SIMULATING CRACK WIDTH DISTRIBUTIONS IN SHCC UNDER TENSILE LOADING

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Key words: Fiber reinforced concrete, Lattice model, Strain-hardening, Durability

Abstract: A three-dimensional lattice model is used to evaluate the fracture behavior of Strain-Hardening Cementitious Composites (SHCC) in tension. Individual fibers are explicitly represented within the random lattice representation of the cement-based matrix. Comparisons are made with experimental measurements of crack openings and their evolution during increasing levels of tensile strain. The preliminary results indicate crack openings are correlated with characteristics of the fiber distribution.

1 INTRODUCTION

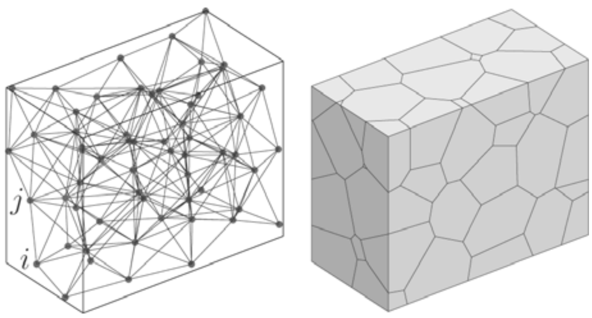
The penetration resistance of structural concrete to water and other deleterious substances is strongly affected by the opening, connectivity, and spacing of cracks [1,2]. The introduction of short dispersed fibers is effective in preventing or reducing crack openings. With proper constituents and design, such materials strain-harden and exhibit numerous fine cracks, rather than few large ones. These strain-hardening cementitious composites (SHCC) are attractive for a variety of applications, including those that require long-term durability.

One relevant area of study regards the sensitivity of SHCC performance to variations in the design parameters, including the potential for non-uniform distributions of the fibers. During tensile loading, SHCC exhibits variations in crack openings [3-6]. It is the larger of the crack openings that control the durability of the SHCC and its substrates. It has been hypothesized that crack openings depend on local variations in the fiber content. Regions with fewer fibers act as defects within the material and promote larger crack openings [7].

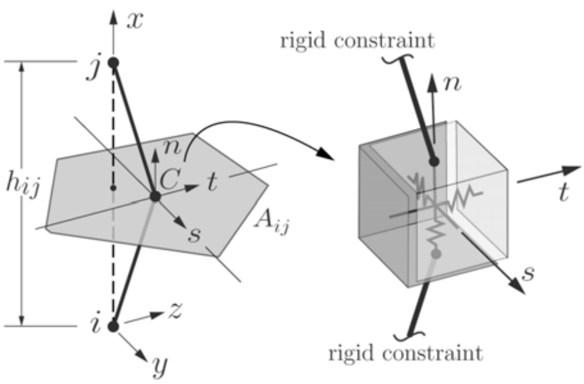
To test this hypothesis, and gain further insights into the fracture mechanics of SHCC, the authors have developed three-dimensional lattice models of SHCC in tension. Individual fibers are explicitly represented within the random lattice representation of the cement-based matrix [8,9]. This form of discrete modeling enables the simulation of crack spacing and crack saturation prior to fracture localization. Comparisons are made with experimental measurements [3,6] of crack opening histograms and their evolution during increasing levels of tensile strain. The results also indicate correlations exist between crack width and characteristics of the fiber distribution. In particular, larger cracks occur in regions with fewer fibers. These simulations provide insights into some of the relationships between material parameters, methods of processing, and composite performance.

2 LATTICE MODELING OF SHCC

Tensile fracture of SHCC is simulated using a lattice model [8,9], in which the matrix and short fibers are modeled as distinct phases. This approach belongs to a growing trend in the modeling of fiber reinforced cement composites [10-14]. In this paper, the matrix elements are connected on a set of randomly distributed nodes. The Delaunay tessellation of this nodal set defines the lattice topology, whereas the dual Voronoi tessellation defines the matrix element properties. Matrix elements are based on the rigid-body-spring concept illustrated in Fig. 1b, where A_{ij} is the area of the associated Voronoi facet and h_{ij} is the distance between nodes i and j. By the A_{ij} / h_{ij} scaling of the spring stiffnesses, the lattice representation of the matrix is elastically homogeneous. Details of the matrix element formulation and solution process are given elsewhere [8,9]. Hereafter, the modeling of short fibers is briefly described.



(a) Delaunay/Voronoi tessellations of material domain



(b) Matrix element *ij* with zero-size spring set at point *C*

Figure 1: Lattice modeling of matrix phase [9]

2.1 Semi-discrete fiber model

A fiber lattice element is formed wherever a fiber intersects a matrix element cross-section (e.g., at point P in Fig. 2, where l_f is the fiber length and θ is the angle between the fiber axis and the loading direction). Analogous to the construction of the matrix elements, nodes i and j are linked via rigid-arm constraints and a zero-length spring. The spring is positioned at point P and aligned with the fiber. This spring represents the axial stiffness of the fiber prior to matrix cracking, according to shear lag theory [8,9].

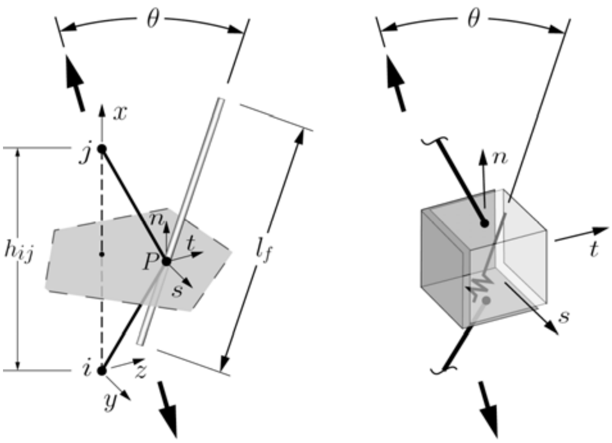


Figure 2: Fiber element and zero-size spring set positioned at point P [9]

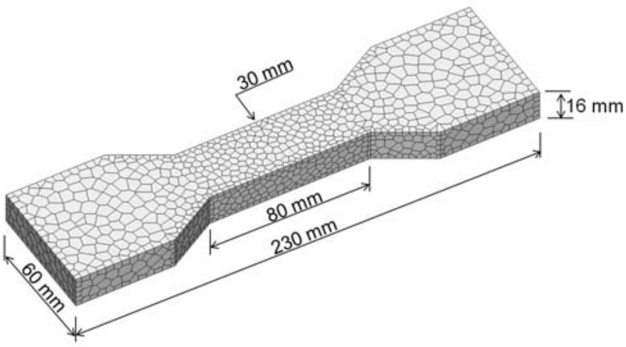
After cracking of the matrix element *ij*, properties of the spring traversing the crack are governed by debonding and frictional pullout, according to the micromechanical model of Naaman et al. [15]. The spring stiffness is nonlinear and gradually reduces to the stress free condition as the fiber pulls out. Fiber rupture in tension is possible, but not considered in this study.

If N fibers intersect the matrix element cross-section, there will be N fiber element contributions to the stiffness coefficients associated with nodes i and j. In this sense, fiber additions do not increase the number of computational degrees of freedom of the model, since fiber elements and matrix elements connect to the same lattice nodes. This enables computations involving a large number of fibers. For example, elastic moduli of micro-fiber composites have been calculated, in which over 1×10^7 fibers were explicitly modeled [9].

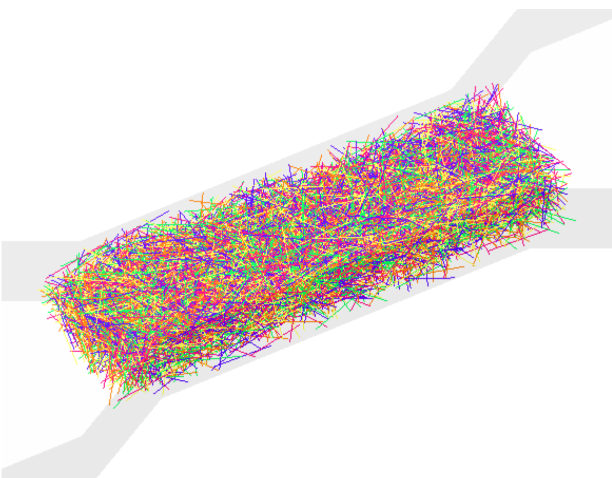
We describe this fiber model as *semi-discrete*, because fiber loading is constrained to the rigid-body kinematics of the associated matrix elements. Alternatively, fibers can possess their own degrees of freedom [16], but that capability is computationally expensive when large numbers of fibers are considered.

2.2 Discretization of tensile test specimen

The SHCC tensile specimens are modeled by the lattice structure shown in Fig. 3, which represents a series of specimens tested by Adendorff et al. [6]. Loads were applied through displacement control of two hinged supports within the end regions of the model, as done in the experiments. Nodal density has been graded, so that a finer discretization is used within the 80 mm gage length.



(a) Lattice discretization of dog-bone specimens



(b) Random placement of fibers within gage length

Figure 3: Modeling of SHCC tensile specimens

Fibers were randomly placed in the central (100 mm) portion of the specimen, where crack formation is expected (Fig. 3b). Based on the dimensions of the Polyvinyl Alcohol (PVA) fibers (l_f = 12 mm, d_f = 40 μm), approximately 64,000 fibers were introduced to achieve a volume fraction V_f = 2%. Typical values were assumed for the matrix and fiber properties that were not reported in [6]. At this preliminary stage of our research, matrix fracture was assumed to be brittle. The consequences of this assumption are discussed later.

3 SIMULATION RESULTS

3.1 Crack patterns

Results presented in this section were obtained using a prismatic specimen, shown in Fig. 4. To compensate for the lack of wider end zones, as previously shown in Fig. 3a, fracture outside of the 80 mm gage length was suppressed. The displacements have been magnified so that, at the 0.8% strain level shown in the figure, non-uniformity of the multiple crack openings is clearly evident. The multi-cracked region of the composite resembles islands of material bridged by individual fibers, which can be seen through close inspection of Fig. 4. Future simulations will make use of the more accurate mesh design shown in Fig. 3a.

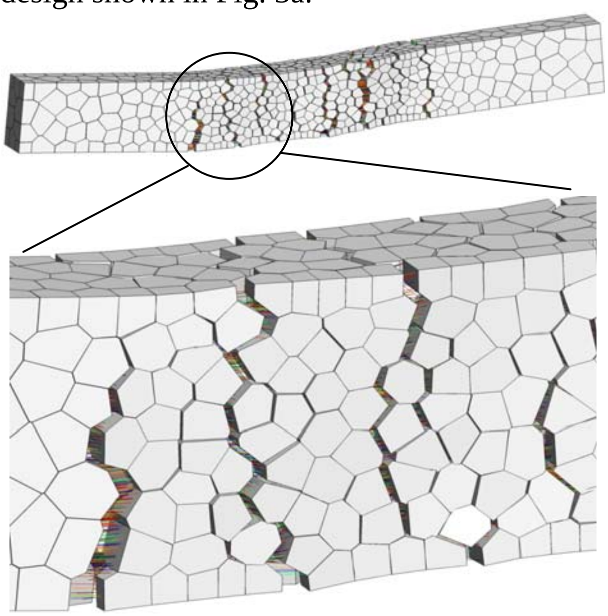


Figure 4: Multiple crack formation within lattice model

3.2 Stress-strain response

The stress-strain response of the lattice model, over the 80 mm gage length, is compared with the results of Adendorff et al. [6] in Fig. 5. Tensile strength of the matrix was adjusted so that the first cracking strength of the model agrees with those of the test specimens. Immediately after first cracking, however, the model exhibits a sudden drop in load resistance that does not appear in the experimental results. This is possibly due to the assumption of brittle fracture of the matrix. Furthermore, although extensive multiple cracking occurs, the model exhibits limited strain capacity prior to fracture localization. By 1% strain, fracture has already localized and softening has initiated in the model. The lack of strain capacity is possibly due to several factors:

- The mesh is relatively coarse when considering the fine crack systems that develop in the test specimens. A finer mesh could allow for continued crack formation and extension of the hardening regime.
- The sudden loss of matrix strength, associated with the brittle fracture assumption, transfers larger loads to the fiber bridges and causes a greater degree of non-uniformity of load paths through the specimen. These effects hasten the localization process.
- Controlled variations in matrix strength are known to promote progressive cracking and strain hardening. However, the matrix strength was assumed to be uniform for this simulation.

Even when considering these factors, however, the lack of toughness of the model is unexpected and points to a need for further study of fiber-matrix interactions in these systems. The simple notion of summing the matrix and fiber contributions to represent composite performance might be insufficient. The presence of the fibers modifies the fracture properties of the matrix; the distributed nature of fracture also affects the efficiencies of the fiber reinforcement. Whereas the explicit modeling of the matrix and fibers as separate phases is an attractive feature of this work, the two-way coupling of matrix and fiber behaviors presents a modeling challenge.

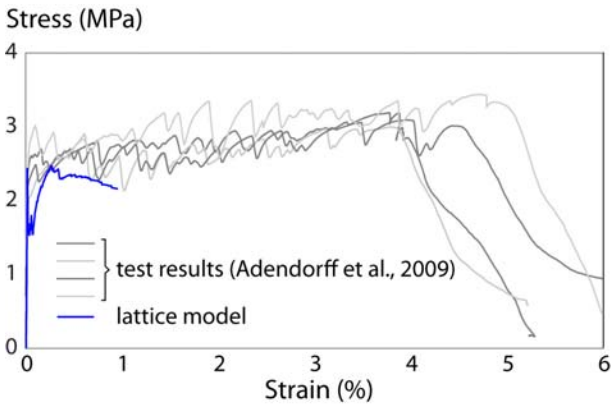


Figure 5: Stress-strain response of SHCC specimens

3.3 Crack opening histograms

Figure 6 presents crack width histograms for several strain levels. In the experiments, crack openings were measured (using Digital Image Correlation) along three longitudinal paths over the gage length. The same approach was used to count and measure crack openings in the model.

At the 0.2% strain level, several cracks have appeared in the model but cracks were not seen in the test results. This difference is likely due to brittle fracture of the matrix in the model. With increasing levels of strain, the cracking trends of the model and test specimens are similar: many fine cracks develop, accompanied by the larger openings of a few cracks. The model tends to support the hypothesis that larger crack openings occur where the local volume fraction of fibers is lower. The histograms of crack width are dependent, to some degree, on how fibers are distributed within the matrix.

In these comparisons, however, the model exhibits larger crack openings. At the 0.8% strain level, for example, the model indicates more than two (equivalent) cracks with openings significantly larger than 100 μm, which is approximately the threshold for large increases in the effective permeability [1]. The overestimation of crack openings, relative to the experimental results, points to a need for careful consideration of the fracture energy of the matrix.

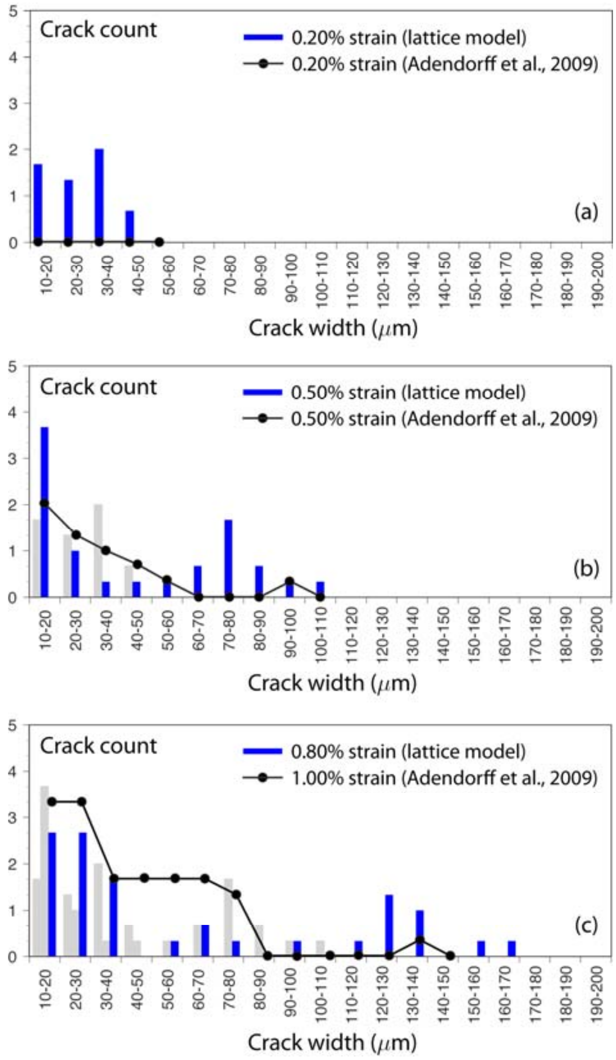


Figure 6: Crack opening histograms

3.4 Future developments

Simulations are currently being run that account for the fracture energy of the matrix phase. The event-by-event solution strategy is stable, but computationally expensive. Furthermore, it appears that much finer discretizations are necessary for resolving the fine crack patterns witnessed during testing. More efficient solution methods are needed to achieve this goal and perform parametric study of SHCC.

The fiber model applied in this paper did not account for snubbing effects, matrix micro-spalling, fiber group effects, and other factors affecting the efficiencies of fiber reinforcement [17]. However, these considerations can be addressed within this form of lattice modeling [13].

Eventually, the attributes of this type of modeling approach will be most apparent when the positions of fibers within the specimen are known, allowing for direct correspondence between the physical and numerical specimens. The positions of steel macro-fibers can be mapped using X-ray computed tomography, for example, but further developments are needed to map micro-fibers and most synthetic fibers. Alternatively, the effects of mixture rheology, boundary conditions and processing techniques on the fiber distribution can be simulated. Svec et al. [18] track the movement of steel fibers within a self-compacting concrete during casting using a fluid dynamics model. The prospect for linking such simulations of the construction process with discrete fiber models of SHCC, such as that presented herein, is intriguing.

4 CONCLUSIONS

A random lattice model has been used to study the tensile fracture behavior of SHCC. The modeling efforts, and comparisons with experimental results, have led to the following observations and conclusions.

- The fine crack systems that develop during strain-hardening are simulated by the model, at least in a qualitative sense. However, quantitative comparisons with test data reveal that the simulated cracks appear earlier in the strain history and there is a tendency for larger crack openings. Brittle fracture of the matrix (in the model) likely contributed to these discrepancies with the test results.
- Strain capacity of the model is low relative to that commonly observed during testing. Finer discretization of the matrix would provide pathways for additional cracking, potentially extending the process of multiple cracking and strain-hardening.
- Although not yet described in quantitative terms, the model results indicate that

correlations exist between local volume fraction of fibers and crack opening. With model improvements, particularly with respect to local interactions between the fibers and matrix, this model can be used to study the effects of fiber distribution non-uniformity on cracking performance.

REFERENCES

[1] Wang, K., Jansen, D.C., Shah, S.P. and Karr, A.F., 1997. Permeability study of cracked concrete, *Cem. Conc. Res.* **27**(3): 381-93.

[2] Hoseini, M., Bindiganavile, V. and Banthia, N., 2009. The effect of mechanical stress on permeability of concrete: A review, *Cem. Conc. Comp.* **31**: 213-20.

[3] van Zijl, G.P.A.G., 2011. Crack distribution characterization towards a framework for durability design of SHCC. *Strain-Hardening Cement Composites (SHCC2-Rio),* RILEM, December 12-14, 2011, Rio de Janeiro, Brazil; pp.149-56 .

[4] Wittmann, F.H., Wang, P., Zhang, P., Zhao, T. and Beltzung, F., 2011. Capillary absorption and chloride penetration into neat and water repellent SHCC under imposed strain. *Strain-Hardening Cement Composites (SHCC2-Rio),* RILEM, December 12-14, 2011, Rio de Janeiro, Brazil; pp.165-72.

[5] Rieger, C., 2010. Micro-Fiber Cement: Pullout Tests, Uniaxial Tensile Tests and Material Scaling, Dr. sc. dissertation, ETH Zürich.

[6] Adendorff, C.J., Boshoff W.P., van Zijl, G.P.A.G., 2009. Characterisation of crack distribution of strain-hardening cement-based composites (SHCC) under imposed load. *Proceedings of the International Conference on Advanced Concrete Materials,* November 17-19, 2009, Stellenbosch, South Africa; pp. 215-21.

[7] Akkaya, Y., Shah, S.P. and Ankenman, B., 2001. Effect of fiber dispersion on multiple cracking of cement composites. *J. Eng. Mech., ASCE*, **127**(4): 311-16.

[8] Bolander, J.E. and Saito, S., 1997. Discrete modeling of short-fiber reinforcement in cementitious composites. *Adv. Cem. Based Mater.* **6**: 76-86.

[9] Bolander, J.E., Choi, S. and Duddukuri, S.R., 2008. Fracture of fiber-reinforced cement composites: effects of fiber dispersion. *Int. J. Fract.* **154**: 73-86.

[10] Leite, J.P.B., Slowik, V. and Mihashi, H., 2004. Computer simulation of fracture processes of concrete using mesolevel models of lattice structures. *Cem. Conc. Res.* **34**(6): 1025-33.

[11] Radtke, F.K.F., Simone, A. and Sluys, L.J., 2010. A computational model for failure analysis of fibre reinforced concrete with discrete treatment of fibres. *Eng. Frac. Mech.* **77**(4): 597-620.

[12] Kunieda, M., Ogura, H., Ueda, N. and Nakamura, H., 2011. Tensile fracture process of SHCC by means of three-dimensional meso-scale analysis, *Cem. Conc. Comp.* **33**(9) : 956-65.

[13] Schauffert, E.A. and Cusatis, G., 2012. Lattice discrete particle model for fiber-reinforced concrete. I: theory. *J. Eng. Mech.* **138**(7): 826-33.

[14] Cunha, V.M.C.F., Barros, J.A.O. and Sena-Cruz, J.M., 2012. A finite element model with discrete embedded elements for fibre reinforced composites. *Comp. Struct.* **94-95**: 22-33.

[15] Naaman, A., Namur, G., Alwan, J., and Najm, H., 1991. Fiber pullout and bond slip. I: analytical study. *J. Struct. Eng.*, **117**(9): 2769–90.

[16] Yip, M., Mohle, J. and Bolander, J.E., 2005. Automated modeling of 3-D structural components using irregular lattices. *Comput.-Aided Civ. Infrastruct. Eng.* **20**(6): 393-407.

[17] Yang, E.-H., Wang, S., Yang, Y. and Li, V.C., 2008. Fibre-bridging constitutive law of Engineered Cementitious Composites. *Adv. Conc. Tech.* **6**(1): 181-93.

[18] Svec, O., Skocek, J., Stang, H., Olesen, J.F. and Thrane, L.N., 2012. Application of a fluid dynamics model to the field of fibre reinforced self-compacting concrete. *Numerical Modeling Strategies for Sustainable Concrete Structures*, May 29-June 1, 2012, Aix en Provence, France.