LIMIT STATES OF RC STRUCTURES: REINFORCEMENT CORROSION, RELIABILITY AND MODELLING

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Abstract: Reinforced concrete is a widely used material for structures of different types as bridges, buildings, underground structures and others. Damage due to reinforcement corrosion is recognized as one of the major causes of deterioration of such structures with service life reduction and considerable additional costs as consequence. An extensive amount of research on these topics has been organized and reported over the last decades and it has been published elsewhere and also reflected in recent international standards and documents – e.g. fib Draft Model Code [1].

The present paper concentrates on the propagation period, i.e. reinforcement corrosion effects, relevant limit states, modelling of degradation mechanism, software tool and service life issue. Several analytical models are briefly presented. The described approach is applied to some illustrative examples showing the feasibility to capture the development of reinforcement corrosion and consequently its effect on the service life and/or reliability of the structure.

1 INTRODUCTION

Reinforced concrete is a widely used material in structures of such different types as bridges, buildings, underground structures and others. Damage due to reinforcement corrosion is recognized as one of the major causes of the deterioration of such structures with service life reduction and considerable additional costs as consequences. Cost-effective decisions need to be made when forecasting the time when an existing structure should be repaired or predicting the service life of a structure during the design stage. Both these activities require some knowledge about the corrosion mechanism and its progress over time, and about the relevant reliability level. The objective is to ensure the functionality and reliability of the structure over its entire service life in order to obtain an optimal result from the whole-life financial point of view. Note that the performance requirements of a structure are generally established by means of performance criteria and associated constraints related to service life and reliability; considering in this respect the reinforcement corrosion of a concrete structure it is evident that a general approach capable of modelling relevant deteriorative effects is needed. An extensive amount of research on these topics has been organized and reported over the last decades and has been published elsewhere; some of these studies are mentioned in the following text.

Owing to either the carbonation of

reinforced concrete or the ingress of chlorides into such a material, the depassivation of the reinforcing steel within it occurs (a process known as the initiation period). This may be followed (under the presence of moisture and oxygen) by a steel corrosion process (known as the propagation period) – see [2]. Suitable methods for the monitoring and reliable mathematical modelling of associated effects are needed – better knowledge in this respect provides the basis for a practical and proactive strategy when designing and maintaining concrete structures. This is also reflected in recent international standards and documents – e.g. [1].

Frequently, the initiation of reinforcement corrosion is supposed to define the end of a structure's service life – see e.g. [3], in spite of the fact that the propagation period may form a significant part of the total service life of a structure. In the case of corroding structures, the residual service life is often taken as the time remaining before a crack of certain width develops at the surface of the concrete.

This is why the present paper concentrates on the propagation period, i.e. reinforcement corrosion effects, relevant limit states, modelling of the degradation mechanism and the use of a software tool to do this, and the service life issue. Note that modelling of the initiation period and associated degradation mechanisms has been dealt with in many other works, e.g. [4-7].

2 LIMIT STATES ASSOCIATED WITH THE PROPAGATION PERIOD

Among the key factors influencing reinforcement corrosion in concrete are the presence of moisture and the ingress of oxygen from the air. The reader may find a detailed description of these factors in e.g. [8-10]. A number of models have been developed which can be roughly divided into two categories – empirical models and more complex electrochemical models. For the purpose of introducing a practical approach for the prognosis of the service life and relevant reliability level of concrete structures the simplest models are utilized in the present work. Inherent uncertainties in the material, technological and environmental characteristics of reinforced concrete structures have to be considered while solving such problems. To respect this, a probabilistic approach has been utilized.

The limit state (LS) concept – in either its Serviceability limit state (SLS) or Ultimate limit state (ULS) variant – is applicable here, governed by a probability condition which in its general form reads:

$$P_f = P\left[A(t) \geq B(t)\right] < P_d \tag{1}$$

where P_d is the design (required) probability, t is time, A is the effect of the analysed action and B is the barrier. Generally, both A and B are time dependent and hence the probability of failure P_f is time dependent too. The combined effect of both structural performance and ageing should be considered wherever relevant. Time t_S corresponds to the limit given by Eq. (1), i.e. it is the predicted service life; time t_D is the design service life. Both t_S and the deteriorative effect A are assessed by utilization of the appropriate degradation models, which apply a probabilistic approach. Note that the index of reliability β is frequently utilized in practice instead of the probability of failure P_f – see e.g. [11, 12].

The level of reliability in the context of durability should be left to the client's decision together with the definition of a target service life, creating in this way a necessary background for the making of critical decisions (e.g. financial optimisation) – see e.g. [1, 13].

Concerning reinforcement corrosion and the appropriate LS, the following cases may be distinguished:

(i) The volume expansion of rust products develops tensile stresses in the surrounding concrete leading to concrete cracking (mainly affecting the concrete cover); for more details about the crack mechanism see e.g. [14]. In this case, B in Eq. (1) represents the critical tensile stress that initiates a crack in concrete at the interface with a reinforcing bar. A is the tensile stress in concrete at design service life t_D;

(ii) alternatively, B is the critical crack width at the concrete surface and A is the crack width at the concrete surface generated by reinforcement corrosion at time t_D. When the progress of corrosion and consequently the opening of cracks continue, a network of cracks is propagated that possibly reaches the surface of the concrete cover. Together with cracks due to mechanical loading [15], a crack network may form and lead to the separation of concrete elements. Such delamination is a complex effect depending e.g. on the diameters of reinforcing bars, their location, concrete quality, coarse aggregate size, cover, the type and amount of loading, and the configuration of the structure. Such a state is either an SLS or a ULS – depending on the location and the severity of this effect;

(iii) given a decrease in the effective reinforcement cross-section due to corrosion and excessive deformation, loss of bearing capacity and finally the collapse of the member/structure in question may occur. Such effects fall either into the SLS (deformation capacity) or ULS (load bearing capacity) categories; in this situation A is the actual (modelled) reinforcement cross-sectional area at time t_D and B is the minimum acceptable reinforcement cross-sectional area with regard to either an SLS or a ULS;

(iv) finally, some changes will occur in the characteristics of the bond between steel and concrete due to corrosion. This may lead either to an excessive deflection of the structure or to a loss of structural strength. The former appears to be a more critical effect [16]. The bond may be modelled by using the time function of bond stress vs. slip affected by the degree of reinforcement corrosion.

3 SOFTWARE TOOL AND MODELS

3.1 Model selection

With respect to durability assessment it is important to develop a relevant approach, evaluation procedures and methods for service life prognosis. In this area, mathematical modelling is often a useful tool together with relevant limit states for the accomplishment of such a task. Several models may be available for the degradation process in question and the engineer can select a suitable one for each specific use. The main criteria in selecting the relevant degradation model for each specific use may be listed as follows:

- the type of relevant limit state and exposure conditions;
- the accuracy and relevance of the model when using the available data in relation to the required design exactness;
- the type of representation (1D, 2D or 3D; spatial and/or temporal variability);
- the level of physical sophistication (macro-, meso- or micro-level);
- the level of mathematical involvement;
- the possibility or feasibility of model combination or conditionality;
- the availability of model data and their statistical characteristics, and/or the availability of relevant testing methods;
- labor and/or time consumption;
- level of model validation and calibration;
- the type of concrete (HSC, FRC, ...) and/or the type of structure;
- the availability of efficient software tools.

There are many predictive computational models for the modelling of degradation caused by the corrosion of reinforcement. They are mainly heuristic, using more or less simplified approaches and data. A common feature of all these models is that the input data are very uncertain. The authors of this paper and their colleagues have developed a software application called *FReET-D* in which all relatively well-known models are summarized within the framework of a unified software environment. In *FReET-D* a combination of analytical models and simulation techniques has been amalgamated to form specialized software for assessing the potential degradation of newly designed as well as existing concrete structures [17, 18]. The included models for carbonation, chloride ingress and corrosion of reinforcement (among other phenomena) can be directly employed in the durability assessment of concrete structures in the form of a durability LS, i.e. the assessment of service life and of the level of the relevant reliability measure. Several features are offered including parametric studies and Bayesian updating. Altogether, 32 models are implemented as pre-defined dynamic-link library functions. In this paper only the included models for corrosion of reinforcement are presented.

FreET-D is actually a specialized module of *FReET* software [19]. *FReET* probabilistic software allows simulations of uncertainties affecting an analyzed problem basically at the level of random variables (in civil/mechanical engineering these are typically material properties, loading, geometrical imperfections). Attention is given to those techniques that have been developed for the analysis of computationally intensive problems; nonlinear FEM analysis being a typical example. The stratified simulation technique Latin hypercube sampling (LHS) is used in order to keep the number of required simulations at an acceptable level. This technique can be used for both the random variable and random field levels. Statistical correlation is efficiently imposed by the stochastic optimisation technique known as simulated annealing, which is described by Vořechovský and Novák [20]. Sensitivity analysis is based on nonparametric rank-order correlation coefficients.

In the following sections the models implemented in *FReET-D* for corrosion of reinforcement in concrete are briefly described. For the purpose of this paper and for simple cross referencing the models are labeled as CorrX in the following sections, where X is an ordinal number.

3.2 Corr1 – rebar diameter

This model by Andrade et al. [21] and Rodriguez et al. [22] is frequently used for the prediction of uniform corrosion. The formula for the time-related net rebar diameter $d(t)$ at exposure time t reads (see Fig. 1a):

$$d(t)=\begin{cases} d_i & t\le t_i \\ \psi\left[d_i-0.0116\ i_{corr}R_{corr}\left(t-t_i\right)\right] & t_i<t\le t_i+\dfrac{d_i}{0.0116\,i_{corr}R_{corr}} \\ 0 & t>t_i+\dfrac{d_i}{0.0116\,i_{corr}R_{corr}} \end{cases} \tag{2}$$

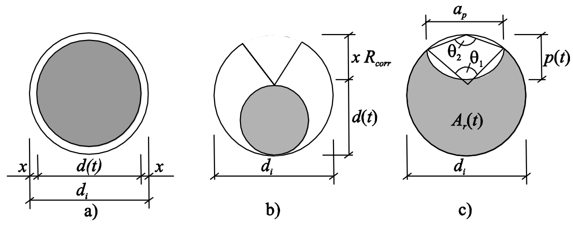

Figure 1: Schematic representation of the damage caused to a reinforcing bar due to: a) uniform corrosion; b) and c) pitting corrosion.

where d_i is the initial bar diameter; $t = t_i+t_p$ where t_i is the time to corrosion initiation (steel depassivation), and t_p is the time of corrosion propagation. Parameter R_{corr} expresses the type of corrosion; ψ is the uncertainty factor of the model. For uniform corrosion the coefficient R_{corr} equals 2. The effects of chloride concentration, pH level or other conditions may also be described by the coefficient R_{corr} whenever applicable. The parameter i_{corr} is the current density; the mean value can be obtained by measurement or testing, or alternatively, calculated according to Eq. (3). This formula describes the dependence of i_{corr} on time, which is derived on the basis of experiments for RC flexural members in [23] as:

$$i_{corr}=0.3686\ln(t)+1.1305 \tag{3}$$

In [24] the parameter i_{corr} was derived from experimental measurements as a function of chloride concentration C for two different pH values of 9 and 12.5 – see Fig. 2. It was recalculated from the corrosion rate found on the basis of experiments and was deliberately rearranged to transform it into the uniform type of corrosion. The pH values of 9 and 12.5 were chosen due to the fact that the pH value of the pore solution is about 12.5 in the case of non-carbonated concrete and pH = 9 in the case of carbonated concrete.

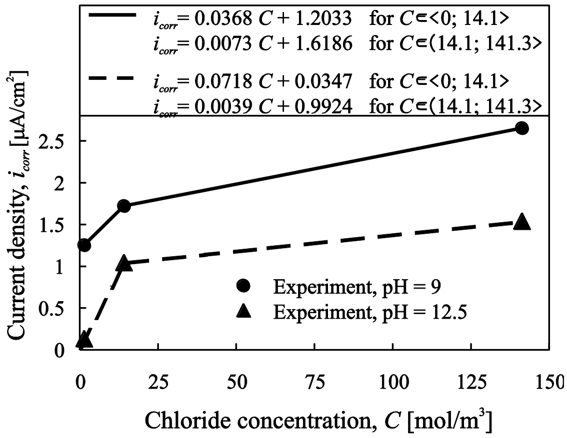

Figure 2: The dependence of current density on chloride concentration according to [24].

3.3 Corr2 – the depth of pit

In cases of localised corrosion the following model can be taken into consideration. The studies by Gonzalez et al. [25] show that the maximum rate of corrosion penetration in the case of pitting corrosion is about 4-8 times that of general corrosion. The depth of the pit $p(t)$ at time t can be estimated by the following equation (see Fig. 1b):

$$p(t) = \psi\left[0.0116\ i_{corr}\ \ R_{corr}\left(t - T_i\right)\right] \qquad (4)$$

The meaning of the parameters is the same as in Eq. (3). The value of coefficient R_{corr} varies from 4 to 8 [25]; note that the effects of chloride concentration, pH level or other conditions may also be described by the coefficient R_{corr} whenever applicable. It is evident that the model given by Eq. (4) can serve as an estimate only. The value of $p(t)$ may serve as an input parameter for model Corr3.

3.4 Corr3 – net cross sectional area

An alternative means of modelling localised corrosion has been presented by Val and Melchers [26]. The section at the pits is predicted by a simplification in the hemispherical form. The net cross sectional area of a corroded rebar A_r at time t is calculated as:

$$A_r(t) = \begin{cases} \psi\left(\pi \dfrac{d_i^2}{4} - A_1 - A_2\right) & p(t) \le \dfrac{\sqrt{2}}{2} d_i \\ \psi\left(A_1 - A_2\right) & \dfrac{\sqrt{2}}{2} d_i < p(t) \le d_i \\ 0 & p(t) > d_i \end{cases} \qquad (5)$$

where

$$A_1 = \frac{1}{2}\left[\theta_1\left(\frac{d_i}{2}\right)^2 - a_p\left|\frac{d_i}{2} - \frac{p(t)^2}{d_i}\right|\right]$$
$$A_2 = \frac{1}{2}\left[\theta_2 p(t)^2 - a_p \frac{p(t)^2}{d_i}\right] \qquad (6)$$

$$a_p = 2p(t)\sqrt{1 - \left(\frac{p(t)}{d_i}\right)^2}$$
$$\theta_1 = 2\arcsin\left(\frac{a_p}{d_i}\right) \qquad (7)$$
$$\theta_2 = 2\arcsin\left(\frac{a_p}{2p(t)}\right)$$

For the description of the parameters from Eqs. (5, 6 and 7) see models Corr1 and Corr2, and Fig. 1c.

It should be noted that pitting corrosion is highly localised on individual reinforcement bars. It is unlikely that many bars could be affected in the same section of a construction member; hence, pitting corrosion will not significantly influence the structural capacity in the early stage of corrosion. The value of $p(t)$ may be obtained from model Corr2.

3.5 Corr4 - time to cracking of concrete cover

The corrosion cracking model was proposed by Liu and Weyers [27] to estimate the critical time to cracking $t_c = t_i + t_{p,cr}$ of concrete cover due to stresses resulting from the expansion of corrosion products from reinforcement. Within this paper only the modelling of $t_{p,cr}$ is described. It is assumed that uniform corrosion causes rust products to form uniformly around the steel surface. Three stages are considered in this model: (1) Free expansion – the porous zone around the reinforcement is filled by rust products. The total amount of corrosion products is less than the amount of corrosion products required to fill the porous zone around the steel-concrete interface W_P and so no stresses are exerted on the surrounding concrete; (2) Stress initiation – the total amount of corrosion products exceeds the amount of corrosion products needed to fill the porous zone W_P and expansive pressure is applied to the surrounding concrete; (3) Cracking – the total amount of corrosion products reaches the critical amount of corrosion products W_{crit} and cracking of the surrounding concrete is induced.

The critical amount of corrosion products W_{crit} consists of two parts: W_P, the amount of corrosion products needed to fill the porous zone around the steel-concrete interface, and W_S, the amount of corrosion products that generate critical tensile stresses. W_{crit} can be estimated from the following formula:

$$W_{crit} = \frac{\rho_{rust}\pi\left[d_i\left(d_s + d_0\right) + 2d_0 d_s\right]}{1 - \rho_{rust}\dfrac{\alpha}{\rho_{st}}} \tag{8}$$

where d_0 is the porous zone thickness, ρ_{rust} and ρ_{st} are the specific gravities of rust and steel, respectively, α is the ratio of steel to the molecular weight of the corrosion products, and d_s is the thickness of the corrosion products needed to generate tensile stresses, which is given by Eq. (9). Note that in [27] the term $2d_0 d_s$ is neglected for W_{crit} derivation, which is not the case assumed here.

$$d_s = \frac{a \cdot f_t}{E_{ef} \cdot 10^3}\left(\frac{x^2 + y^2}{x^2 - y^2} + \nu_c\right) \tag{9}$$

In this formula a represents the concrete cover, f_t is the tensile strength of concrete, ν_c is the Poisson ratio of concrete and E_{ef} is the effective modulus of elasticity given by Eq. (10); $x = \left(d_i + 2d_0\right)/2$ and $y = a + \left(d_i + 2d_0\right)/2$.

$$E_{ef} = \frac{E_c}{1 + \varphi_{cr}} \tag{10}$$

E_c is the modulus of elasticity of concrete and φ_{cr} is the creep coefficient. For a constant corrosion rate, the resulting time to cracking at the rust-concrete interface since corrosion initiation $t_{p,cr}$ can be derived as:

$$t_{p,cr} = \psi\frac{W_{crit}^2}{2k_p} \tag{11}$$

where k_p is related to the rate of metal loss and is expressed as:

$$k_p = 0.092\left(\frac{1}{\alpha}\right)\pi\, d_i\, i_{corr} \tag{12}$$

3.6 Corr5 - corrosion induced crack width

This model, proposed by Li et al. [28], provides an estimation of corrosion-induced crack width – essential information for the prediction of the serviceability of corrosion-affected RC structures.

A concrete element with an embedded reinforcing steel bar can be modelled as a thick-walled cylinder – see [19-31]. This is shown schematically in Fig. 3a. Usually d_0 is constant once the concrete has hardened. When the reinforcing steel corrodes, its products fill the pore band completely. As the corrosion propagates in the concrete, a ring of corrosion products forms, the thickness of which, $d_s(t)$ (Fig. 3b), can be determined from the following relation according to [27]:

$$d_s = \frac{W_{rust}(t)\left(1 - \dfrac{\alpha\rho_{rust}}{\rho_{st}}\right) - \rho_{rust}\pi d_i d_0}{\pi\rho_{rust}(d_i + 2d_0)} \tag{13}$$

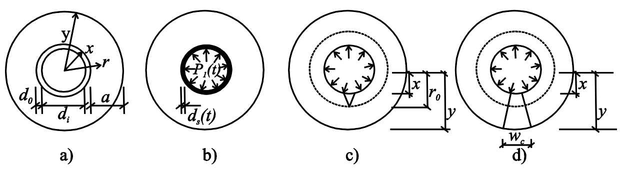

Figure 3: Schematic diagram of the corrosion-induced concrete cracking process.

where $W_{rust}(t)$ is the mass of corrosion products. Obviously, $W_{rust}(t)$ increases with time and can be determined from the following relation according to [27]:

$$W_{rust}(t)=\left(2\int_{t_i}^{t}0.092(\frac{1}{\alpha_{rust}})\pi\cdot a\cdot i_{corr}(t)dt\right)^{1/2} \quad (14)$$

The growth of the ring of corrosion products (known as a rust band) exerts outward pressure on the concrete at the interface between the rust band and the concrete. Under this expansive pressure the concrete cylinder undergoes three phases in terms of cracking: (i) no cracking; (ii) partially cracked; and (iii) completely cracked. In the no cracking phase the concrete cylinder can be considered to be elastically isotropic so that the theory of elasticity can be used to determine the radial stress $\sigma_r(r)$ and tangential stress $\sigma_\theta(r)$ at any point (r) in the cylinder [32]. From the radial stress $\sigma_r(r)$, the expansive pressure at the interface between the rust band and the concrete can be obtained as:

$$P_1=-\sigma_r(x)=\frac{E_{ef}d_s(t)}{x\left(\frac{y^2+x^2}{y^2-x^2}+\nu_c\right)} \quad (15)$$

From the tangential stress $\sigma_\theta(r)$ at $r = a$, the initial cracking time can be determined by satisfying the condition $\sigma_\theta(x)=f_t$. After initiation, the crack in the concrete cylinder propagates in a radial direction and stops arbitrarily at r_0 (which varies between the radii x and y) to reach a state of self-equilibrium. The crack divides the thick-walled cylinder into 2 co-axial cylinders: an inner, cracked cylinder and an outer uncracked one, as shown in Fig. 3c. In the case of the outer uncracked concrete cylinder the theory of elasticity still applies. Let us assume that the cracks in the inner cracked concrete cylinder are smeared and uniformly circumferentially distributed [30], and that the concrete is a quasibrittle material. According to Bažant and Jirásek [33] and Noll [34], a residual cracked surface exists along the radial direction dependant on the tangential strain of that point; it is a function of the radial coordinate r. It is assumed in this model that the residual tangential stiffness is constant along the cracked surface, i.e. in the interval [x, r_0] and represented by $\alpha_{stiff}E_{ef}$, where $\alpha_{stiff}<1$ is the tangential stiffness reduction factor. According to Bažant and Planas [35] the stiffness reduction factor α_{stiff} is dependent on the average tangential strain ε_θ over the cracked surface and can be determined as follows:

$$\alpha_{stiff}=\frac{f_t\exp[-\gamma(\varepsilon_\theta-\varepsilon_\theta^c)]}{E_{ef}\varepsilon_\theta} \quad (16)$$

where ε_θ^c denotes the average tangential cracking strain and $\gamma=f_t/G_f$ is a material constant where G_f is fracture energy. The cracking in the radial direction makes the concrete an anisotropic material in the local vicinity of the cracks. Therefore, the elastic modulus in the radial direction is different from the tangential one. Based on the described concept, the corrosion-induced concrete crack width can be expressed as follows:

$$w_c=\frac{4\pi d_s(t)}{(1-\nu_c)(x/y)^{\sqrt{\alpha_{stiff}}}+(1+\nu_c)(y/x)^{\sqrt{\alpha_{stiff}}}}-\frac{2\pi y f_t}{E_{ef}} \quad (17)$$

3.7 Corr6 - time to cracking

A mathematical model for the prediction of time to cracking due to corrosion $t_c=t_i+t_{p,cr}$ was proposed by El Maaddawy and Soudki [36] aiming for the further improvement of model Corr4. The authors developed a relationship between steel mass loss and the internal radial pressure caused by rust growth. The time to corrosion cracking is estimated on the basis of Faraday´s law.

Four basic assumptions were used in the currently-described model to determine the internal radial pressure caused by the expansion of corrosion products:

1) The corrosion products are formed uniformly around the steel reinforcing bar, which results in uniform expansive stresses around the steel bar.

2) There is a porous zone around the steel reinforcing bar which the corrosion products

must first fill before they start to induce pressure in the surrounding concrete.
3) The volume expansion caused by corrosion only creates strain in the concrete (i.e. strain in the steel is neglected).
4) The concrete around the steel reinforcing bar is modelled as a thick-walled cylinder with a wall thickness equal to the thinnest concrete cover. The concrete ring is assumed to crack when the tensile stresses in the circumferential direction at every part of the ring have reached the tensile strength of the concrete.

A uniform layer of corrosion products would create uniform expansive stresses at the interface surface between steel and concrete that would result in a uniform radial displacement at the surface of the rust layer. The relationship between the percentage steel mass loss and the corresponding radial pressure on the concrete is given by Eq. (18). It is assumed that the corrosion products must first fill the porous zone at the steel-to-concrete interface before their expansion starts to create pressure on the surrounding concrete. A linear relationship is considered to exist between radial pressure and displacement. The relationship between M_r, the mass of rust per unit length of one bar, and M_{loss}, the mass of steel per unit length consumed to produce M_r, is expressed as $M_{loss} = 0.622 M_r$, where 0.622 is the considered value of the ratio of steel to rust molecular weight, α. The relationship between the mass density of rust ρ_{rust} and steel ρ_{st} is considered as $\rho_{rust} = 0.5\rho_{st}$, where $\rho_{st} = 7.85\,\mathrm{g/cm^3}$.

$$P_{corr} = \frac{100\dfrac{M_{loss}}{M_{st}} E_{ef} d_i}{90.9\left(1+\nu_c+k\right)\left(d_i+2d_0\right)} - \frac{2d_0 E_{ef}}{\left(1+\nu_c+k\right)\left(d_i+2d_0\right)} \quad (18)$$

where k is given by the following equation:

$$k = \frac{\left(d_i+2d_0\right)^2}{2a\left(a+d_i+2d_0\right)} \quad (19)$$

By combining Eq. (18) with the radial pressure that causes cracking (Eq. 20), the governing criterion is obtained for the pressure when cracking occurs (Eq. 21).

$$P_{cr} = \frac{2af_t}{d_i} \quad (20)$$

$$\frac{100\dfrac{M_{loss}}{M_{st}} E_{ef} d_i}{90.9\left(1+\nu_c+k\right)\left(d_i+2d_0\right)} - \frac{2d_0 E_{ef}}{\left(1+\nu_c+k\right)\left(d_i+2d_0\right)} = \frac{2af_t}{d_i} \quad (21)$$

To introduce time in Eq. (21), Faraday's well known law will be used. When Faraday's law is applied to a steel geometry, the units are adjusted and it is combined with Eq. (21), the time from corrosion initiation to corrosion cracking, $t_{p,cr}$, is obtained:

$$t_{p,cr} = \psi \frac{1}{365.25}\left[\frac{7117.5\left(d_i+2d_0\right)\left(1+\nu_c+k\right)}{i_{corr} E_{ef}}\right]\left[\frac{2af_t}{d_i} + \frac{2d_0 E_{ef}}{\left(1+\nu_c+k\right)\left(d_i+2d_0\right)}\right] \quad (22)$$

4 EXAMPLES

4.1 Comparison of steel corrosion models

The above-mentioned models operate using different sets of input parameters. To compare them completely is not a simple or straightforward task. For this reason only three models are compared here – Corr4, Corr5 and Corr6, all in deterministic form only for the purpose of a clearer comparison. Corr4 and Corr6 enable the calculation of the time to crack initiation in concrete while Corr5 allows the tracking of the whole cracking process. As assumed in these models, concrete with an embedded reinforcing steel bar is modelled as a thick-walled cylinder. The models are compared in Fig. 4a via the dependence of r_c on d_s, where r_c is the radial distance from the steel-concrete interface to the outer face of the cracks and d_s is the thickness of the ring of corrosion products (see Fig. 4b). A time axis is also given in the figure. A full description of all input values can be found in Table 1.

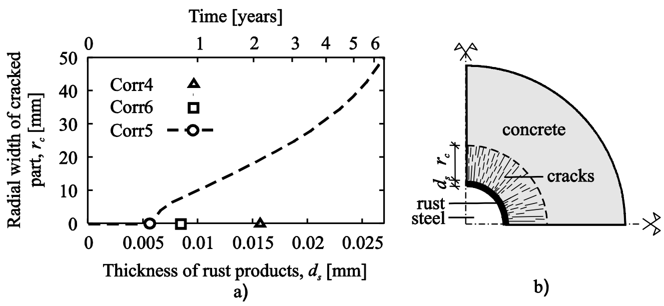

Figure 4: (a) Comparison of models Corr4, Corr5 and Corr6; (b) a quarter of the section of the model.

Table 1: Input parameters for corrosion model comparison

Input parameter	Nom.	Unit	Value	Model
Initial bar diameter	d_i	mm	25	4,5,6
Porous zone thickness	d_0	mm	0	4,5,6
Concrete cover	α	mm	50	4,5,6
Current density	i_{corr}	μA/cm^2	1	4,5,6
Specific gravity of rust	ρ_{rust}	kg/m^3	3925	4,5,6
Specific gravity of steel	ρ_{st}	kg/m^3	7850	4,5,6
Ratio of steel to rust molecular weight	α	-	0.622	4,5,6
Tensile strength of concrete	f_t	MPa	2.4	4,5,6
Elastic modulus of elasticity of concrete	E_c	GPa	30.5	4,5,6
Poisson´s ratio of concrete	ν_c	-	0.24	4,5,6
Creep coefficient	φ_{cr}	-	2	4,5,6
Material constant	γ	-	1	5

In Fig. 4a, zero time is assumed to be the time of corrosion initiation. According to the Carb4, Carb5 and Carb6 models, cracks in concrete initiate in 2.10 (triangular point), 0.61 (rectangular point) and 0.27 (circular point) years, which corresponds to d_s = 0.0156, 0.0084 and 0.0056 mm, respectively. These d_s values, which correspond to times of crack initiation, agree with linear tangential stresses in concrete (at the steel-concrete interface) of 9.60, 5.17 and 3.41 MPa, respectively. Note that the input tensile stress of concrete has been taken to be equal to 2.4 MPa.

4.2 Critical crack width on the concrete surface

A parametric study utilizing the Corr5 model is demonstrated in this example. The limit state condition is constructed in this case in the form:

$$P_f(t_D) = P\left\{w_{cr} - w_a\left(t_D\right) \le 0\right\} \le P_d \qquad (24)$$

where w_{cr} is the limit value of a crack width equal to 0.3 mm – one of the essential limits for the serviceability assessment of corrosion-affected RC structures recommended in CEN [37]. The actual corrosion-induced crack width w_a over time is computed according to Eq. (16). In this example, time t_D represents the propagation period only; to gain a service life prediction the appropriate initiation period must be added.

In Fig. 5 the results of statistical analysis are depicted, i.e. the values of crack width on the concrete surface after 15 years of steel corrosion development as the function of concrete cover *a*. Next, Fig. 6 shows the reliability index β vs. concrete cover for three different propagation time values; the limit value of β = 1.5 prescribed typically for SLS is also shown in the figure. It appears that e.g. for t_D = 25 years the cover should be greater than 50 mm to satisfy the serviceability requirements; for t_D = 15 years 30 mm of cover would be satisfactory.

Table 2: Input parameters for crack width analysis

Nom.	Unit	Distribution	Mean	COV
d_i	mm	Normal	20	0.02
d_0	mm	Deterministic	0.0125	
a	mm	Deterministic	30-70	
t	years	Deterministic	15, 20, 25	
i_{corr}	μA/cm^2	Normal	1.5	0.2
ρ_{rust}	kg/m^3	Normal	3600	0.02
ρ_{st}	kg/m^3	Normal	7850	0.01
α	-	Deterministic	0.57	
f_t	MPa	Lognormal (2 par)	3.3	0.12
E_c	GPa	Lognormal (2 par)	27	0.08
ν_c	-	Deterministic	0.18	
φ_{cr}	-	Deterministic	2	
γ	-	Deterministic	1	
ψ	-	Lognormal (2 par)	1	0.15

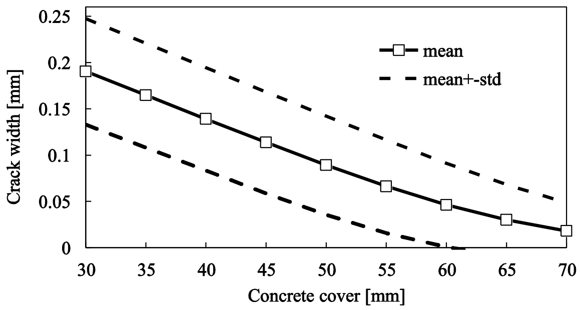

Figure 5: Crack widths on the concrete surface after 15 years of steel corrosion development.

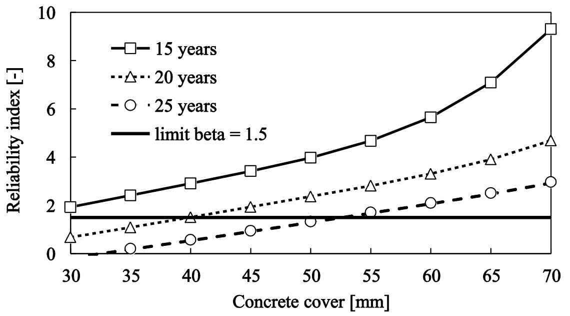

Figure 6: Reliability index β after 15, 20 and 25 years of steel corrosion development (the condition given by Eq. 24).

5 CONCLUDING REMARKS

- The use of modelling in the field of reinforcement corrosion in RC structures has recently increased; nevertheless, it is recommended that users be aware of the conditions under which the model in question was developed – see comments by Otieno et al. [38] or Ahmad [39].
- A broader choice of models is useful e.g. due to problems with the availability of statistical data for the input variables of some models.
- The usefulness of effective degradation modelling and hence reliable design/assessment for durability may enhance decision processes and, as a consequence, bring positive financial and sustainability impacts.

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