INFLUENCE OF MEASUREMENT UNCERTAINTIES ON RESULTS OF CREEP PREDICTION OF CONCRETE UNDER CYCLIC LOADING

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Abstract. During the last five decades, several experimental models and numerical models have been developed to predict the time-dependent deformation of concrete under cyclic loading. It is well known to engineers that creep accounts for a majority of structural deformation and failures. In plain and reinforced concrete structures, cyclic creep may lead to excessive deformation, excessive crack widths or structural collapse. The deformation in concrete under cyclic loading depends on a number of parameters, such as, the concrete composition, environmental condition, strength of concrete, modulus of elasticity, the stress amplitude, the mean stress, number of cycle, wave forms and time under load. In this work, the quantification of uncertainty of creep models under cyclic loading is computed using four different creep models, (BP model, Neville und Whaley model, modified MC90 model and modified Hyperbolic model) by considering different uncorrelated and correlated parameters responsible for cyclic creep. Four sources of uncertainty parameters uncertainty, model uncertainty, data uncertainty, and uncertainty of the creep phenomenon are considered for all of the models and compared showing significant differences. A general probabilistic method is developed for the prediction quality of creep models under cyclic loading. The Latin Hypercube Sampling (LHS) numerical simulation method (Monte Carlo type method) was used. Further, global sensitivity analysis considering the uncorrelated and correlated parameters are used to quantify the contribution of each source of uncertainty to the overall prediction uncertainty and to identify the important parameters. The error in determining the input quantities and model itself can produce significant changes in creep prediction values. The variability influence of input random quantities on the cyclic creep was studied by means of the stochastic uncertainty and sensitivity analysis. All input imperfections were considered to be random quantities. The Latin Hypercube Sampling (LHS) numerical simulation method (Monte Carlo type method) was used. It has been found by the stochastic sensitivity analysis that the cyclic creep deformation variability is most sensitive to the Elastic modulus of concrete, compressive strength, mean stress, cyclic stress amplitude, number of cycle, in that order.
1 INTRODUCTION

Because of the uncertainty associated with concrete and creep modeling under cyclic loading, uncertainty should be accounted for in model application and evaluation [1,2]. The analysis and consideration of uncertainty is particularly important because decisions regarding design of concrete structures, and the repair and deflection of concrete structures are increasingly based on creep modeling. A recent change is the application of monitoring and experimental data for a safety analysis in various fields of engineering. The most relevant question of the application of experimental and monitoring data is: Which uncertainties should be applied to the experimental and monitoring data when utilised in a safety and serviceability analysis? This question is answered with this paper using the existing framework for the determination of measurement uncertainties based on ISO/IEC Guide 98-3 (2008a) [3].

Uncertainty in creep and shrinkage modeling has been classified by [1,4] into four categories: model uncertainty, parameter uncertainty, measurement uncertainty and phenomenon uncertainty. The uncertainty introduced by model structure and parameterisation has received much attention in recent years [1,4,5]. Simply speaking, model uncertainty arises from incomplete understanding of the system being modeled and/or the inability to accurately reproduce creep processes with mathematical and statistical techniques. This in contrast, to parameter values, ranges, physical meaning, and temporal and spatial variability. But parameter uncertainty also reflects the incomplete model representation of cyclic creep phenomenon and inadequacies of parameter estimation techniques, and often limited, measured data.

Although the uncertainty inherent in measured data used to calibrate and validate model predictions is commonly acknowledged, measurement uncertainty is rarely included in the evaluation of model performance. One reason for this omission is the lack of data on the uncertainty inherent in measured cyclic creep data.

For additional information on models and parameter uncertainty, which is carried out, one method of importance measurement of models by considering the uncorrelated and correlated parameters is proposed by [6]. The distinction between uncorrelated and correlated contribution of uncertainty for an individual variable is very important and output response and input variables is approximately linear in this method.

In this work, is a description of method for the determination of the measurement uncertainty for creep strain measurements following ISO/IEC Guide 98-3 (2008a) [3]. Subsequently these methods are extended to account for model uncertainties and a probabilistic model assignment uncertainty. Furthermore, the measurement uncertainties based on a process equation and based observation are derived. In the third section the core of the introduction concept is derived and discussed, namely the posterior measurement uncertainty and sensitivity analysis.

For the detailed description of four cyclic creep models refer to [7-13].

2 SOURCES OF UNCERTAINTY

This section describes the different sources of uncertainty in the cyclic creep prediction. These sources of uncertainty can be classified into three different types-physical or natural uncertainty, data uncertainty and model uncertainty - as shown in Fig.1. Fig.1 shows the different sources of error and uncertainty considered in this paper for the sake of illustration of the proposed methodology. There are several other sources of uncertainty that are not considered here. Each of these different sources of uncertainty is briefly discussed below.
2.1 Physical or natural uncertainty

Physical or natural uncertainty refers to the uncertainty or fluctuations in the environment, test procedures, instruments, observer, etc. Hence, repeated observations of the same physical quantity do not yield identical results. This paper considers the physical uncertainty in loading and materials properties. The uncertainty in the systematic errors to the measurement, human error, the variability in other materials properties such as Poisson ratio, supplementary cementing materials, the curing time period, temperatures, etc. is not considered. The probabilistic analysis considered the relationship of concrete compliance function $J(t - t_0)$ defined using Eq.(1) with the input variables that cover both intrinsic and extrinsic factors.

$$J(t - t_0) = \frac{1}{\sigma(t_0)}[\epsilon_{el}(t_0) + \epsilon_{cr}(t, t_0)]$$  (1)

where $\sigma(t_0)$ is the creep stress, $\epsilon_{el}(t_0)$ is the instantaneous elastic strain, and $\epsilon_{cr}(t, t_0)$ is the creep strain between time of load application $t_0$ and time of evaluating creep $t$. The selected model can be described by Eq.(2) where the strain can be defined using by Eq.(4).

$$J(t - t_0) = \sum_{k=1\ldots m} x_k \epsilon_k = x^T \epsilon$$  (2)

$$\epsilon = (X^T X)^{-1} X^T J$$  (3)

$J$ is the vector of $N$ concrete compliance observations and the matrix $X$ consists of the $m$ effects corresponding to the $N$ observation as:

$$J = \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_N \end{bmatrix}; \quad X = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,m} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N,1} & x_{N,2} & \cdots & x_{N,m} \end{bmatrix}$$

2.2 Measurement uncertainties

The measurement uncertainty following ISO/IEC Guide 98-3, 2008a [3] is determined with a measurement equation which yields the measurand. The uncertainties of the input quantities, i.e. the (random) variables, determine the uncertainty of the measurand. One type of measurement uncertainties is derived by assigning a statistical model to observations using the definition of probability. Other types of measurement uncertainties are derived with the help of a process equation modeling the measurement process physically. The probabilistic models of the associated random variables are evaluated by scientific judgment based on all of the available information (ISO/IEC Guide 98-3 [3] implying a Bayesian definition of probability. It is possible to characterise three type of uncertainty during the measurement of the cyclic creep strain: uncertainty related to measurement, uncertainty due to the positioning of the gauges and uncertainty due to the installation.

Based upon the physical properties of the measurement process, the process equation is derived and uncertainty models are introduced for the associated random variables. This derivation takes basis in the concept for the determination of uncertainties according to ISO/IEC Guide 98-3 [3]. In addition to this concept, a model uncertainty and an assignment uncertainty are introduced. The starting point for the derivation of the process equation is the measurement equation Eq (1). The introduction of the model uncertainty in strain measurement, which describes the uncertainty associ-
ated with the physical formulation of the problem, leads to Eq (1). With an associated uncertainty model, the measurement uncertainty can be derived e.g. with a Monte Carlo simulation. The uncertainty model can be derived by considering the product information of the measurement system. The product information is usually valid for all strain gauges of the same type, all amplifiers of the same type and for all surrounding and application conditions as documented in the manufacturer specifications according to standardised rules [16]. The measurement uncertainty based on observations of the cyclic creep strain follow a normal distribution with the parameters \(\mu\) and \(\sigma\) Eq 4.

\[
\epsilon_{\text{total}} \sim N(\mu, \sigma)
\] (4)

The parameters of the distribution are estimated with the method of Maximum Likelihood. For the calculation of the marginal distribution, the statistical uncertainties of the parameters are integrated Eq 5.

\[
f_{\epsilon_{\text{cr,cyc}}}(\epsilon_{\text{cr,cyc}}) = \int_{-\infty}^{\infty} f(\epsilon_{\text{cr,cyc}} | (\mu, \sigma)) f(\mu) f(\sigma) d\mu d\sigma
\] (5)

With Bayesian updating the posterior measurement uncertainty, i.e. the distribution of the measurement uncertainty accounting for prior knowledge and observations, is derived Eq 6.

\[
f''(\epsilon_{\text{cr,cyc}}) = f'(\epsilon_{\text{cr,cyc}}).L(\epsilon_{\text{cr,cyc}})
\] (6)

The uncertainty model for the variation of the cyclic creep strain calculation based on the measurement is summarised in Table 1. A necessary condition for Bayes’ theorem is that the probability of observing any particular data outcome for a given state must be known. This information is often available from laboratory testing, product literatures, or past experiences. Information about an input quantity X consists of a series of indications regarded as realisations of independent, identically distributed random variables characterised by a PDF, but with unknown mean and variances. Calculation therefor proceeds in two steps:- first, a non-informative joint prior- (pre-data) PDF is assigned to the unknown mean and variances. This joint prior PDF is then updated, based on the information supplied by the series of indications, to yield a joint posterior (post-data) PDF for the unknown parameters, which is shown in Figure 2. The desired posterior PDF for the unknown mean is then calculated as a marginal PDF by integrating over the possible values of unknown variances. The updating is carried out by forming the product of a likelihood function and the prior PDF.

Table 1: Uncertainty model for the cyclic creep measurement and prediction

<table>
<thead>
<tr>
<th>Random Variable</th>
<th>Mean</th>
<th>Std.</th>
<th>CoV</th>
<th>Distribution</th>
<th>Models</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_{c,28})</td>
<td>52.00 MPa</td>
<td>3.12</td>
<td>0.06</td>
<td>Log-normal</td>
<td>1,2,3,4</td>
<td>[17]</td>
</tr>
<tr>
<td>(f_d)</td>
<td>50.70 MPa</td>
<td>3.00</td>
<td>0.06</td>
<td>Log-normal</td>
<td>1,2,3,4</td>
<td>Assumed</td>
</tr>
<tr>
<td>(E_{c1,28})</td>
<td>34144 MPa</td>
<td>3414.4</td>
<td>0.10</td>
<td>Log-normal</td>
<td>1,2,3,4</td>
<td>[17]</td>
</tr>
<tr>
<td>(E_d)</td>
<td>33290 MPa</td>
<td>3329.0</td>
<td>0.10</td>
<td>Log-normal</td>
<td>1,2,3,4</td>
<td>Assumed</td>
</tr>
<tr>
<td>Humidity</td>
<td>0.65</td>
<td>0.026</td>
<td>0.04</td>
<td>Normal</td>
<td>1,2,3,4</td>
<td>[15]</td>
</tr>
<tr>
<td>Cement content</td>
<td>362 kg/m³</td>
<td>36.20</td>
<td>0.10</td>
<td>Normal</td>
<td>1,3</td>
<td>[1]</td>
</tr>
<tr>
<td>Water.cement ratio</td>
<td>0.50</td>
<td>0.05</td>
<td>0.10</td>
<td>Normal</td>
<td>1</td>
<td>[1]</td>
</tr>
<tr>
<td>Sand-cement ratio</td>
<td>5.16</td>
<td>0.156</td>
<td>0.10</td>
<td>Normal</td>
<td>1</td>
<td>[1]</td>
</tr>
<tr>
<td>Frequency</td>
<td>9 Hz</td>
<td>0.72</td>
<td>0.08</td>
<td>Normal</td>
<td>1,3</td>
<td>Assumed</td>
</tr>
<tr>
<td>Mean stress</td>
<td>0.35(f_c)</td>
<td>0.035</td>
<td>0.10</td>
<td>Normal</td>
<td>1,2,3,4</td>
<td>Assumed</td>
</tr>
<tr>
<td>Stress amplitude</td>
<td>0.3(f_c)</td>
<td>0.03</td>
<td>0.10</td>
<td>Normal</td>
<td>1,2,3,4</td>
<td>Assumed</td>
</tr>
<tr>
<td>Number of cycles</td>
<td>(10^5)</td>
<td>80000</td>
<td>0.08</td>
<td>Normal</td>
<td>1,2</td>
<td>Assumed</td>
</tr>
</tbody>
</table>

1 = BP, 2 = modified MC90/CE 2, 3 = modified Hyperbolic, 4 = Neville
The likelihood function is the product of functions, one function for each indication, and is identical in form, e.g., to a Gaussian PDF with expectation equal to the indication and variance formally equal to the unknown variance.

In Figure 2, plots are shown for the prior and the posterior probability density for mean observed cyclic creep function.

![Figure 2: Illustration of prior and posterior probability density for the observed mean of cyclic creep. Also the likelihood for the test results is shown.](image)

All three (prior, likelihood and posterior uncertainty) are applied to determine the measurement uncertainty. For the process equation based measurement uncertainty, the uncertainty associated with the assignment of a probabilistic to a measured cyclic creep strain is modeled. In Figure 2 the probability densities for the mean of the statistical model are depicted. The posterior density, calculated with Bayesian updating, is then orientated closer to the likelihood with a slightly higher density.

The measurement uncertainty obtained by observation has different boundary conditions associated with the probability models calculated separately. The process equation based measurement uncertainty is seen as the accumulation of prior knowledge of the measurement process. It becomes clear that the measurement uncertainty for a specific application is not exactly determinable and that furthermore, both concepts for the determination of the measurement uncertainty have their different boundary conditions and their limitation [16].

In order to carry out MCM of the Bayesian updating by running the program in MATLAB, for \( n_{\text{digit}} = 1 \) it performed \( 10^6 \) evaluations of the different models until the stabilisation of the results. It gives the estimate cyclic creep with associated standard uncertainty, and measurement uncertainty, \((CV_{\varphi, \beta}) \text{ or } u(Ex))\); For simplification in this work, (standard uncertainty \( u(Ex) \), is written as measurement uncertainty \((CV_{\varphi, \beta})\), which is shown in the last row Table 2, with a shortest 95 percentage coverage interval. Noted in this work, is the method of calculating the measurement error and predicted values to consider measurement uncertainty with the goal of facilitating enhanced evaluation of cyclic creep models. The basis of this method was the theory that cyclic creep models should not be evaluated against the values of measured data, which are uncertain, but against the inherent measurement uncertainty. Especially for, the deviation calculation of the probability distribution of measured data, the value of internal uncertainty is assumed. Apparently, the output has a Gaussian shape, see Figure 4, which is usually predicted. However, in detail the statistical study, results show a deviation from normally, due to the excess kurtosis coefficient value of 0.5. In order to study the behaviour of the output PDF and the relations with the several model parameters, a sensitivity analysis focusing on the measuring uncertainties was

![Figure 3: Output PDF of cyclic creep obtained for MCM](image)
also carried out, allowing the identification of critical parameters for the measurement uncertainty magnitude and the output PDF shape.

\begin{table}[h]
\centering
\caption{Uncertainties in cyclic creep models}
\begin{tabular}{|c|c|c|c|c|}
\hline
Model & 1 & 2 & 3 & 4 \\
\hline
U(E(total)) & 0.283 & 0.306 & 0.300 & 0.380 \\
U(E(internal.)) & 0.080 & 0.080 & 0.080 & 0.080 \\
U(E(posterior)) & 0.062 & 0.086 & 0.093 & 0.121 \\
\hline
\end{tabular}
\end{table}

1 = BP, 2 = modified MC90/CE 2 , 3 = modified Hyperbolic , 4 = Neville

2.3 Model uncertainty

Model uncertainty is the uncertainty related to imperfect knowledge or idealisations of the mathematical models used or uncertainty related to the choice of probability distribution types for the stochastic variables. Even when there occur no measurement uncertainty (or when it is negligible), there may be some discrepancies between the predicted and observed values in most situations. This is called model error or uncertainty.

3 EVALUATION OF MODELS QUALITY CONSIDERING MEASUREMENT UNCERTAINTY

The mean value of the predicted cyclic function of the four models for a short time is presented in Fig 4. Because the initial elastic strains were not reported, due to pronounced short-time creep duration, they had to be assumed, and thus the compressions are relevant only to the part of strain representing the creep increase due to the part of strain cycling. Significant errors have often been caused by combining the creep coefficient with an incompatible value of the conventional elastic modulus. Thus analysis must be properly based on the cyclic creep function. In Fig. 4 the data of all four models show quite different values in the first hour of testing and at 100 hours the difference shown is minimal despite the use of a similar concrete and testing condition. This may be due to fluctuation in time to the physical mechanism of creep. The modified MC90/EC2, Neville and modified hyperbolic models are based only on the set of data and may not be applicable to conditions substantially different than these during the experiments.

Fig. 5 and 6 shows that the results of the uncertainty analysis of four different models. Both Figs. showed that the correlated and uncorrelated contribution of input variables have made important contributions to the uncertainty in model output. The uncorrelated input variables uncertainty of model Neville is very small, only the contribution of four variables. On the other hand, the input variables are notable effects on the output, because there are more variables and the complex model and model uncertainty is small. The correlated and uncorrelated input variables for the model Neville shows the largest uncertainty $CV_{par,cyc}(t - t_0) = 0.08$ at $t = 1$ h and uncertainty $CV_{par,cyc}(t - t_0) = 0.06$ at $t = 100$ h, the uncertainty goes to decreasing with the increasing time under load. The uncorrelated input quantities uncertainty of model mod. MC90 and mod. Hyperbolic $CV_{par,cyc}(t - t_0) = 0.10$ and are almost independent with time. Model BP has strongly time-dependent uncertainty varying in the range of $CV_{par,cyc}(t - t_0) = 0.11 \cdots 0.08$. Taking into account the input variables real correlation of the Neville model the input variables increase significantly $CV_{par,cyc}(t - t_0) = 0.08$.
and may cause the effect of strong correlation of strength and young modulus of elasticity. Comparing the total uncertainty of the models from Fig. 6, we conclude that the model and measurement play the important role on the uncertainty behaviour of models. In comparison of all models, BP has the lowest total uncertainty $CV_{par,cyc}(t-t_0) = 0.30$ and Neville model data, such that long-term values cannot be estimated with confidence. Generally, the longer the time over which creep has actually been measured, the better the prediction. The CV in the initial time of loading shows a higher figure and with increasing time, because the initial time shows more uncertainty in measurement. The most important variable at short-time creep is model uncertainty factor for all models.

Figure 5: Input variables uncertainty of cyclic creep prediction

has highest total uncertainty $CV_{tot,cyc}(t-t_0) = 0.40$. The models mod. MC90, mod. Hyperbolic and Neville are based on the experimental data and also, assumed time strain equation always satisfactorily fit the experimental

Figure 6: Total uncertainty of cyclic creep prediction

Total model quality (MQ) can be used to balance the better response of the model to its uncertainty in order to select the model that is most suitable for a certain response. Fig. 7 show the time-dependent model quality. MQ which is dependent upon total uncertainty con-
considering the correlated input quantities. The MQ is slight time dependent. For this reason, the time interrogation is according to [4] and results are given in Fig 5. In all these comparisons, model BP is found to be the best. CEB-MC90/EC2 model, which modifies its original model MC90/EC2 by co-opting key aspects of cyclic loading (the mean stress and stress amplitude function and dependence on the number of cycles would simply mean a loading frequency), comes out as the second best. Considerably worse but the third best overall is seen to be the modified Hyperbolic model. Since the current the Neville model, labelled Neville, is the simplest, introduced in 1973 on the basis of Neville’s research, it is not surprising that it comes out as the worst, because it is based on only four variables and there is no consideration of concrete composition and environmental variables.

![Figure 7: Total uncertainty of cyclic creep prediction](image)

**3.1 Sensitivity analysis (SA)**

For the quantification of the influences of the individual parameters on the cyclic creep strain, a sensitivity analysis is performed. SA is required to find out the dominant effect of the variability of input random variables on the cyclic creep strain. Fig. 8 shows the results of the sensitivity analysis of uncorrelated and correlated variables. For the calculation of the sensitivity, the model uncertainty is not considered. It is assumed that the sensitivity indices are up to \( \sum_{p=1}^{nK} S_p = 1 \). The normalisation is necessary due to consideration of correlation, which may be the results of sensitivity indices \( S_p \geq 1 \). From this arises the difficulties in the comparison between the uncorrelated and correlated indices. High value of sensitivity \( S_p \) indices means highly influential on the uncertainty. For example \( S_p = 1 \) means only this quantities affect the output. [6] method is used in this paper for the global sensitivity analysis.

![Figure 8: Uncorrelated and correlated sensitivity indices of model BP](image)

In the model BP sees a more time dependent sensitivity indices over time. The main reason behind this is the increased combination of time
function with the input quantities. It is seen that the most sensitive quantities turn out to be concrete strength. In second place is the content of the cement when quantities are assuming the uncorrelated. Further, the stress amplitude and frequency is the third and fourth influence quantities. The influence of water-cement ratio, aggregate-sand ratio and humidity are also considerable. The concrete strength is most dominating in quantities when considering the quantities correlation. The second dominant quantity is the cement content and stress amplitude. The sensitivity indices of cement content and stress amplitude show a small decrease with increasing time. In cyclic parameter it is observed that there are considerable influences.

4 CONCLUSION

This paper investigated the various sources of uncertainty in a cyclic creep prediction and illustrated the proposed methods to the quality of the overall uncertainty in cyclic creep prediction for structures with complicated geometry and cyclic loading. Several sources of uncertainty measurement uncertainty, physical variability, data uncertainty and modeling errors - were included in the prediction. A framework for the determination of uncertainties is introduced. This framework, which is elaborated on the example of cyclic creep strain measurement, accounts explicitly for the assignment of the uncertainty of a probabilistic model to a measurement value and for model uncertainties.

The new concept of measurement uncertainty, the posterior measurement uncertainty and cyclic creep strain measurement uncertainty are derived by Bayesian updating. The prior and the likelihood are informative distribution as the prior measurement uncertainty and the likelihood is associated with probabilistic models of observations. Physical variability included loading conditions and materials properties such as stress intensity. The success of the evaluation of measurement uncertainties depends on the nature of the metrogological problems considered, being particularly relevant to the nature of the mathematical models used. The uncertainty in data used to characterise these parameters was taken into account. The variability influence of input random quantities on the cyclic creep was studied by means of the stochastic uncertainty. The Latin Hypercube Sampling (LHS) was used.

The uncertainty and sensitivity analysis is computed using the LHS sampling technique. It is seen from the uncertainty analysis that the complex cyclic creep model BP has the good MQ and less uncertainty but the simple Neville model has higher uncertainty and lower model quality. In contrast, the complex model needs computational effort and more input variables. Accounting for measurement uncertainty and model uncertainty with this methodology can improve model calibration by reducing the likelihood.

Also, the proposed approach for uncertainty quantification is applicable to several engineering disciplines and the domain of cyclic creep analysis was used only as an illustration to develop the methodology. In general, the proposed methodology provides a fundamental framework in which multiple models can be connected through a Bayes network and the confidence in the overall model prediction assessed quantitatively.

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