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MULTI-LAYER SHEAR LAG MODEL FOR POST-INSTALLED ANCHOR USED IN RETROFIT OF CONCRETE STRUCTURES

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Abstract: An analytical model is presented for the pull-out behavior of an anchor-infill assembly structure which consists of multi-layer infill materials separated by steel layers. The use of an anchor-infill assembly structure of multi-layer internal structure is for the retrofit of a concrete bridge pier by the concrete jacketing method for example. The present analytical model is a combination of shear-lag models with an end spring at the end of each steel layer. The advantage of the present model for an anchor-infill assembly structure is discussed and the influence of the material parameters on the behavior of the anchor-infill assembly is shown.

1 INTRODUCTION

In the retrofitting of concrete bridge piers it is desirable to minimize the residual deflection and the damage in the structure in particular the foundation structure[1-5]. In this study it will be investigated analytically to obtain such a condition by adjusting the mechanical properties of anchor-infill assembly structure used in the concrete jacketing method.

In this study a multi-layer anchor-infill assembly structure for the post-installed anchor is presented with the analytical model to simulate its pull-out deformational response. The advantage of the present analytical model to the previous ones[1-3] is that it can have a desired number of infill layers. Then, it becomes possible to have longer de-bonded length at the interface between the steel layer and the infill material layer. Thus the energy absorbing capacity increases. The de-bonding at the interface between the steel layer and the infill material layer is represented by the interface failure criterion for each infill material.

The anchor-infill assembly structure for a post-installed anchor is such that consists of an anchor bar and surrounding infill materials having a multi-layer structure. The mechanical properties of the infill material used for the present anchor-infill assembly structure are characterized by nonlinear interfaces. Because of the mechanical properties of the infill layer the existing pull-out model of deformed bars is not applicable in this case.

Interface de-bonding is examined using the stress criterion. The influence of the mechanical properties such as stiffness and strength of the infill material on the pull-out behavior of an anchor-infill assembly structure is examined numerically.

2 MODELING OF ANCHOR-INFILL ASSEMBLY

In the present model, the shear-lag model with an end spring to express the end constraining effect[6] is used. The end constraining effect means the resistance to the pull-out of an anchor or a steel layer provided by the various types of end shape.

2.1 Pull-out model

The pull-out model of an anchor-infill assembly is shown in Fig.1. The anchor-infill assembly is assumed symmetric and the right half is shown in Fig.1. The assembly structure consists of a number of steel layers and infill layers between the steel layers. The pull-out load is assumed to be given vertically.

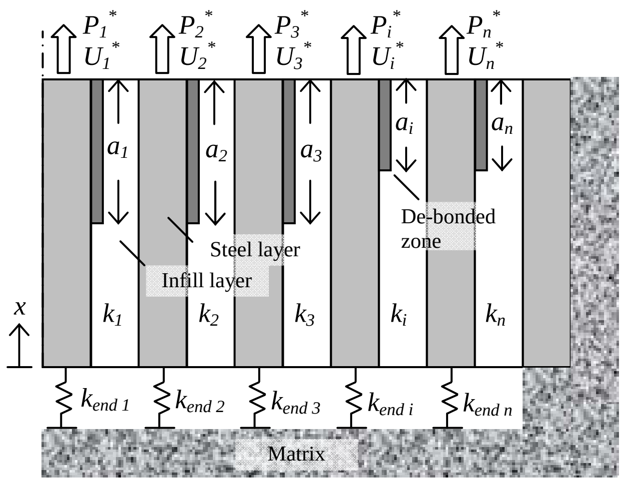

Figure 1: Multi-layer anchor-infill assembly.

The shear traction q_i acting on the interface between a steel layer and an infill layer is characterized by the following equations for the i-th layer.

$$q_i = k_i U_i(x) \quad , \quad 0 < x < L - a_i \ ;$$

$$q_i = q_{0i} \quad , \quad L - a_i < x < L \tag{1}$$

where k_i is the stiffness of the infill layer, U_i is the relative pull-out displacement in the i-th infill layer, x is the coordinate measured from the bottom of the assembly structure, L is the length of the assembly structure, and a_i is the length of de-bonded interface between the steel layer and the infill layer.

The relationship between the pull-out force P_i and the shear traction q_i is given by the following equation where $(\ )_{,i}$ represents the differentiation with respect to x.

$$P_{i,x} - q_i = 0 \tag{2}$$

The pull-out force P_i is expressed in terms of the relative pull-out displacement U_i as follows.

$$P_i = E_i A_i U_{i,x} \tag{3}$$

where E_i is the elastic modulus and A_i is the cross-sectional area of steel of i-th layer. Then, the following differential equations are obtained.

$$U_{i,xx} - \omega_i^2 U_i = 0 \quad , \quad 0 < x < L - a_i \ ;$$

$$U_{i,xx} - \frac{q_{0i}}{E_i A_i} = 0 \quad , \quad L - a_i < x < L \tag{4}$$

where

$$\omega_i = \sqrt{\frac{k_i}{E_i A_i}} \tag{5}$$

The boundary conditions are given by the following equations where $k_{end\ i}$ is the spring constant to express the end constraint at the bottom of a steel layer and P_i^* is the pull-out force acting at the top of a steel layer.

$$k_{end\,i} U_i = P_i \quad , \quad x = 0 \ ;$$

$$E_i A_i U_{i,x} = P_i^* \quad , \quad x = L \tag{6}$$

The continuity conditions at the boundary between the de-bonded interface and the unde-bonded interface for the displacement and the displacement gradient are given as follows.

$$U_i^- = U_i^+ \quad , \quad x = L - a_i \ ;$$

$$U_{i,x}^- = U_{i,x}^+ \quad , \quad x = L - a_i \tag{7}$$

The solution for the relative pull-out displacement is obtained by solving Eq.(4) with the boundary and continuity conditions, Eqs.(6) and (7), as follows.

$$U_i(x) = \frac{P_i^* - q_{0i} a_i}{E_i A_i \omega_i} \left\{ \frac{\cosh \omega_i x}{\alpha_{1i}} + \frac{\sinh \omega_i x}{\alpha_{2i}} \right\}, 0 < x < L - a_i \ ;$$

$$U_i(x) = \frac{P_i^* - q_{0i}a_i}{E_i A_i \omega_i}\left\{\frac{\cosh\omega_i(L-a_i)}{\alpha_{1i}} + \frac{\sinh\omega_i(L-a_i)}{\alpha_{2i}}\right\} + \frac{q_{0i}}{2E_i A_i}\left(x^2 - (L-a_i)^2\right) + \frac{P_i^* - q_{0i}L}{E_i A_i}\left(x - (L-a_i)\right), \quad L - a_i < x < L \tag{8}$$

where

$$\alpha_{1i} = \sinh\omega_i(L-a_i) + \beta_i\cosh\omega_i(L-a_i);$$
$$\alpha_{2i} = \beta_i^{-1}\sinh\omega_i(L-a_i) + \cosh\omega_i(L-a_i); \tag{9}$$
$$\beta_i = \frac{k_{end\,i}}{E_i A_i \omega_i}$$

The pull-out displacement at the top of the steel layer U_i^* is given by the following equation.

$$U_i^* = \frac{P_i^* - q_{0i}a_i}{E_i A_i \omega_i}\left\{\frac{\cosh\omega_i(L-a_i)}{\alpha_{1i}} + \frac{\sinh\omega_i(L-a_i)}{\alpha_{2i}}\right\} + \frac{P_i^* - \frac{1}{2}q_{0i}a_i}{E_i A_i}a_i \tag{10}$$

2.2 Infill material model

The mechanical behavior of an infill material is shown in Fig.2. The vertical axis shows the shear traction which is a force acting on the interface between a steel layer and an infill layer per unit length. The horizontal axis shows the shear slip displacement along the interface. Before yielding of the infill material, the relationship is assumed to be linear. After yielding a certain amount of decrease occurs. Then, the shear traction is assumed to be constant, although as the shear slip displacement increases, the shear traction tends to decrease in general because of the decrease of shear resistance of the interface[6].

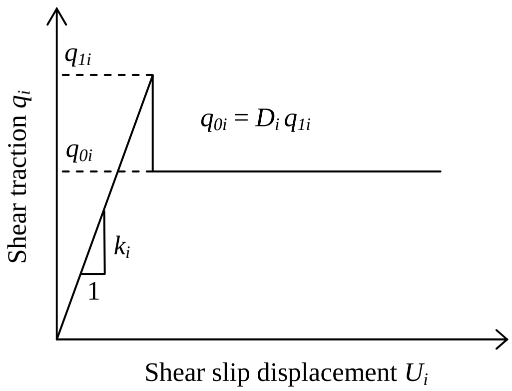

Figure 2: Mechanical behavior of infill material.

The shear traction after reaching the yield point q_{1i} is given as follows.

$$q_{0i} = D_i q_{1i};$$
$$D_i = D_{0i}, \quad 0 \le D_{0i} \le 1 \tag{11}$$

The shear traction along the interface between a steel layer and an infill layer is given as follows.

$$q_i(x) = (P_i^* - D_i q_{1i} a_i)\omega_i\left\{\frac{\cosh\omega_i x}{\alpha_{1i}} + \frac{\sinh\omega_i x}{\alpha_{2i}}\right\}, \quad 0 < x < L - a_i$$
$$q_i(x) = D_i q_{1i}, \quad L - a_i < x < L \tag{12}$$

3 CRITERION OF DE-BONDING

As the shear traction acting on the interface between a steel layer and an infill layer increases, de-bonding of the interface occurs. The criterion of de-bonding is based on the stress criterion in this study.

3.1 Stress criterion

The stress criterion for de-bonding is based on the assumption that the de-bonding occurs when the maximum shear traction acting on the interface between a steel layer and an infill layer reaches the critical value. Thus, the de-bonding of infill material starts when the shear traction of the infill material becomes equal to the strength of the infill material.

$$q_i(L - a_i) = q_{1i} \tag{13}$$

In this study it is assumed that the critical value q_{1i} is constant.

3.2 Load at de-bonding

Using the condition of the shear traction for de-bonding, the load at de-bonding is given as follows from the solution of the shear traction, Eq.(12).

$$\begin{aligned} P_i^* = q_{0i}a_i + \frac{q_{1i}\alpha_{1i}\alpha_{2i}}{\omega_i} \\ \{\alpha_{1i}\sinh\omega_i(L - a_i) \\ + \alpha_{2i}\cosh\omega_i(L - a_i)\}^{-1} \end{aligned} \tag{14}$$

Eq.(14) is substituted into Eq.(10) when the pull-out displacement U_i^* is to be calculated. The pull-out load P_i^* and the relative pull-out displacement U_i^* at the top of i-th steel layer are summed to get the total pull-out load and the total pull-out displacement.

4 LOAD-DISPLACEMENT RELATIONSHIP

The pull-out load-displacement relationship for i-th steel layer is obtained by Eqs.(10) and (14) by solving these equations for either given pull-out load or pull-out displacement for the de-bonded length a_i. When the anchor-infill assembly structure consists of n sets of steel-infill layers, the total pull-out force P^* and the total pull-out displacement U^* are expressed as follows.

$$\begin{aligned} P^* = \sum_{i=1}^{n} P_i^*; \\ U^* = \sum_{i=1}^{n} U_i^* \end{aligned} \tag{15}$$

The solution for the whole anchor-infill assembly structure is obtained by solving the above nonlinear simultaneous equations for given boundary conditions. The boundary conditions may be given by the pull-out force distribution or by the pull-out displacement distribution.

5 INFLUENCE OF PARAMETERS

The influence of the material properties on the pull-out behavior of the anchor-infill assembly is studied numerically.

The constants used for the anchor-infill assembly are shown in Table 1. The ratio between the modulus of elasticity of steel E_{si} and that of infill E_{Ii} is set E_{si} / E_{Ii} =10. The Poisson's ratio of infill is ν_{Ii} = 0.3. The yield point of infill q_{1i} , defined as the force acting on the steel-infill interface per unit length, is determined by the balance with the yield point of steel, 300 N/mm^2. The reduction coefficient D_i to be multiplied to the yield point of infill q_{1i} to get the shear traction after de-bonding q_{0i} is 0.5. The reduction coefficient D_i represents the shear traction transfer capability of the de-bonded steel-infill interface.

The ratio between the length of steel layer L and the half thickness of the center steel layer, and the thickness of other steel layer t_{si} is set L / t_{si} = 40. This value of the L / t_{si} ratio is chosen by considering the value of the embedment length and the diameter of a post-installed anchor used in actual retrofit work. The ratio between the thickness of steel layer t_{si} and the thickness of infill layer t_{Ii} is set t_{si} / t_{Ii} = 2.

Table 1: Constants of anchor-infill assembly

Elastic modulus of steel ($E_{si} = E_i$)	2.0 x 10^4 N/mm^2
Elastic modulus of infill (E_{Ii})	2.0 x 10^3 N/mm^2
Yield point of infill (q_{1i})	7.5 N/mm
Reduction coefficient (D_i)	0.5
Length of anchor-infill assembly (L)	200 mm

The parameters to be examined in this study are the shear strength or the yield point of an infill material, the shear stiffness of an infill material, and the steel layer thickness.

The anchor-infill assembly structure to be examined in this study consists of 8 infill layers separated by steel layers. The relative pull-out displacement at each steel layer U_i^* is given as an imposed displacement as a displacement boundary condition. The distribution of the relative pull-out displacement at each steel layer U_i^* is expressed by a quadratic polynomial function so that the total pull-out displacement distribution would be a cubic polynomial function. The total pull-out force P^* is applied as a concentrated force at the top of the center steel layer and it is assumed to be distributed to the other steel layers by a sufficiently stiff cantilever-like structure connected to the top of each steel layer and fixed at the most external steel layers to be contact with surrounding concrete matrix. The relative pull-out displacement given at the top of the first steel layer U_1^* = 0.08mm. The influence of each parameter on the total de-bonded length for all interfaces between steel layers obtained by summing up all the de-bonded length at each interface a_i and infill layers and the total pull-out force obtained by summing up all the pull-out force P_i^* at each steel layer will be discussed in the following.

For all the parameters to be examined, the distribution of the parameter is assumed to be linear, that is, the present anchor-infill assembly structure has a property of a functional gradient material.

5.1 Influence of shear strength

The shear strength of an infill layer q_{1i} is assumed to distribute linearly from the center layer to the outer layer. The influence of shear strength of infill material on the de-bonded length is shown in Fig.3. The horizontal axis shows the shear strength ratio q_{1n} / q_{11} and the vertical axis shows the change of de-bonded length compared to the base value obtained from the values shown in Table 1.

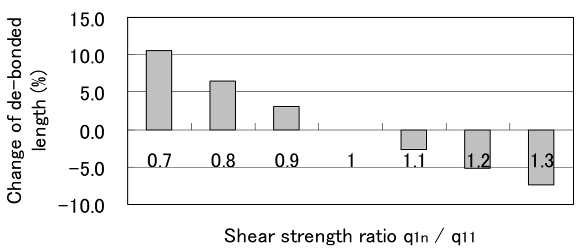

Figure 3: Influence of shear strength of infill material on de-bonded length.

It is shown that the de-bonded length decreases as the shear strength ratio increases, that is, when the shear strength of the outer layer becomes larger than that of the center layer.

The influence of the shear strength of infill material on the pull-out force is shown in Fig.4. It is shown that the pull-out force increases as the shear strength of the outer layer becomes larger than that of the center layer.

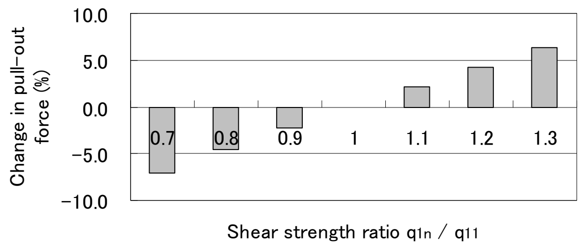

Figure 4: Influence of shear strength of infill material on pull-out force.

5.2 Influence of shear stiffness

The shear stiffness of an infill layer k_i is assumed to distribute linearly from the center layer to the outer layer. The influence of shear stiffness of infill material on the de-bonded length is shown in Fig.5. It is shown that the de-bonded length becomes larger as the shear stiffness of the outer layer increases.

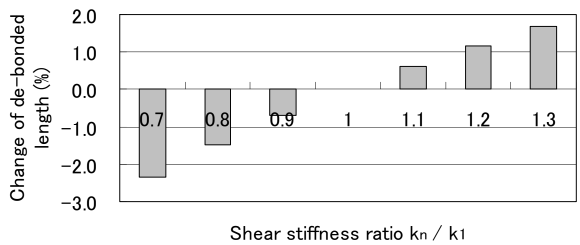

Figure 5: Influence of shear stiffness of infill material on de-bonded length.

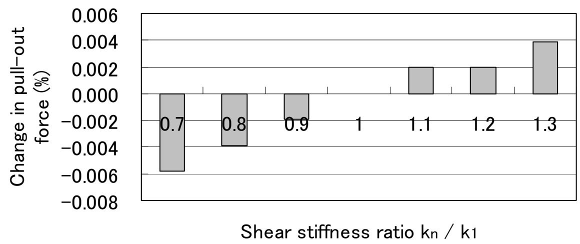

Figure 6: Influence of shear stiffness of infill material on pull-out force

Fig.6 shows the influence of the shear stiffness of an infill material on the pull-out force. Although there is a tendency that the pull-out force increases as the shear stiffness of the outer layer increases, the influence is small.

5.3 Influence of steel layer thickness

The influence of the steel layer thickness on the de-bonded length and the pull-out force is shown in Fig.7 and Fig.8. The steel layer thickness changes linearly from the center to the outer layers. It is shown that the de-bonded length and the pull-out force increase as the steel layer thickness increases.

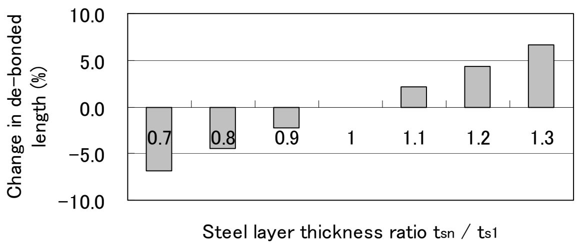

Figure 7: Influence of steel layer thickness on de-bonded length.

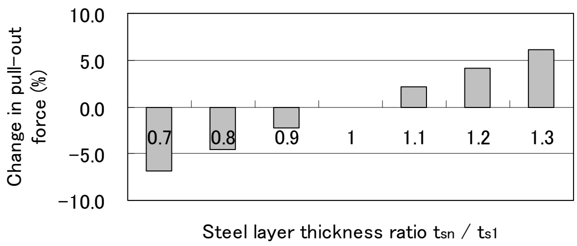

Figure 8: Influence of steel layer thickness on pull-out force.

6 CONCLUSIONS

A pull-out model for the anchor-infill assembly with the multi-layer infill structure and the end constraint of an anchor and steel layers is presented in this study.

(1) The present multi-layer shear-lag model can be used to express the pull-out behavior of an anchor-infill assembly with a number of sets of steel-infill layers.

(2) The multi-layer structure of steel-infill layers is effective to increase the energy absorbing capacity because of larger de-bonded length for selected material constants.

(3) With the present model it is possible to estimate the influence of parameters included in the model on the pull-out behavior of an anchor-infill assembly structure.

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