameter and Materials

As shown in Table 1, the parameters of the test are steel diameters, cover concrete depths and water-cement ratios. The rebar diameters are D16 (SD295A) and D22 (SD295A). There are three concrete cover depths per a diameter of each rebar. The water-cement ratios are 60% and 30%. As shown in Figure 6, specimens are prismatic shape of 200 × 200 × 500mm, set a rebar in place. The displacement gauge type π is placed in the center of the axial cross section rebar in order to measure of corrosion crack width on concrete surface.

Table 1: Parameter of Experiment.

Specimen No.	Steel diameter D (mm)	Water-cement ratio (%)	Cover depth C (mm)	Modulus of elasticity of concrete (kN/mm^2)
1	22	60	20	19.0
2			35	19.0
3			55	19.0
4		30	20	27.2
5			35	27.8
6			55	27.8
7	16	60	20	22.5
8			25	22.5
9			40	22.5
10		30	20	27.1
11			25	27.1
12			40	27.1

Table 2: the mix proportion of the concrete.

Gmax [mm]	W/C [%]	Sl [cm]	Air [%]	Quantity of material per unit volume of concrete[kg/m^3] W	C	S	G	AE	NaCl
20	60	10	5	165	290	798	1003	1.2	8.7
20	60	10	5	157	550	617	992	2.2	8.3

3.2 Specimen

Mix proportion of specimen is shown in Table 2. High-early-strength Portland cement is used in this experiment. Water is used 5% NaCl aqueous solution in order to promote corrosion of rebar. Figure 7 shows corrosion test of the electrolytic corrosion. Specimen is immersed in a tank with an aqueous solution of 5% NaCl after curing for 7 days. At this time, water is prevented from immersing concrete surface.

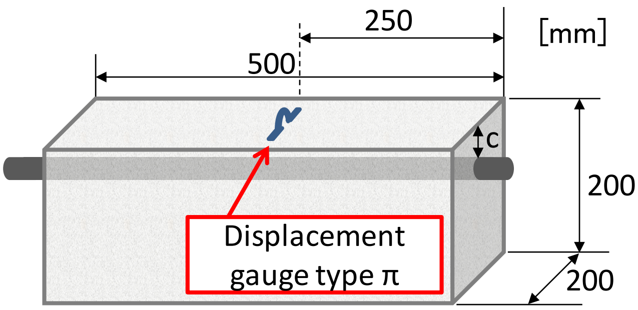


Figure 6 : Specimen.

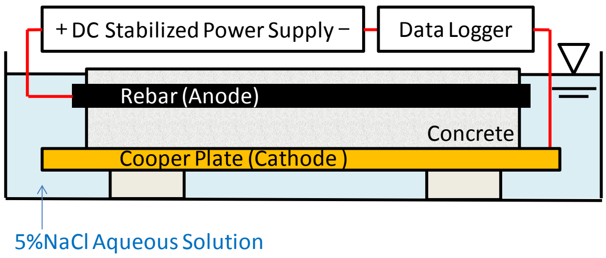


Figure 7 : Corrosion test of the electrolytic corrosion.

Table 3 : Each parameter in the analysis.

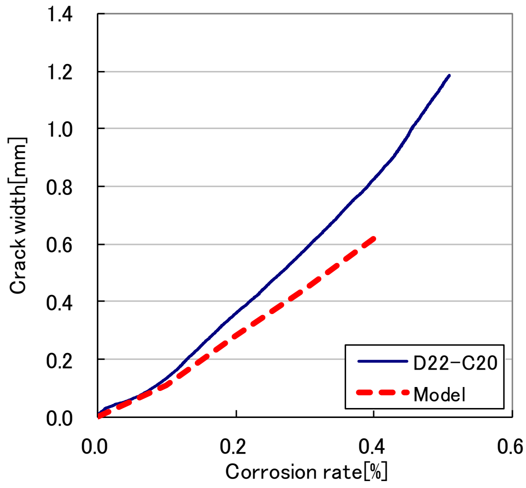
	Modulus of elasticity (kN/mm^2)	Poisson's ratio
Concrete	Shown Table 1	0.20
Crack Concrete	10.0	0.20
Old-Corrosion	0.40	0.17
New-Corrosion	0.20	0.17
rebar	210.0	0.17

The copperplate is set bottom of specimen. Then, we use DC stabilized power supply, corroded until corrosion ratio for the target.

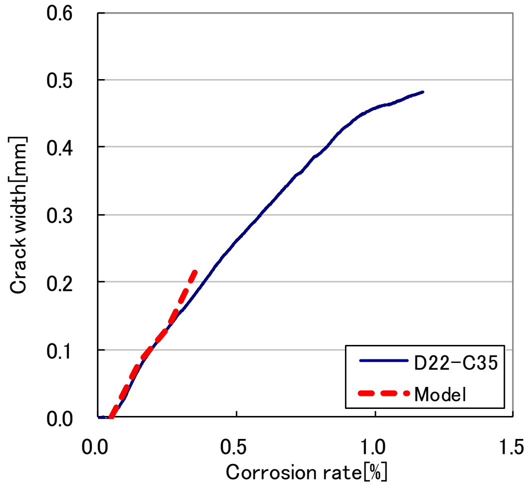
4 RESULTS AND DISCUSSION

4.1 Results of Experiment

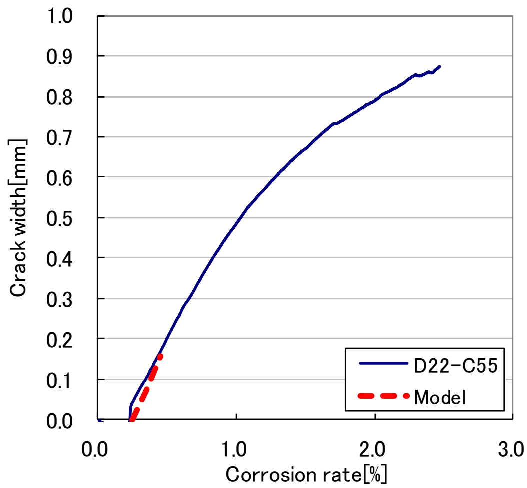
Figure 8 shows the relationship between the crack width on the concrete surface and the corrosion ratio of rebar in each specimen. The solid line is the experimental values obtained from the relationship between the measured value and the integrated current amount, and the dashed line is the predicted value calculated by this model. Each parameter used in the analysis is the experimental value and it is shown in Table 3.



(a)20mm cover depth



(b)35mm cover depth



(c)55mm cover depth

Figure 8 : Relationships between crack width on concrete surface and corrosion ratio of rebar.

Table 4 : Relationship of the concrete cover depth, the corrosion ratio and the crack width.

cover concrete depth	20mm	35mm	55mm
Corrosion ratio crack occurs	0.03%	0.10%	0.27%
Crack width rapidly opened	0.01mm	0.03mm	0.05mm

Table 4 shows the relationship of the thickness of the concrete cover, the corrosion ratio and the crack width at early stage. Cracks occur on the concrete surfaces at the corrosion ratio of 0.03%, 0.10% and 0.27%. And the cover concrete depths are 20mm, 35mm and 55mm, respectively. At this time, crack widths rapidly opened, which are 0.01mm, 0.03mm and 0.05mm, respectively. It can be seen they increase according to the cover concrete depth.

4.2 Applicability Evaluation for Model

Examine the accuracy of the model developed in Section 2, and evaluate the applicability of the model using the experimental parameters.

The value of model shows the same trend of increasing in parabolic according to the ratio of corrosion in both the theoretical crack width and the experimental crack width, after the crack occurs on the concrete surface. It can be seen very consistent.

However, the predicted values differ from the experimental values in the beam whose concrete cover depth is 20mm.

One of the reasons is that the expansion pressure of corrosion is smaller because corrosion products flow out to the concrete surface and into the water through the crack at an early stage.

5 CONCLUSIONS

In this study, the prediction model for expansion pressure and crack width due to corrosion has been developed. Furthermore, the relationship between the crack width and the corrosion ratio can be compared with the experiment of crack proceeding.

First, both of the crack width rapid opening and the corrosion ratio as crack occurs on surface concrete tends to increase with larger concrete cover.

Second, it is possible to predict the corrosion crack width by the reduction of the pressure because the predicted values and experimental values are fit well.

Finally, this model is not able to take into account Step 1 and Step 5 i.e. during latent Primary crack and Increasing of Secondary crack. We would like to consider them for future work.

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