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PARAMETRIC DAMAGE OF CONCRETE UNDER HIGH-CYCLE FATIGUE LOADING IN COMPRESSION

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Abstract: Concrete elements degrade due to the continuous application of compressive fatigue loads. The irreversible deformation or induced compressive strains evolve; hence, the progressive stress redistribution between concrete and embedded steel reinforcing bars should be accounted for in fatigue analysis of concrete structures.

Experimental investigations were conducted to study the behavior of four small-scale reinforced concrete deep beams with shear-span to effective-depth ratio of 1.25. The principal compressive strain evolutions from the beam struts were obtained from the strain transformation analysis of LVDT data, and compared with the predicted principal strain evolutions. Each predicted strain evolution was obtained by substituting the initial strut compressive stress estimated from the static analysis of the fatigue load into a strain evolution model from the literature. The comparison between the measured compressive strain evolution from the experiments and the predicted strain evolutions indicated that the design approach in the literature is overly conservative for concrete. However, this is generally attributed to the neglect of the contribution of the tensile strength of concrete in fatigue resistance.

The strut-and tie method was used for the fatigue resistance verification of deep beams by modifying the constitutive models and effectiveness factor of concrete with fatigue damage models. The irreversible compressive strain is considered as a pseudo-load, and the progressive crack growth of steel reinforcement resulting from the evolving stresses is accounted for using an equivalent cycle concept with the Paris crack growth model. Within the developed algorithm, failure will occur when one of the evolving stresses in either the concrete strut or steel reinforcement approaches the corresponding residual strength.

INTRODUCTION

Deep beams are usually designed in practice for structural elements such as large wind turbine foundations, offshore structures, transfer girders, and pile caps. A majority of these structures are prone to fatigue loading, hence are susceptible to fatigue damage.

Three basic modes of fatigue failure, as reported in the literature, govern the fatigue behavior of reinforced concrete deep beams. These include: the diagonal splitting of concrete, the fracture of the longitudinal reinforcing bars, and the crushing of concrete struts [1, 2].

In the design of beams for fatigue resistance, the concrete section size and the amount of reinforcement required are initially obtained using static analysis and in accordance with the ultimate and the serviceability limit states. Thereafter, the stresses induced in the provided materials are estimated from the static analysis of the maximum and minimum fatigue loads that the beam may resist. The stresses at critical sections are further normalised with the ultimate strengths, and substituted into the fatigue stress-life models (S-N curves), in order to obtain the number of cycles leading to failure. The design is deemed safe provided the number of cycles to failure estimated is greater than that expected for the service life [3-6].

Reports in the literature have shown that irreversible strains accumulate in concrete (struts) under fatigue loading and, as such, the stresses in the reinforcing bars (ties) evolve correspondingly resulting in crack propagation and ultimately fracture. This indicates that the fatigue analysis involving the estimation of the number of cycles to failure by substituting constant stresses into S-N models may not be conservative [7, 8]. However, the tensile strength of concrete is usually neglected in designs and, as such, a conservative result may be obtained.

Basically, the aim of the fatigue resistance design of deep beams is to ensure that the observed failure modes, especially the crushing of concrete and fracture of the reinforcing bars, do not occur during the service life of the structure. As such, it is expedient that the designs are verified in terms of the evolving deformation parameters, such as the average principal compressive strain evolution of the concrete struts.

To achieve the aforementioned objective, four small-scale deep beams were tested under fatigue loading and the average principal compressive strains were obtained progressively from the transformation of the measured concrete strains in three directions, within a plane, in each shear span. Analytically, the stresses induced in the compressive struts were estimated and substituted into the strain evolution models available in the literature. The obtained evolutions were used to corroborate the experimentally observed average principal compressive strain evolutions.

A rigorous analytical approach which involves a modification of the strut and tie analysis for deep beams to account for the fatigue damage is proposed. The progressive crack growth and the corresponding increase in strain in the reinforcing bars are accounted for using an equivalent-cycle concept with the Paris crack growth model. By considering the progressive deformation per cycle, one of two criteria governs the failure of the beam. That is, failure may occur by crushing of concrete strut when the residual strength is equal to the induced stress in the strut, or by fracture of the reinforcing bars (ties) when the induced tensile stress at the intersection with the concrete crack is equal to the corresponding yield stress.

Fatigue life of a deep beam

Depending on the reinforcement ratio, the stability of a deep beam under fatigue loading is governed by the progressive crack growth in a reinforcing bar at its intersection with a concrete crack, or by the degradation of the concrete struts [9].

Reinforcement fracture

From the Paris crack growth law (Eq. 1), the propagation of a reinforcing bar crack, up to a depth resulting in fatigue fracture, can be predicted using a parameter representing the stress intensity factor range (ΔK) (Eq. 2). This parameter depends on the size of the crack tip plastic zone in comparison with the stress intensity field. In the literature, the use of this parameter in Linear Elastic Fracture Mechanics (LEFM) is deemed realistic, since the plastic zone size is usually very small for high-cycle fatigue loading. The parameter ΔK is generally expressed as a function of the fatigue stress range ($\Delta\sigma$), crack size (a) and a shape factor (Y) for the reinforcing bar [10-13].

$$\frac{da}{dN} = C.\Delta K^n \qquad (1)$$

$$\Delta K = Y.\Delta\sigma.\sqrt{\pi a} \qquad (2)$$

By substituting Eq. 2 into Eq. 1 and integrating N with respect to the crack depth, the number of cycles (N_{ij}) required to propagate a crack from an initial point to another due to a given stress can be estimated. Hence, the crack depth (a_j) for a given number of cycles can be expressed as (Eq. 3).

$$a_j = \left(\frac{a_i{}^{\alpha}}{1-\left[N_{ij}\left(C.\alpha.\pi^{\frac{n}{2}}.Y^n.\Delta\sigma^n.a_i{}^{\alpha}\right)\right]}\right)^{\frac{1}{\alpha}} \qquad (3)$$

where α = (n/2)-1.
a_i and a_j are the smallest and largest crack depth for the interval of cycles considered (N_{ij}). However, the estimation of a_j requires the value of a_i, which is the previous crack depth [11].

The initial minimum crack depth can be calculated using a backward crack growth calculation with increments of cycles. It can also be obtained iteratively from (Eq. 4) [13]:

$$a_o = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{Y\Delta\sigma_{lim}}\right)^2 \qquad (4)$$

where $\Delta\sigma_{lim}$ corresponds to the fatigue limit stress at which fatigue damage will not initiate, and ΔK_{th} is the threshold stress intensity factor. The crack does not propagate for stress intensity values lower than ΔK_{th}. The ΔK_{th} values are given as [12, 14]:

$$191 \text{ Nmm}^{-3/2}, \text{ for } R \leq 0.17 \qquad (5)$$

$$222.4\ (1\text{-}0.85R) \text{ Nmm}^{-3/2}, \text{ for } R \geq 0.17$$

where R is the stress ratio ($\sigma_{min}/\sigma_{max}$).
C and n for steel are taken as 2 x 10^{-13} and 3 respectively [15].

An equation for the shape factor, proposed in BS 7910 (1999) as a function of the crack depth, is given in Eq. 6 [16]. By substituting Eq. 6 into Eq. 3, an iterative approach may be used to estimate the crack depth.

$$Y = \frac{\frac{1.84}{\pi}\left\{tan\left(\frac{\pi a}{4r}\right)/\left(\frac{\pi a}{4r}\right)\right\}^{0.5}}{cos\left(\frac{\pi a}{4r}\right)} \cdot \left[0.75 + 2.02\cdot\left(\frac{a}{2r}\right) + 0.37\cdot\left\{1 - sin\left(\frac{\pi a}{4r}\right)\right\}^3\right] \qquad (6)$$

In Equation 6, r is the radius of the reinforcing bar and a, is the crack depth.

The area of reinforcement at the intersection with a concrete crack reduces as the crack grows. The fracture of a reinforcing bar may result when the induced stress in the bar, at its intersection with the concrete crack, is equal to the residual strength of its cross section. The reinforcing bar area $A_i(a_y)$ at failure is given as [13]:

$$A_i\ (a_y) = A_o - A(a_y) \qquad (7)$$

A_o : cross-sectional area of uncracked rebar.
$A(a_y)$: area of the fractured surface
f_y : yield stress of steel.

Provided an equation for the area of a reinforcing bar can be expressed in terms of the crack depth, then the progressive area reduction under fatigue loading can be estimated. An expression suitable for this purpose will be discussed in a subsequent section.

Concrete damage

The fatigue loading of concrete elements may result in the evolution of damage parameters such as the irreversible strain. In addition, the strength and stiffness of concrete may degrade correspondingly. As such, the stress induced in the steel reinforcement (tie) at the intersection with the concrete crack also increases as a result of the accumulated irreversible concrete strains. Basically, there is a succession of strain increase influence from the strut to the tie and vice versa, as the fatigue loading progresses.

Models have been developed for concrete strength degradation, stiffness degradation and irreversible strain evolution in the literature. In addition, some assumptions have been used to modify the monotonic constitutive models for concrete in order to account for fatigue damage. These include the assumption of the convergence of the centerlines of fatigue hysteresis loops at a common point, and the intersection of the peak stress of a fatigue-damaged concrete element with the monotonic stress-strain envelope [17-20].

The validity of the assumptions have been reported in the literature [21, 22], hence the compressive stress and strain of a degrading strut at any instance of cycle can be estimated, taking into account the degradation of the strength and stiffness.

In the proposed modification for the strut and tie analysis, the irreversible strain in the concrete strut is considered as a progressively increasing pseudo-load. On the other hand, the progressive crack growth and corresponding area reduction of steel reinforcement is obtained from fracture mechanics. In all, the induced stresses in concrete strut and ties for damage evolution are based on modified equilibrium, compatibility and constitutive equations.

EXPERIMENTAL PROGRAM

Test specimens

The overall dimensions of 175 x 250 x 700 mm were used for the deep beams cast for the experimental investigation. The beam properties and set up are given in Table 1 and Figure 1 respectively. It was ensured that the properties of the reinforcement (shear and flexure) were higher than the minimum required by CSA A23.3-04 11.2.8.1 and 11.2.8.2 for shear, 10.5.1.2 for flexure, and Eurocode 2-1-1(2004) 9.2.1.1 and 9.2.1.1 for shear and flexure respectively.

In order to control the bond fatigue between steel and concrete, adequate anchorage was provided based on code requirements CSA- N12.13.1, N12.13.2 (shear reinforcement anchorage), N12.5.2 (flexural reinforcement anchorage). The anchorage provision also satisfied EC 2-1-1 (2004) clause 8.5(1) and (2) for shear reinforcement anchorage requirement and 2-1-1 clause 8.4.1 (1) P for longitudinal reinforcement anchorage requirements.

Materials

A design compressive strength of 50 MPa was selected for the plain concrete used, with a maximum aggregate size of 10 mm. The concrete slumps obtained during the casts were between 80 to 150 mm. All specimens cast were removed from the curing room at 28 days and placed in a dry compartment. The average compressive strength of concrete for the beams tested are given in Table 1. The value given in the second column of Table 1, for the fatigue loading phase, is equivalent to the average compressive strength within the time frame for testing the four beams.

Canadian standard 15M, 10M (high-strength deformed steel reinforcing bars) and D4 (cold-worked) were used as reinforcing bars for the beams. The average yield strength obtained for the reinforcing bars were 430, 480, and 610 MPa respectively. The yield strength of the cold-worked steel rebar corresponded to the proof stress. Two 15M reinforcing bars were used as main reinforcement for C’ beams, while two 10M reinforcing bars were used for C beams. In all, 2-10M reinforcing bars were provided at the top. D4 reinforcing bars were used as web reinforcement.

Test procedure

Initially, two control beams (T1 and T2) with the same reinforcement provisions as in the C' and C beams were tested monotonically to failure. The corresponding failure loads observed were 310 kN and 270 kN respectively. As indicated in the fifth column of Table 1, percentages of the failure load were used for the fatigue tests conducted.

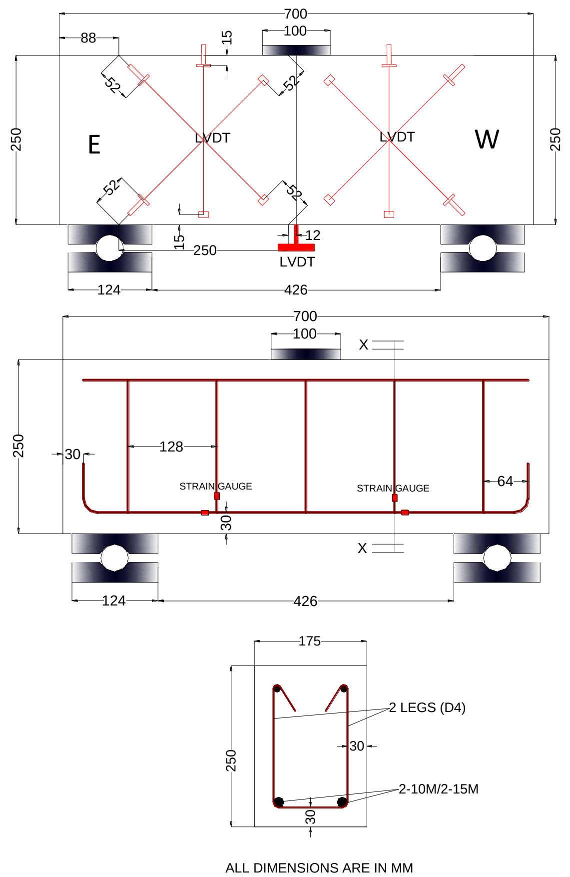

Figure 1: Details of deep beam specimen.

The fatigue tests were conducted using servo-hydraulic testing equipment having a loading capacity of 350 kN. The loading equipment was used to generate a pulsating load of a continuous sinusoidal waveform throughout the test duration. All fatigue tests were conducted at a frequency of 5 Hz, and a constant minimum load of 5 kN was used.

Instrumentation

The average principal strains (Eq. 8), the average shear strains and the average strains in the x and y directions within the two shear spans of each beam were obtained from strain transformation of the LVDT data (Fig.2).

Table 1: Properties of beam specimens

Sp.	f_c', MPa	$\rho_l(\%)$	ρ_v (%)	P% (kN)
C7	63	0.45	0.2	70
C'7	63	0.9	0.2	70
C8	63	0.45	0.2	80
C'8	63	0.9	0.2	80
T1	55	0.9	0.2	Mono
T2	55	0.45	0.2	Mono

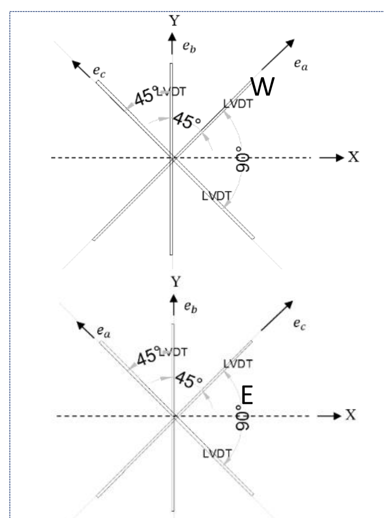

Figure 2: LVDTs on beam specimens.

A program was written in FORTRAN to generate the evolutions from the laboratory data.

Considering the West LVDTs, (γ_{xy} is positive)

$e_x = e_c - e_b + e_a$

$e_y = e_b$

$\gamma_{xy} = e_a - e_c$

Considering the East LVDTs, (γ_{xy} is negative)

$e_x = e_c - e_b + e_a$
$e_y = e_b$
$\gamma_{xy} = e_c - e_a$

The average principal concrete strains were obtained thus:

$$e_{1,2} = \frac{1}{2}\left(e_x + e_y\right) \pm \frac{1}{2}\left(\sqrt{\left(e_x - e_y\right)^2 + \gamma_{xy}{}^2}\right) \qquad (8)$$

To verify the accuracy of the instrumentation, the approximate inclination of each perpendicular plane to the failure plane or observed crack orientation within a shear span was compared with the estimated evolution of the inclination angle of the average principal tensile strains within the shear spans using the LVDT data (Figure 4). The comparison shows that the instrumentation was of an acceptable accuracy.

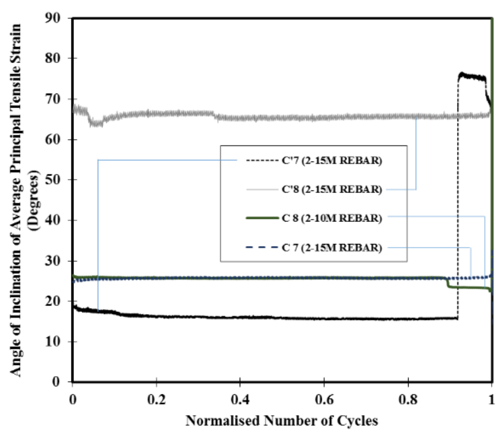

Figure 3: Inclination of average principal tensile strain.

TEST RESULTS ANALYSIS AND DISCUSSION

Failure mode

Figure 4 shows the failure modes of all the beams tested under monotonic and fatigue loading conditions. The failure mode of T2 was observed to be a combination of shear and flexure, as the fracture of the reinforcing bars occurred at the mid-span region. On the other hand, crushing of the compressive strut was observed in T1. Under fatigue loading, the fracture of the longitudinal reinforcing bars at the intersection with the concrete crack were observed in beams C8,C7,C'8, and C'7 (regions with thick crack paths).

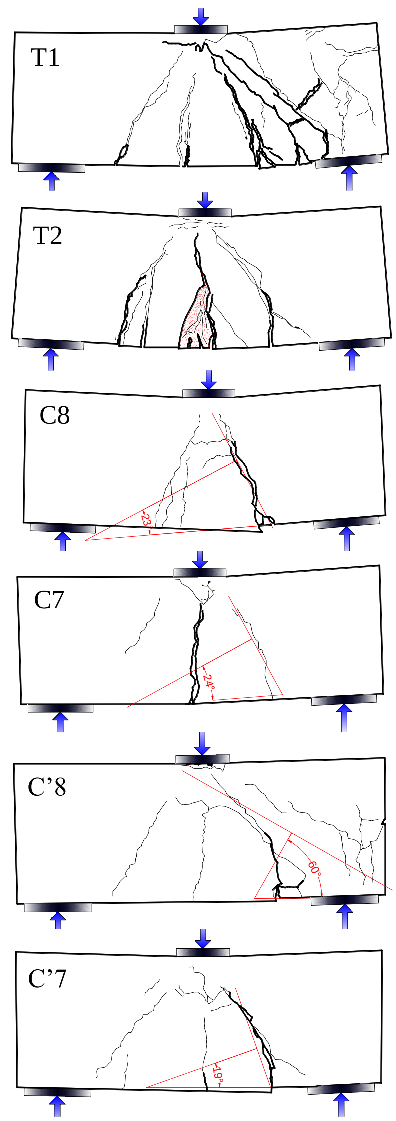

Figure 4: Crack patterns of beams at failure.

In beam C'8, severe crushing of the compressive strut also accompanied the fracture of the longitudinal reinforcing bars.

The observed failure mode in the tests conducted was attributed to the resistance mechanism within the shear span (arch mechanism) after concrete cracks.

Compressive forces are usually transferred through the concrete strut to the support, while the tensile forces are resisted by the tie. The shear reinforcement within the shear spans prevent the splitting of the concrete strut. This has also been observed and reported in the literature on monotonic loading [23].

Since the strut and tie analysis is commonly used in the design of deep beams, it is imperative that a practical means of observing the strut or tie damage accumulation, which verifies the design approach for fatigue resistance, be developed. In this investigation, the evolution of concrete compressive strain and the number of cycles leading to failure due to the crack growth of the reinforcement were used to verify the conservative level of the fatigue design approach.

Strut and tie analysis under monotonic loading

The failure loads for the two monotonically loaded beams (T1 and T2) were predicted using the strut and tie analysis [24] (Table 2). The induced stresses in the strut and tie required for fatigue analysis were also estimated by assuming a static condition of the fatigue load. For the fatigue analysis, the estimated compressive stresses in the struts of the beams were normalised with the concrete strengths. Subsequently, the normalised values were substituted into the total strain evolution models proposed by Ganuy et al. [7] and Holmen [8].

Average principal compressive strain

Figures 5 and 6 show the average compressive strain evolutions for the beams tested under fatigue loading.

The evolutions obtained from the models were plotted alongside the average principal compressive strain evolution from the fatigue tests conducted on the beams. In addition, the number of cycles leading to the fracture of a reinforcing bar under a constant tensile stress,

Table 2: Failure loads of beams under monotonic loading

Sp.	Exp. Failure Load (kN)	Predicted (kN)
CONT-15	310	330
CONT-10	270	280

using Hanson's model [25], is shown in Figure 5 and 6. The intersection between the compressive strain evolution and the number of cycles to failure of the reinforcing bars using Hanson's model, correspond to the point of failure in design.

As observed in all cases, the designs were conservative for concrete damage, since the rates of concrete strain evolution predicted were higher than those observed from the experiments. In addition, the predicted numbers of cycles at which the reinforcement will fracture were lower than those observed from the experiments. The results also reveal the influence of the reinforcement ratio in the fatigue behavior of reinforced concrete beams. In under-reinforced concrete beams, fatigue damage solely depends on the reinforcement. This can be observed in beams C7 and C8 (Figure 7), where no increase in the compressive strain evolution occurred with increase in fatigue cycles. This has also been reported in the literature [4, 7]. This behaviour is attributed to the increased rotation and deflection of under-reinforced concrete beams after concrete cracks. As such, less force is transmitted through the compressive force path to the support.

Crack growth on reinforcing bar

As indicated in Eq. 7, the area of a fractured surface can be expressed as a function of the crack depth. Hence, the progressive reduction in the area of the cross section can be estimated as the number of cycles increases. However, it is assumed that the stresses induced in the reinforcing bars at

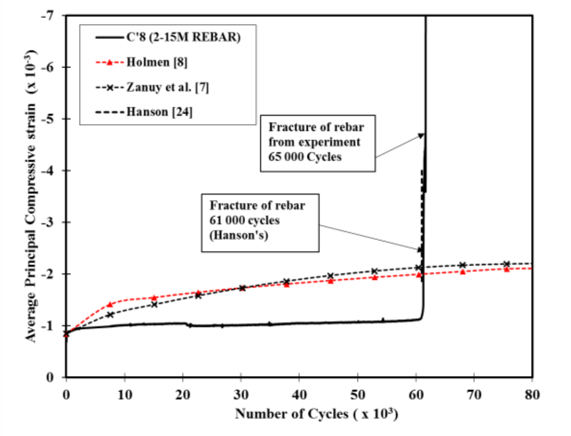

Figure 5: Average compressive strain evolution for C'8.

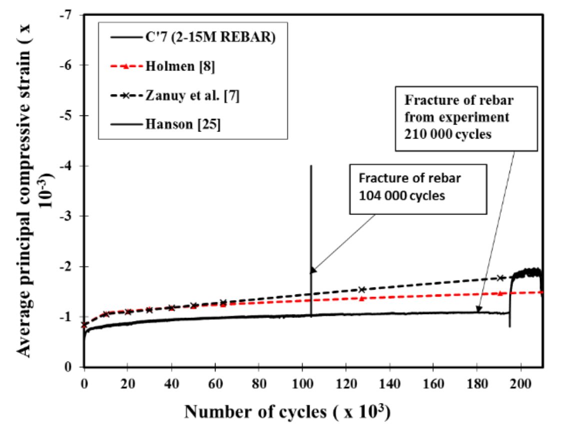

Figure 6: Average compressive strain evolution for C'7.

a beam cross section are equal.

With the assumption that the shape of the cracked surface perpendicular to the steel bar axis is of the form of a segment and its mirrored shape (Figure 8), then:

$$A(a_y) = \frac{\theta}{90}\pi r^2 - r sin\theta\left(2r - a_y\right) \quad (9)$$

$$\theta = cos^{-1}\left(\frac{r-0.5a_y}{r}\right) \quad (10)$$

A (a_y) is the area of the fractured surface, and r is the radius of the reinforcing bar. The value can be obtained iteratively since it also depends on the crack depth (a_y). The residual area A_i is the difference between the initial undamaged area A_o and A (a_y). Using the maximum and minimum stresses in the ties, the evolutions of the crack depths of the reinforcing bars up to failure were estimated

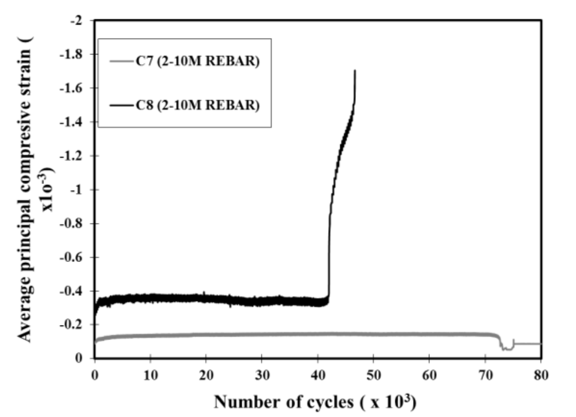

Figure 7: Average compressive strain evolution for C7 and C8.

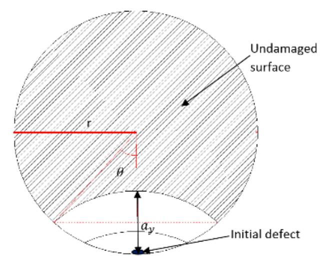

Figure 8: Crack growth on a reinforcing bar surface.

(Eq.1 to 8). Figure 9 shows the plot of the crack depth evolution against the fatigue loading cycles, and the predictions from Hanson's S-N model [25].

From the predictions, it can be observed that the numbers of cycles at which fracture occurred were conservative for the under-reinforced concrete beams (C7 and C8) and the beam with lower fatigue load when compared with the actual number of cycles to failure from the experiments. Hanson's model [2] gave highly conservative estimates in the beams except in C`8. The crack growth approach prediction was also not conservative for beam C'8. This is attributed to the neglect of the progressive increase in the stress induced in the reinforcement at its crack location due to the evolving irreversible compressive strain in concrete. However, the accumulated compressive strain is negligible when the fatigue loads are small or beams are under-reinforced. A realistic approach for the analysis of the fatigue behaviour of reinforced concrete deep beams, which considers the influence of concrete damage and accumulated irreversible compressive strain is given subsequently.

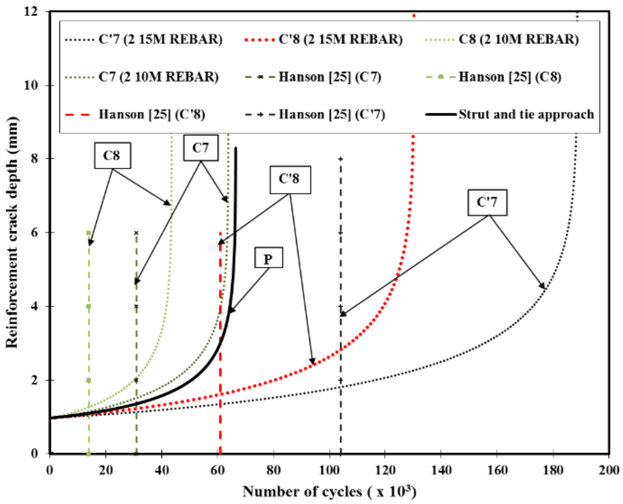

Figure 9: Average compressive strain evolution for C7 and C8.

Proposed analysis approach

In the design of deep beam structures prone to high-cycle fatigue damage, the load required for the fatigue resistance verification is usually small compared to the monotonic failure load; hence, the estimated stresses in the strut and tie are lower than the limiting or yield stresses.

The approach for the fatigue life prediction involves a continuous repetition of the resolution of the equilibrium of forces in the strut and tie, taking into consideration the progressive degradation of concrete parameters such as the strength and stiffness, the accumulation of irreversible compressive strain in the strut, and the crack growth on reinforcement at the intersection with a major concrete crack on the deep beam.

The effectiveness factor required for the limiting strut stress per fatigue cycle or interval and the constitutive model for concrete are modified with a concrete strength damage model which is a function of the strut stress. The irreversible compressive strain is considered as a pseudo-load in the resolution of the equilibrium of the strut and tie forces, while the progressive reduction in the area of reinforcement at the intersection with the major concrete crack is modified using a crack growth model as a function of the tie stress.

As the resolution of the equilibrium of forces is repeated per loading cycle or interval, the stress in the tie increases, the size or area of the concrete strut changes progressively, and the limiting stress of concrete also reduces. The number of repetition of the analysis at which either the strut stress is equal to the modified limiting stress or the evolving tie stress equals its corresponding yield stress gives the fatigue life of the deep beam.

The forces in the horizontal and vertical ties at each instance of fatigue loading can be expressed as (Figure 10):

$$F_o = A_i E_s \varepsilon_x \tag{11}$$

$$T_o = A_{vi} E_s \varepsilon_y \tag{12}$$

In Figure 10, D_i is the compressive force in the concrete strut due to the applied maximum fatigue load, ε_x and ε_y are the strains in the ties (longitudinal and vertical reinforcement respectively), while E_s is the elastic modulus of steel.

Table 3: Number of cycles to failure

Sp.	Hanson [25]	Crack growth	Experiment
Number of cycles at reinf. first fracture			
C7	31 000	64 000	72 000
C8	14 000	43 000	46 000
C'8	61 000	130 000	65 000
C'7	104 000	182 000	210 000

From the approach proposed by [24], the residual concrete strength due to fatigue damage and the modified limit stress are expressed as:

$$f'_{cf} = f'_c(1\text{-}D_s) \tag{13}$$

$$f'_{c(i)} = \frac{f'_{cf}}{0.8+170\varepsilon_1} \tag{14}$$

where $f'_{c(i)}$ is the limiting stress and f_c' is the initial strength. D_s is the concrete strength damage model and ε_p is the initial strain corresponding to the undamaged concrete strength (peak stress).

The effective compressive strain (ε_{c2}) in the concrete strut can be obtained from the stress-strain equation. From the Hognestad's equation for normal concrete, the strain corresponding to the peak stress and the lateral strain in the compressive strut are estimated using Equation 15 and 16 respectively.

$$\varepsilon_c^* = \varepsilon_p\ (1+\sqrt{D_s}) - \varepsilon_o \qquad (15)$$

where ε_o is the irreversible compressive strain.

$$\varepsilon_1 = \varepsilon_x + (\varepsilon_x + \varepsilon_{c2})\ cot^2\theta \qquad (16)$$

The analysis is done iteratively since the value of ε_1 required in modifying the compressive strength is initially unknown; hence, a compatibility equation which relate ε_1, ε_{c2}, ε_x and ε_y is required to obtain unknown parameters.

Damage models for concrete strength, concrete stiffness and irreversible strain in concrete are available in the literature, and, as such can be implemented in this approach [17, 26, 27].

The concrete damage and the irreversible strain at the first cycle are insignificant, while the initial crack on the reinforcement is estimated using Equation 4. For subsequent cycles, previous damage in concrete is converted to an equivalent number of cycles by substituting the current strut stress and the damage value into the fatigue damage model.

The equivalent number of cycles estimated is subsequently added to the next cycle or interval of cycles in order to estimate the current damage.

In the analysis, ε_{c2} and ε_1 are initially assumed. Subsequently, ε_x and ε_y and the corresponding forces (F_o and T_o) can be estimated using a strain compatibility equation, Eq. 11 and 12, since the fatigue load is known.

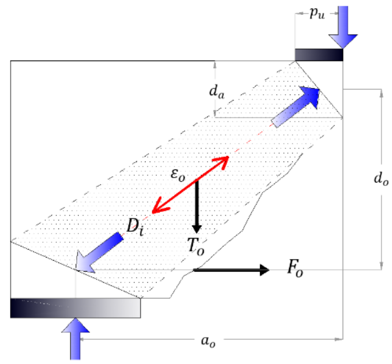

Figure 10: Strut and tie model for a deep beam under fatigue loading.

The limiting concrete stress, effective strain and lateral tensile strain (ε_1) are estimated using Equations 13 to 16. The iteration continues until the final values of the effective strain and ε_1 converge.

The values of the strut and tie stresses estimated by dividing the forces by the corresponding areas are substituted into concrete damage and crack growth models respectively. Hence, the irreversible compressive strain, concrete damage and reinforcement crack depth required for the next cycle or interval of cycles can be estimated. The analysis is repeated until either the strut stress is equal to the limiting stress or the tie stress is equal to the yield stress.

A damage model [17] was modified for concrete strength degradation, stiffness degradation and the irreversible strain accumulation.

As an assumption, the ratio of the minimum to maximum compressive stress due to fatigue loading is equivalent to the ratio of the applied minimum to maximum fatigue load; hence, the influence of the minimum fatigue load is accounted for in fatigue damage models in terms of the stress ratio. The value of A_i and A_{vi} (residual area of longitudinal and shear reinforcement) can be estimated as illustrated previously using fracture mechanics.

Using the delineated approach, a fatigue verification analysis was conducted for specimen C'8. The number of cycles to failure predicted (66 000 cycles) (indicated as curve P, in Figure 9) is reasonably close to the actual value (65 000 cycles) observed from the experiment. The result further reinforces the influence of irreversible strain accumulation for an appropriate prediction of fatigue life.

CONCLUSION

An experimental investigation was conducted in order to study the behavior of reinforced concrete deep beams under fatigue loading. The failure mode observed in all the beams tested was by fracture of the longitudinal reinforcement, and the damage accumulation based on strain readings was more significant in beams with the higher reinforcement ratio.

The general approach for fatigue damage verification in the literature is highly conservative, especially for concrete. However, an analysis which considers the interwoven influence of the irreversible strain accumulation in concrete and reinforcement crack growth is deemed imperative since the actual stress resulting in crack propagation is required for appropriate fatigue failure prediction using fracture mechanics.

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